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# Stochastic-based pavement rehabilitation model at the network level with prediction uncertainty considerations

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**ABSTRACT**

An optimum network-level pavement rehabilitation model has been developed for generating a long-term rehabilitation schedule comprised of a specified number of annual rehabilitation cycles. The optimum model deploys the discrete-time Markov model to predict the performances of both original and rehabilitated pavements wherein the pavement improvement rates are incorporated into the transition probability matrix. The model implements continuous cyclic improvements in the long-term performance curve compared to the traditionally assumed vertical improvements. A Markov chain with (m) condition states can incorporate (m-1) rehabilitation treatments with an expected improvement outcome being the upgrade to condition state (**1**), the state with best pavement condition. The optimum model deploys an effective decision-making policy that maximises the long-term performance while minimising rehabilitation cost. The optimum model can be solved using exhaustive search with functional evaluations. The sample results obtained for a pavement network comprised of (12) highways indicated the efficiency of proposed model in yielding practical long-term rehabilitation schedules. The sample results also provided the minimal annual budget required to progressively remove the 'very poor' pavements that greatly affect the life-cycle cost. Furthermore, investigation of prediction uncertainty resulted in a relatively mild impact when considering lower and upper-limit performance values using 95% confidence level.

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Pavement network; pavement performance; pavement rehabilitation; pavement management; Markov chains

## 1. Introduction

The highway system is the vital transportation system for the movement of goods and people in all nations around the World. It is crucial for the development and prosperity of any nation. Also, the highway system represents a huge economic investment that should be maintained to provide the public with safe and efficient driving conditions. The pavement structure is the highway element that carries the traffic loads, thus suffering from pavement deterioration over time. This calls for pavement maintenance and rehabilitation (M&R) remedies aimed to preserve/upgrade the pavement condition over time. Because nations have to deal with large pavement networks in the presence of limited resources, a mathematical tool called pavement management system (PMS) has emerged to provide the pavement engineers with optimal solutions concerning the best M&R plan that should be implemented. Classically, the M&R treatment types, the pavements receiving the M&R treatments, and the appropriate M&R timings are essential elements of any M&R plan (Abaza, 2007; Hafez et al., 2021; Khattak

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et al., 2008; Kim et al., 2018; Li et al., 2006; Sebaaly et al., 1996; Torres-Machí et al., 2015). Therefore, any reliable PMS is expected to provide answers to these important questions.

Typically, there are four major components associated with any sound PMS (Abaza & Murad, 2007; Jorge & Ferreira, 2012; Mandiartha et al., 2012; Santos et al., 2017). The first one is a performance prediction model that can detect the pavement future conditions. The second one is an M&R module that can provide information on the potential M&R treatments in terms of cost and effectiveness. The third one is an effective decision-making policy formulated to yield optimal M&R plans capable of maximising the pavement long-term performance and minimising life-cycle cost. The last component is an appropriate optimisation method that can generate reliable optimal solutions for the formulated decision-making strategy. Along these guidelines, several PMSs have been developed using advanced optimisation methods (Abaza & Murad, 2007; Jorge & Ferreira, 2012; Khavandi Khiavi & Mohammadi, 2018; Mathew & Isaac, 2014; Santos et al., 2017, 2019; Zhang & Gao, 2012). However, very few ones have gained international recognition because they are deemed as either too complex to use in practice or too demanding in terms of data requirements with some being labelled as 'data hungry'.

It is typical that highway agencies develop their own PMSs that can meet their personal requirements and specifications. Therefore, locally developed PMSs can significantly vary with respect to data requirements and methodological approach used. However, the majority of advanced PMSs are designed to provide optimal M&R solutions at the network-level for both highway and airfield pavements. Generally, the advanced PMSs vary in complexity depending on the type of performance model used to predict future pavement conditions, efficiency of the formulated decision-making policy, and optimisation method used to yield optimal M&R plans. Also, they vary in terms of data requirements, and details associated with the derived optimal M&R solutions. For instance, the solution details may include the specific M&R treatments to be applied to particular pavement sections for a given year over a specified analysis period. However, as the size of the pavement network increases, optimal convergence becomes a critical issue in terms of computation time and solution reliability. Some recent publications have provided review/overview of current practices and methodologies used in PMSs (Miah et al., 2020; Peraka & Biligiri, 2020).

The main data requirement for any PMS is related to the observed pavement performance over time. The traditional approach for obtaining pavement condition records requires conducting manual or automatic pavement distress assessment on annual or biennial basis (Vyas et al., 2021a). The manual assessment method is considered as time-consuming, costly, error-prone and hazardous compared to automatic distress assessment. Sholevar et al. (2022) presented a survey on automated and semi-automated pavement condition data collection methods, and pavement condition indices to be estimated from collected data. Typically, a unified pavement condition index is derived to represent the overall pavement condition to be used in pavement management applications (Abaza, 2017). A unified condition indicator is normally required when defining and analyzing pavement serviceability over time. The serviceability concept has been adopted by the pavement engineering community to evaluate the overall condition of a pavement structure, and it defines the ability of a specific pavement segment to support traffic loads in its current condition. Road serviceability also denotes the quality of road surface as perceived by users, and it has been quantified using different measurable distresses of road surface (Fuentes et al., 2021).

## 2. Research objectives

In this paper, it is proposed to develop a simplified network-level pavement management model that mainly focuses on pavement rehabilitation actions expected to produce major improvement outcomes. The main objective is to design a 'macroscopic' model wherein a limited number of rehabilitation variables are used to represent a limited number of potential rehabilitation strategies. In the macroscopic approach, the rehabilitation variables represent the pavement proportions to be treated in the relevant condition states using specific rehabilitation strategies for every calendar year within the analysis period. The actual pavement segments to be scheduled for rehabilitation are to be field selected in each condition state according to worst-first criterion. This is considered as a major advantage compared to the 'microscopic' approach used by the vast majority of PMSs wherein optimal M&R solutions are to be generated for individual pavement segments, thus making the pavement management problem highly complicated. The other main research objectives include:

- It is proposed to apply the discrete-time Markov model in predicting pavement performance wherein the rehabilitation improvement rates are incorporated into the transition probability matrix. However, the proposed approach allows to incorporate different deterioration rates for both original and rehabilitated pavements, which is considered an enhancement compared to other similar models. This represents a new contribution.
- It is proposed to apply continuous cyclic improvements in the long-term performance curve under the assumption that rehabilitation work at the network-level is to be spanned over the entire year as usually implemented in practice, which is an improvement compared to the traditionally assumed immediate vertical improvements typically applicable to individual pavement projects. This is also considered as a new contribution.
- It is proposed to apply an effective decision-making policy that attempts to minimise a cost-effectiveness index defined as the ratio of pavement rehabilitation cycle cost and performance rating change. This is considered as a simplified but yet effective approach compared to other complex ones. This is again a new contribution.
- It is proposed to apply a simple exhaustive search approach to yield reliable optimal solutions with minimal data requirements. Again, this is a major advantage when compared to other advanced PMSs using complex optimisation methods.
- It is proposed to generate optimal long-term rehabilitation schedule for a specified analysis period wherein the annual rehabilitation variables represent the pavement proportions in various condition states to be treated every year using relevant rehabilitation strategies.
- It is finally proposed to investigate the impact of prediction uncertainty on the optimal solutions. This is mainly related to uncertainties associated with the transition probabilities estimated for a given network comprised of a number of pavement projects.

All previously outlined features of the proposed rehabilitation model should make it attractive to the pavement engineering community. In summary, it is attractive because of its simplicity, effectiveness, and minimal data requirements.

## 3. Overview of performance prediction models

Probably, the most critical component of any sound PMS is the performance prediction model. This is because there are uncertainties involved in condition data collection methods, and uncertainties inherited in the prediction models themselves wherein some models are superior to others. Therefore, a model capable of accurately predicting the pavement future conditions is a key requirement for yielding reliable optimal M&R solutions. There are several types of performance prediction models that have been used in relation to pavement deterioration modelling. The most widely used prediction models can generally be classified as deterministic, probabilistic, and Artificial Neural Networks (ANN) (Abed et al., 2019; Alaswadko & Hwayyis, 2022; Vyas et al., 2021b; Xiao et al., 2022). The deterministic models are mainly regression-based, while the probabilistic ones include Bayesian and Markov models. Pavement deterioration is a complicated process and stochastic in nature due to variable climate, traffic and material characteristics (Pranav et al., 2020). Hence, the probabilistic-based models have been extensively used in modelling pavement performance.

The ANN models are also capable of predicting future pavement conditions with a high degree of accuracy, however the majority of used ANN models are deterministic in nature (Xiao et al., 2022). Justo-Silva et al. (2021) provided a review on machine learning techniques used to develop pavement performance prediction models, and presented their advantages and disadvantages. Hu et al.

(2022) reviewed the empirical methods of pavement performance modelling, and presented their main features, strengths, and weaknesses. Sholevar et al. (2022) provided a comprehensive review of machine learning methods used in analyzing pavement condition data. However, only recently some researchers have proposed to integrate machine learning methods into pavement management modelling (Barua & Zou, 2022).

The Markov models have been extensively used in pavement management applications (Abaza & Murad, 2007; Lethanh & Adey, 2012; Lethanh et al., 2015; Yamany & Abraham, 2021). Generally, the Markov models have gained wide publicity in infrastructure asset management (Abaza, 2022; Meidani & Ghanem, 2015; Wang et al., 2022; Yamany et al., 2021). Different types of Markov model have been cited in the literature including discrete-time Markov chain, semi-Markov chain, exponential hidden Markov chain, Poisson hidden Markov chain, random Markov chain, and recurrent Markov chain (Abaza & Murad, 2007; Lethanh & Adey, 2012; Lethanh et al., 2015; Meidani & Ghanem, 2015; Yang et al., 2006; Zhang & Gao, 2012). However, the most popular one is the discrete-time Markov model with discrete number of transitions and condition states. It is popular due to its simplicity and efficacy, minimal data requirements, and ease of integration into pavement management modelling. Markovian modelling has been also used in several pavement management applications such as overlay thickness design, life-cycle analysis, and pavement design (Abaza & Murad, 2009, 2017, 2021; Galvis Arce & Zhang, 2021; Pittenger et al., 2012).

Generally, the Markov model provides a convenient approach to predict the future based on the present only, so the past has no impact, an indication of the model's 'memoryless' property. The main elements of the Markov chain are state probabilities, duty cycle (i.e. transition period), and transition probabilities. The discrete-time Markov model requires using a discrete number of condition states typically defined using an appropriate pavement condition indicator (Abaza, 2021, 2022), and a discrete duty cycle typically taken as one-year in pavement management applications. The state probabilities define the pavement proportions that exist in the various condition states at a given discrete-time, while the transition probabilities denote the probabilities of pavement transiting from one state to another during one duty cycle. The discrete-time Markov model can use either homogeneous or non-homogeneous Markov chains.

The homogeneous Markov chain assumes the transition probabilities remain unchanged during each duty cycle, while the non-homogenous Markov chain applies different transition probabilities for each duty cycle. The semi-Markov chain requires estimating a new set of transition probabilities for each holding time, which is the time the pavement takes to leave its current state. The pavement lifetime is typically divided into uneven holding times based on the pavement performance curve (Yamany et al., 2021). Semi-Markov models assume the holding times could follow any continuous-time distribution, so they are more flexible than the traditional Markov models which assume holding times follow exponential distribution (Thomas & Sobanjo, 2013). Generally, semi-Markov models outperform the homogeneous Markov models, but they require more extensive data to estimate the distribution of the holding times. However, they could be computationally less expensive than the non-homogeneous Markov models (Yamany et al., 2021).

## 4. Research methodology

The main objective of developing the proposed network-level management model is to generate a long-term pavement rehabilitation schedule comprised of (n) annual rehabilitation cycles as shown in Figure 1. Each rehabilitation cycle is achieved by applying a number of potential rehabilitation treatments. The discrete-time Markov chain is used to model the associated long-term pavement deterioration and improvement mechanisms in the search for an optimal rehabilitation schedule. Three types of deterioration rates (i.e. transition probabilities) are considered in the modelling procedure, namely original, improvement and hypothetical transition probabilities. Practically, they should be estimated from observed pavement condition data collected on both original and rehabilitated pavements. The improvement transition probabilities are incorporated into the transition probability

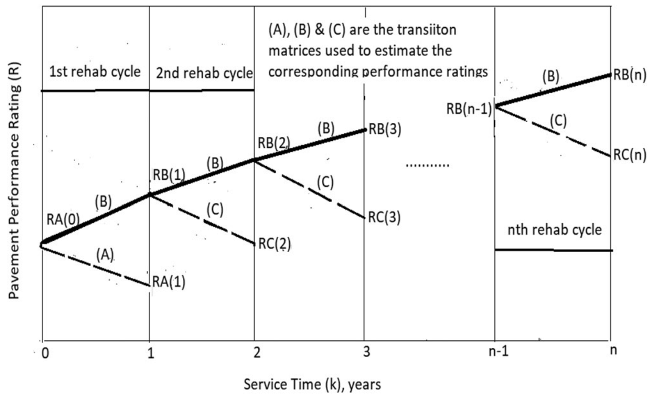


**Figure 1.** Typical long-term pavement rehabilitation schedule with (n) annual rehabilitation cycles.

matrix. A reliable and efficient decision-making policy is proposed which aims to minimising annual rehabilitation cycle cost while maximising pavement improvement.

### 4.1. Pavement performance prediction

Pavement performance has been typically evaluated using several performance indices such as present serviceability index (PSI), pavement condition index (PCI), and international roughness index (IRI). The PSI is highly related to pavement roughness and can be estimated from the IRI using regression-based models. However, the PCI is mainly dependent on the extent and severity of prevailing pavement distresses typically assessed using visual inspection with simple linear measurements. Pavement sections with small length are usually used in the assessment procedure. Assessment of a pavement project can be done in two ways. The first one requires assigning a performance rating for each pavement section, and then computing the average of all section ratings to represent the project performance rating. The second one requires the estimation of pavement proportions for a number of deployed condition states/classes. The condition states are typically defined using equidistance ranges of a particular performance index. The project performance rating is then computed from the multiplication product of pavement proportions, $S_i(k)$, and expected state performance ratings ($\bar{R}_i$) as defined in Equation (1) (Abaza, 2017, 2021, 2022). Equation (1) yields the expected project performance rating, $R(k)$, for the kth year considering ($m$) condition states. The expected state performance ratings are simply the mid-values of the equidistance ranges.

$$R(k) = \sum_i \bar{R}_i \times S_i(k) \quad (i = 1, 2, \ldots, m) \tag{1}$$

where: $\sum_i S_i(k) = 1.0$.

For example, using the PCI with 100-point scale, 5 condition states (m), and equidistance ranges of (100-80, 80-60, 60-40, 40-20, 20-0) results in expected state performance ratings ($\bar{R}_i$) of (90, 70, 50, 30, 10). The future pavement performance can stochastically be predicted using the discrete-time Markov model wherein Equation (1) is used to estimate the kth year performance rating, R(k), as a function of the pavement proportions, $S_i(k)$, called state probabilities as outlined next.

### 4.2. Original pavement performance

The state probabilities associated with original pavement can be estimated using Equation (2), which is an application of the discrete-time Markov model. The corresponding transition probability matrix (**A**) only incorporates two state movements that can take place during a duty cycle, namely either remaining in the current condition state (i) with probability ($A_{i,i}$) or transiting to the next worst state ($i+1$) with probability ($A_{i,i+1}$). This transition matrix form has been used by several researchers and found to be reliable in predicting pavement performance provided it deploys a reasonably small number of condition states (Abaza, 2022; Abed et al., 2019; Galvis Arce & Zhang, 2021; Wang et al., 2022; Yamany et al., 2021). Equation (2) only predicts the original state probabilities in the absence of any rehabilitation work as all entries below the main diagonal are assigned zero values. The initial/present state probabilities, $SA_i(0)$, are required to estimate the corresponding values, $SA_i(1)$, after one transition. One transition (i.e. duty cycle) is typically taken equal to one year in pavement management applications. Both $SA(k)$ and $SA(k\text{-}1)$ are row vectors.

$$SA(k) = SA(k-1)A \quad (k = 1, 2, \ldots..) \tag{2}$$

where: $SA(k) = [SA_1(k), SA_2(k), \ldots., SA_m(k)]$

$$A = \begin{bmatrix} A_{1,1} & A_{1,2} & 0 & 0 & 0 & 0 & \ldots & 0 \\ 0 & A_{2,2} & A_{2,3} & 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & A_{3,3} & A_{3,4} & 0 & 0 & \ldots & 0 \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ 0 & 0 & \ldots & 0 & A_{m-1,m-1} & A_{m-1,m} & & \\ & 0 & 0 & 0 & \ldots & \ldots & 0 & 1.0 \end{bmatrix}$$

The model presented in Equation (2) deploys (m) condition states, therefore the state probabilities represent the pavement proportions that exist in the various deployed states at the kth transition. Whereas the transition probabilities indicate the pavement deterioration rates prevailing during every transition assuming homogeneous Markov chain. It is recommended that the deterioration transition probabilities ($A_{i,i+1}$) be estimated from observed condition data collected on original pavement. Abaza (2021, 2022) proposed simplified but yet effective approaches to estimate the corresponding transition probabilities. The initial and first-year performance ratings, RA(0) & RA(1), shown in Figure 1 are estimated from Equation (1) using the corresponding original state probabilities, SA(0) & SA(1).

### 4.3. Rehabilitated pavement performance

The expected improvement of rehabilitated pavement in terms of the rehabilitation state probabilities, $SB_i(k)$, can be estimated using Equation (3). The corresponding transition probability matrix (**B**) incorporates both deterioration transition probabilities ($B_{i,i}$ & $B_{i,i+1}$) in the presence of rehabilitation, and improvement transition probabilities ($Q_{i,1}$). The solid lines depicted in Figure 1 indicate continuous pavement improvement through the application of (n) annual rehabilitation cycles. It is continuous because the rehabilitation work is assumed to be spanned over the entire year in consistency with the definition of transition probabilities. Equation (3a) indicates that the first-year rehabilitation state probabilities, $SB_i(1)$, are dependent on the initial original state probabilities, $SA_i(0)$. The $SB_i(1)$ are used to estimate the corresponding rehabilitation performance rating, RB(1), shown in Figure 1. However, Equation (3b) is used for the subsequent years wherein the rehabilitation performance ratings [RB(k), $k \geq 2$] are estimated from Equation (1) using the corresponding rehabilitation state probabilities.

$$SB(1) = SA(0)B \quad (k = 1) \tag{3a}$$

$$SB(k) = SB(k-1)B \quad (k = 2, 3, \ldots., n) \tag{3b}$$

where: $SB(k) = [SB_1(k), SB_2(k), \ldots., SB_m(k)]$.

$$B = \begin{bmatrix} B_{1,1} & B_{1,2} & 0 & 0 & 0 & 0 & \ldots & 0 \\ Q_{2,1} & B_{2,2} & B_{2,3} & 0 & 0 & 0 & \ldots & 0 \\ Q_{3,1} & 0 & B_{3,3} & B_{3,4} & 0 & 0 & \ldots & 0 \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ Q_{m-1,1} & 0 & \ldots & 0 & B_{m-1,m-1} & B_{m-1,m} & & \\ Q_{m,1} & 0 & 0 & \ldots & \ldots & 0 & 1.0 & \end{bmatrix}$$

The pavement improvement rates ($Q_{i,j}; i = 2, 3, \ldots ., \text{m}, \text{j} = 1$) incorporated into the rehabilitation transition probability matrix (**B**) assumes the improvement outcomes will be the upgrading of pavements from condition states (2, 3, … ., m) to state (1). Each improvement rate denotes a specific rehabilitation treatment that can achieve the intended pavement upgrade. The deterioration transition probabilities associated with the first row ($B_{1,1}$ & $B_{1,2}$) are the same as the corresponding values ($A_{1,1}$ & $A_{1,2}$) provided in matrix (**A**). However, the deterioration transition probabilities provided in the other matrix rows need to be adjusted so that the sum of any row remains equal to one as shown in Equation (4).

$$Q_{i,1} + B_{i,i} + B_{i,i+1} = 1.0 \tag{4}$$

It also requires to assume that the ratio of original transition probabilities remains the same as the ratio of the corresponding transition probabilities in the presence of rehabilitation as stated in Equation (5).

$$\frac{A_{i,i+1}}{A_{i,i}} = \frac{B_{i,i+1}}{B_{i,i}} \tag{5}$$

Enforcement of the above two conditions results in Equation (6) to be used in computing the deterioration transition probabilities ($B_{i,i}$ and $B_{i,i+1}$) in the presence of pavement improvement. However, Equation (6) is only applicable to the first-year rehabilitation cycle ($k = 1$). This is because pavement deterioration for the subsequent cycles may be associated with deterioration transition probabilities that are different from the ones provided in matrix (**A**) as outlined next.

$$\begin{aligned} B_{i,i+1} &= A_{i,i+1}(1 - Q_{i,1}) \\ B_{i,i} &= A_{i,i}(1 - Q_{i,1}) \\ B_{m,m} &= 1 - Q_{m,1} \end{aligned} \tag{6}$$

### 4.4. Hypothetical pavement performance

The deterioration of rehabilitated pavement had the pavement not been rehabilitated is shown in Figure 1 as broken lines starting at the second transition ($k \geq 2$). The relevant hypothetical deterioration rates ($C_{i,i}$ and $C_{i,i+1}$) may not be the same as for the original pavement, therefore Equation (7) is similar to Equation (2) but it incorporates a different transition probability matrix (**C**). The hypothetical state probabilities, $SC_i(k)$, are also dependent on the rehabilitation state probabilities, $SB_i(k-1)$, associated with the preceding transition as can be verified from Figure 1. The hypothetical state probabilities are used in Equation (1) to determine the corresponding hypothetical performance ratings, RC(k), as depicted in Figure 1.

$$SC(k) = SB(k-1)C \quad (k = 2, 3, \ldots, n) \tag{7}$$

where: $SC(k) = [SC_1(k), SC_2(k), \ldots, SC_m(k)]$.

$$C = \begin{bmatrix} C_{1,1} & C_{1,2} & 0 & 0 & 0 & 0 & \ldots & 0 \\ 0 & C_{2,2} & C_{2,3} & 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & C_{3,3} & C_{3,4} & 0 & 0 & \ldots & 0 \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ 0 & 0 & \ldots & 0 & C_{m-1,m-1} & C_{m-1,m} & & \\ 0 & 0 & 0 & \ldots & \ldots & 0 & 1.0 & \end{bmatrix}$$

Equation (7) is used to calculate the hypothetical state probabilities, $SC_i(k)$, had the pavement not been rehabilitated, which are then used to estimate the relevant hypothetical performance ratings [RC(k), $k \geq 2$] as shown in Figure 1. The improvement transition probabilities, ($Q_{i,1}$), should therefore be incorporated in matrix (**C**) for rehabilitation cycles ($k \geq 2$). This simply results in modifying Equation (6) so that the deterioration transition probabilities ($A_{i,i}$ & $A_{i,i+1}$) are replaced by the corresponding values ($C_{i,i}$ & $C_{i,i+1}$) to still yield the required improvement transition probabilities ($B_{i,i}$ and $B_{i,i+1}$). Similarly, the hypothetical deterioration probabilities ($C_{i,i+1}$) should be estimated from condition data collected on rehabilitated pavement.

### *4.5. Pavement rehabilitation modelling*

The rehabilitation work is considered to be uniformly spanned over the entire (k) transition when dealing with a large pavement network, which is consistent with the definition of improvement transition probabilities to be taken place during the full transition. Therefore, Figure 1 shows improvements achieved by rehabilitation cycles are represented by continuously increasing straight lines, and not instantaneous vertical rises as traditionally assumed. The assumption of immediate vertical rises in the long-term performance curve is mainly applicable when rehabilitation work can be carried out in a short-time period. However, when considering a large pavement network, the rehabilitation work cannot practically be achieved in a short-time period, and it is logical to assume its extension over the entire year.

The amount of rehabilitation work to be carried out during the kth year depends on the rehabilitation state probabilities, $SB_i$(k-1), associated with the preceding transition, and deterioration state probabilities, $SC_i(k)$, had the pavement not been rehabilitated. It cannot only depend on $SB_i$(k-1) because additional pavement deterioration would have taken place during the kth year. Therefore, it is assumed that the state probabilities, $\overline{S}_i(k)$, available for rehabilitation during the kth year are the averages of rehabilitation state probabilities, $SB_i$(k-1), and deterioration state probabilities, $SC_i(k)$, had the pavement not been rehabilitated as provided in Equation (8). The only exception is for the 1st rehabilitation cycle ($k = 1$) wherein the corresponding average state probabilities, $\overline{S}_i(1)$, are computed from the original state probabilities, $SA_i(0)$ & $SA_i(1)$, as indicated by Equation (8a) and Figure 1.

$$\overline{S}_i(1) = [SA_i(0) + SA_i(1)]/2 \quad (k = 1) \tag{8a}$$

$$\overline{S}_i(k) = [SB_i(k-1) + SC_i(k)]/2 \quad (k = 2, \ldots, n) \tag{8b}$$

The amounts of rehabilitation work as state probabilities (i.e. proportions) cannot exceed the averages computed using Equation (8). Therefore, the amounts of rehabilitation work in terms of the rehabilitation variables, $X_{i,1}(k)$, are estimated from the multiplication of improvement transition probabilities, $Q_{i,1}(k)$, and average state probabilities, $\bar{S}_i(k)$, as defined in Equation (9). Each rehabilitation variable represents a specific rehabilitation treatment so that a particular rehabilitation cycle can include (m-1) potential rehabilitation treatments.

$$X_{i,1}(k) = Q_{i,1}(k)\,\bar{S}_i(k) \quad (i = 2, 3, \ldots, m) \tag{9}$$

Consequently, the rehabilitation variables**,** $X_{i,1}(k)$, represent the pavement proportions to be deducted from the average state probabilities. The rehabilitation variables are to be used in the proposed optimum network-level rehabilitation model outlined next.

### *4.6. Optimum network-level rehabilitation model*

The successful implementation of any pavement management model requires deploying an effective decision-making policy. Therefore, the optimum network-level rehabilitation model proposes to minimise a cost-effectiveness index, $I_{CE}(k)$, defined as the ratio of total rehabilitation cycle cost, TC(k), and performance rating change, $\Delta$R(k), as presented in Equation (10) considering the kth rehabilitation cycle. The total rehabilitation cycle cost is computed as a multiplication of the total pavement surface area ($A_p$), unit costs of applicable rehabilitation treatments, $UC_i(k)$, and rehabilitation variables, $X_{i,1}(k)$. The performance rating change, $\Delta$R(k), is defined as the difference between the rehabilitated performance rating, RB(k), and hypothetical performance rating had the pavement not been rehabilitated. The hypothetical performance rating is equal to the original performance rating, RA(1), for the 1st rehabilitation cycle, and equal to RC(k) for the subsequent cycles. Therefore, the model proposed in Equation (10) attempts to yield the optimal annual rehabilitation plan that minimises the total rehabilitation cycle cost while maximising expected performance improvement.

$$\text{Minimize}: \ I_{CE}(k) = TC(k)/\Delta R(k) \ (k = 1, 2, \ldots.., n) \tag{10}$$

where: $TC(k) = A_P \times \sum_{i=2}^{m} UC_i(k) \times X_{i,1}(k)$, $X_{i,1}(k) = Q_{i,1}(k) \times \bar{S}_i(k)$, $\Delta R(k) = \mathrm{RB}(k) - \mathrm{RA}(k)$ $(k = 1)$, $\Delta \mathrm{R}(k) =$
$\mathrm{RB}(k) - \mathrm{RC}(k)$ $(k \geq 2)$

Subject to:

(1) $0.0 \leq Q_{i,1} \leq 1.0$
(2) $0.0 \leq X_{i,1}(k) \leq \bar{S}_i(k)$
(3) $\Delta R(k) > 0$
(4) $TC(k) \leq AB(k)$

The optimum model outlined in Equation (10) is subject to four sets of constraints. The first set requires the improvement transition probabilities ($Q_{i,1}$) to be between zero and one. The second set maintains the non-negativity values of rehabilitation variables ($X_{i,1}$) and assures they don't exceed the average state probabilities, $\bar{S}_i(k)$. The third one simply enforces the performance rating change, $\Delta$R(k), to be greater than zero. Lastly, the fourth set imposes that the total rehabilitation cycle cost be less than or equal to the allocated annual budget, AB(k). The optimisation method that is best suitable to solve the optimum model previously described is the one that deploys functional evaluations using an appropriate exhaustive search algorithm as deployed next in the sample presentation section. This is especially true when dealing with a limited number of rehabilitation variables.

## 5. Sample presentation

A case study is presented in this section to demonstrate the potential use of the proposed optimum network-level rehabilitation model. The case study considers a sample of 12 minor highways with

**Table 1.** Sample highway initial and terminal transition probabilities.

| Highway number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $P_{1,2}$ | 0.18 | 0.25 | 0.26 | 0.29 | 0.32 | 0.35 | 0.38 | 0.39 | 0.47 | 0.52 | 0.58 | 0.64 |
| $P_{4,5}$ | 0.38 | 0.36 | 0.69 | 0.51 | 0.74 | 0.31 | 0.49 | 0.75 | 0.85 | 0.49 | 0.52 | 0.50 |

low traffic volume wherein the minor highways are used to connect villages in the northern district of Nablus, West Bank, Palestine, with nearby major highways. The minor highways were constructed using a flexible pavement structure with two layers, namely asphalt concrete surface and aggregate base. The initial and terminal transition probabilities ($P_{1,2}$ & $P_{4,5}$) associated with the highway sample were previously estimated and used in a former publication as provided in Table 1 (Abaza, 2021). There are different methods proposed to estimate the transition probabilities mainly relying on historical records of pavement distress (Abaza, 2022; Costello et al., 2016; Yamany & Abraham, 2021). In particular, a simplified and efficient method has been proposed to estimate the transition probabilities for individual projects based on two consecutive annual performance ratings (Abaza, 2022). The initial/present state probabilities, $SA_i(0)$, associated with the sample pavement network are estimated to be equal to (0.192, 0.257, 0.354, 0.126, 0.071) considering a Markov chain with ($m = 5$) condition states designated as very good, good, fair, poor and very poor, respectively.

### 5.1. Sample transition probability matrices

The application of the proposed optimum network-level rehabilitation model requires the estimation of three transition probability matrices, namely the original transition matrix (**A**), rehabilitation matrix (**B**), and hypothetical matrix (**C**). These matrices need to represent the deterioration of the entire pavement network, which can be accomplished by using the averages of the initial and terminal transition probabilities provided in Table 1. The resulting averages are (0.386 & 0.549), thus representing the network initial and terminal transition probabilities ($A_{1,2}$ & $A_{4,5}$), respectively, which are two main entries in matrix (**A**). The remaining entries in matrix (**A**) can be estimated using linear interpolation as indicated by Equation (11). Linear interpolation was used in former publications and found to be reliable in pavement performance prediction (Abaza, 2017, 2022).

$$A_{i,i+1} = A_{1,2} + \frac{A_{m-1,m} - A_{1,2}}{m-2} \quad (i = 2, 3, \ldots, m-2) \tag{11}$$

Therefore, the remaining deterioration probabilities ($A_{2,3}$ & $A_{3,4}$) of matrix (**A**) are estimated from Equation (11) using ($m = 5$) condition states. The resulting transition probability matrix (**A**) is provided in Equation (12) wherein the sum of any row adds up to one. This sample matrix (**A**) follows the general form presented in Equation (2).

$$A = \begin{bmatrix} 0.614 & 0.386 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.560 & 0.440 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.505 & 0.495 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.451 & 0.549 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.000 \end{bmatrix} \tag{12}$$

Estimation of the hypothetical deterioration transition matrix (**C**) can be done using the same procedure outlined in constructing matrix (**A**) wherein the corresponding initial and terminal transition probabilities are to be estimated from historical distress records. However, a much simpler approach is proposed for the purpose of sample presentation. It is proposed to estimate matrix (**C**) from matrix (**A**) by simply multiplying the transition probabilities ($A_{i,i+1}$) by an adjustment factor (F) to yield the transition probabilities ($C_{i,i+1}$) as stated in Equation (13). The adjustment factor is expected to be lower than one if the rehabilitated pavement is stronger than the original pavement implying the corresponding deterioration rates ($C_{i,i+1}$) are lower than original values ($A_{i,i+1}$), thus resulting in superior performance. It is also expected to be greater than one if the rehabilitated pavement is weaker indicating the corresponding deterioration rates are larger than original values, thus resulting in inferior performance. Therefore, experience and engineering judgement can be deployed to estimate the appropriate adjustment factor. It can also be estimated from the structural capacities associated with both original and rehabilitated pavements (Abaza, 2017). Alternatively, it can be computed as the ratio of average annual condition ratings to be estimated over a specified service period considering both pavements.

$$C_{i,i+1} = F \times A_{i,i+1} \tag{13}$$

For the purpose of sample presentation, the adjustment factor is assumed equal to (1.1), an indication of weaker rehabilitated pavement. The resulting transition matrix (**C**) is provided in Equation (14). Again, sample matrix (**C**) deploys only two state movements similar to matrix (**A**).

$$C = \begin{bmatrix} 0.575 & 0.425 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.516 & 0.484 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.456 & 0.544 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.396 & 0.604 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.000 \end{bmatrix} \tag{14}$$

The rehabilitation matrix (**B**) can now be computed for the 1st rehabilitation cycle ($k = 1$) from Equation (6) using sample matrix (**A**) and improvement transition probabilities ($Q_{i,1}$). A trial sample matrix (**B**) is presented in Equation (15) which only represents the 1st rehabilitation cycle. The trial improvement transition probabilities are assumed equal to ($Q_{3,1} = 0.7$, $Q_{4,1} = 0.5$, $Q_{5,1} = 0.2$) implying that rehabilitation work is only applied to pavements in condition states (**3**, **4**, **5**), respectively. Please note the first two rows in matrix (**B**) are the same as the corresponding ones in matrix (**A**). Matrix (**B**) for the subsequent rehabilitation cycles ($k \geq 2$) is to be determined in the same way but using matrix (**C**) instead of matrix (**A**).

$$B = \begin{bmatrix} 0.614 & 0.386 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.560 & 0.440 & 0.000 & 0.000 \\ 0.700 & 0.000 & 0.152 & 0.148 & 0.000 \\ 0.500 & 0.000 & 0.000 & 0.226 & 0.274 \\ 0.200 & 0.000 & 0.000 & 0.000 & 0.800 \end{bmatrix} \tag{15}$$

The improvement transition probabilities ($Q_{i,1}$) are to be varied over their theoretical range [0,1] in the search for the optimal rehabilitation plan (i.e. cycle) consisting of (m-1) rehabilitation treatments as applied to condition states (**2**, **3**, … ., m) as outlined earlier. However, only three potential rehabilitation treatments are considered in this sample presentation. The first rehabilitation treatment is applied to pavements in condition state (**3**), which involves removal of (3 cm) asphalt surface by cold milling to be replaced by (3 cm) new asphalt mix at an estimated unit cost ($UC_i$) of (\$13/m$^2$). The second rehabilitation treatment is applied to pavements in condition state (**4**), which requires removal and replacement of (5 cm) asphalt at a unit cost of (\$20/m$^2$). The third treatment is applied to state (**5**) requiring as a minimum the complete removal of existing asphaltic surface and replacement of new asphalt surface at an estimated unit cost of (\$30/m$^2$). The use of only three sample potential rehabilitation treatments is consistent with the common practice worldwide, and appropriate for a Markov chain with 5 condition states. The total surface area ($A_P$) associated with the sample pavement network is estimated to be ($2.52 \times 10^5$ m$^2$).

### *5.2. Sample optimal rehabilitation schedules*

The optimum network-level rehabilitation model outlined in Equation (10) has been used to obtain three sample optimal rehabilitation schedules with different annual budget. Each sample optimal

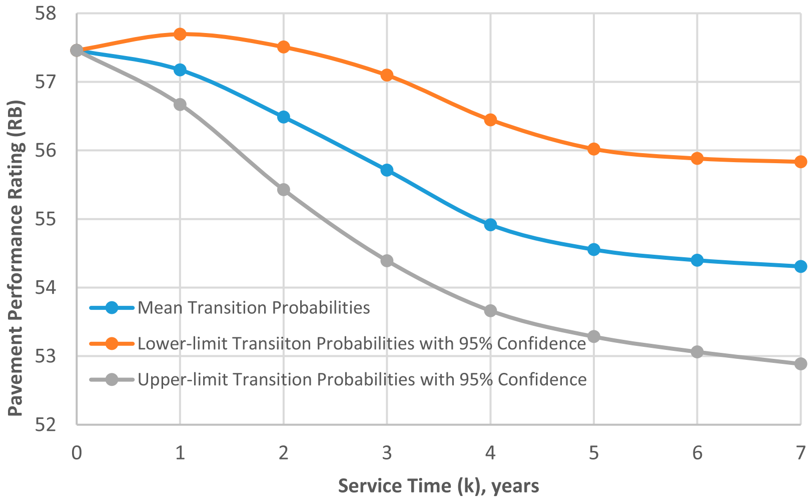


**Figure 2.** Sample optimal rehabilitation schedule using \$0.50 million annual budget.

schedule is derived for 7 years analysis period, thus resulting in 7 annual rehabilitation cycles. In the search for the optimal solution, an exhaustive search approach with functional evaluation has been applied wherein the search is simultaneously made with respect to the three deployed improvement transition probabilities ($Q_{i,1}$) until the minimum cost-effectiveness index value, $I_{CE}(k)$, is reached. This requires the calculation of the total rehabilitation cycle cost, TC(k), as outlined in Equation (10). It also requires to compute the performance improvement change, $\Delta R(k)$, with the relevant performance ratings RA(k), RB(k) and RC(k) are determined based on matrices (**A**), (**B**) and (**C**), respectively. The corresponding state probabilities are first computed using relevant discrete-time Markov models as outlined in the methodology section, then Equation (1) is used to calculate the three involved performance ratings.

Table 2 provides the optimal solutions for the sample rehabilitation schedule associated with \$0.5 million annual budget. The provided optimal solutions include the rehabilitation variables, average state probabilities, and corresponding performance improvement change. The average state probabilities are computed as outlined in Equation (8) and used to define the rehabilitation variables, $X_{i,1}(k)$, as per Equation (9). Therefore, the optimal improvement transition probabilities, $Q_{i,1}(k)$, can simply be computed from dividing the rehabilitation variables by the corresponding average state probabilities. This means the rehabilitation variables represent pavement proportions to be taken from the relevant average state probabilities. Table 2 indicates that the 1st rehabilitation treatment ($X_{3,1}$) has dominated the optimal solutions with the 2nd rehabilitation treatment ($X_{4,1}$) slightly used in rehabilitation cycles 3 and 4. However, the 3rd rehabilitation treatment ($X_{5,1}$) has not been used at all because of its least cost-effectiveness. Figure 2 displays the long-term performance curve associated with the sample rehabilitation schedule presented in Table 2. The middle curve is the one corresponding to the optimal solutions provided in Table 2 because average network transition probabilities have been used in constructing the deployed transition probability matrices. The relevant curve starts with a relatively sharp decline in the pavement performance rating, RB(k), and begins levelling at around the 5th rehabilitation cycle.

Table 3 provides the sample optimal rehabilitation schedule associated with \$0.75 million annual budget. The optimal solutions have made more use of the 1st and 2nd rehabilitation treatments, and some limited use of the 3rd treatment in only two rehabilitation cycles. In these two cycles (i.e. cycles 3 & 4), it can be noticed the full use of 1st and 2nd rehabilitation treatments wherein the values of rehabilitation variables are equal to the corresponding values of the average state probabilities. Overall, higher performance improvements have been achieved with \$0.75 million annual budget compared

**Table 2.** Sample optimal rehabilitation schedule with \$0.50 million annual budget.

| | Optimal rehab. variables | | | Optimal average state probabilities, $\bar{S}(k)$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Year ($k$) | $X_{3,1}(k)$ | $X_{4,1}(k)$ | $X_{5,1}(k)$ | $\bar{S}_1(k)$ | $\bar{S}_2(k)$ | $\bar{S}_3(k)$ | $\bar{S}_4(k)$ | $\bar{S}_5(k)$ | Optimal $\Delta R(k)$ |
| 1 | 0.1526 | 0.0000 | 0.0000 | 0.1550 | 0.2374 | 0.3230 | 0.1790 | 0.1056 | 8.35 |
| 2 | 0.1526 | 0.0000 | 0.0000 | 0.2246 | 0.2257 | 0.2039 | 0.1606 | 0.1852 | 7.91 |
| 3 | 0.1507 | 0.0013 | 0.0000 | 0.2516 | 0.2447 | 0.1507 | 0.09623 | 0.2567 | 6.66 |
| 4 | 0.1443 | 0.0054 | 0.0000 | 0.2476 | 0.2608 | 0.1443 | 0.0546 | 0.2927 | 6.00 |
| 5 | 0.1526 | 0.0000 | 0.0000 | 0.2342 | 0.2643 | 0.1545 | 0.0423 | 0.3047 | 6.23 |
| 6 | 0.1526 | 0.0000 | 0.0000 | 0.2312 | 0.2617 | 0.1578 | 0.0391 | 0.3102 | 6.36 |
| 7 | 0.1526 | 0.0000 | 0.0000 | 0.2315 | 0.2597 | 0.1572 | 0.0384 | 0.3132 | 6.39 |

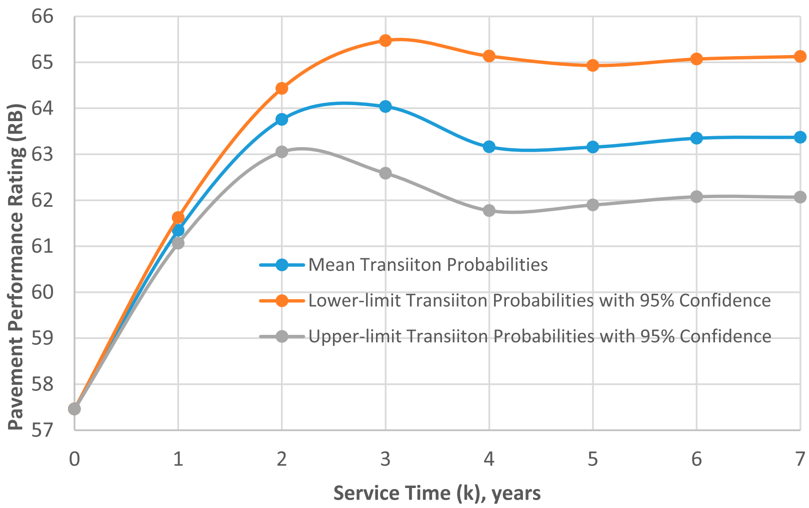


**Figure 3.** Sample optimal rehabilitation schedule using \$0.75 million annual budget.

**Table 3.** Sample optimal rehabilitation schedule with \$0.75 million annual budget.

| | Optimal rehab. variables | | | Optimal average state probabilities, $\bar{S}(k)$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Year ($k$) | $X_{3,1}(k)$ | $X_{4,1}(k)$ | $X_{5,1}(k)$ | $\bar{S}_1(k)$ | $\bar{S}_2(k)$ | $\bar{S}_3(k)$ | $\bar{S}_4(k)$ | $\bar{S}_5(k)$ | Optimal $\Delta R(k)$ |
| 1 | 0.2289 | 0.0000 | 0.0000 | 0.1550 | 0.2374 | 0.3230 | 0.1790 | 0.1056 | 12.52 |
| 2 | 0.1731 | 0.0363 | 0.0000 | 0.2905 | 0.2435 | 0.1731 | 0.1202 | 0.1727 | 10.76 |
| 3 | 0.1420 | 0.0495 | 0.0047 | 0.3230 | 0.2909 | 0.1420 | 0.0495 | 0.1946 | 7.88 |
| 4 | 0.1706 | 0.0354 | 0.0017 | 0.2960 | 0.3168 | 0.1706 | 0.0354 | 0.1812 | 6.76 |
| 5 | 0.1880 | 0.0266 | 0.0000 | 0.2743 | 0.3170 | 0.1880 | 0.0412 | 0.1795 | 7.71 |
| 6 | 0.1890 | 0.0260 | 0.0000 | 0.2771 | 0.3121 | 0.1890 | 0.0423 | 0.1795 | 7.91 |
| 7 | 0.1857 | 0.0281 | 0.0000 | 0.2819 | 0.3116 | 0.1857 | 0.0413 | 0.1795 | 7.72 |

to \$0.50 million budget as evident from the optimal performance changes, ΔR(k), and middle curve depicted in Figure 3. Similarly, Table 4 provides the optimal results for \$1.0 million annual budget where it can be noticed the full use of 1st and 2nd rehabilitation treatments in cycles (2–7), and more use of the 3$^{rd}$ rehabilitation treatment. It can also be noticed that the average state probability, $\bar{S}_5(k)$, associated with condition state (**5**) has consistently decreased over time. The same average state probability has steadily increased in Table 2, and remained almost stable in Table 3. Therefore, the optimal rehabilitation schedule associated with \$1.0 million annual budget is superior to the other two schedules not just because it yields better overall pavement improvements as per Figure 4, but it also gradually eliminates the pavements in 'very poor' condition, thus substantially reducing routine maintenance cost and added user cost.

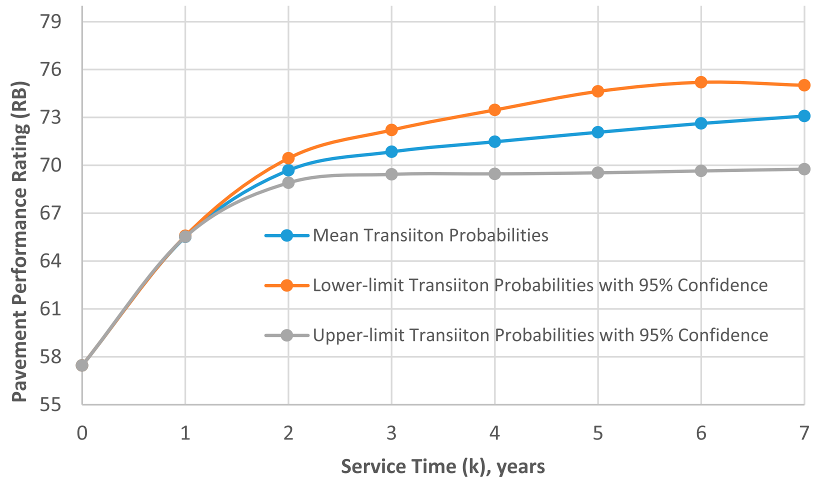


**Figure 4.** Sample optimal rehabilitation schedule using $1.0 million annual budget.

**Table 4.** Sample optimal rehabilitation schedule with $1.0 million annual budget.

| | Optimal rehab. variables | | | Optimal average state probabilities, $\bar{S}(k)$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Year ($k$) | $X_{3,1}(k)$ | $X_{4,1}(k)$ | $X_{5,1}(k)$ | $\bar{S}_1(k)$ | $\bar{S}_2(k)$ | $\bar{S}_3(k)$ | $\bar{S}_4(k)$ | $\bar{S}_5(k)$ | Optimal $\Delta R(k)$ |
| 1 | 0.3053 | 0.0000 | 0.0000 | 0.1550 | 0.2374 | 0.3230 | 0.1790 | 0.1056 | 16.69 |
| 2 | 0.1423 | 0.0799 | 0.0174 | 0.3564 | 0.2612 | 0.1423 | 0.0799 | 0.1602 | 12.26 |
| 3 | 0.1506 | 0.0287 | 0.0479 | 0.3663 | 0.3294 | 0.1506 | 0.0287 | 0.1250 | 9.20 |
| 4 | 0.1932 | 0.0401 | 0.0218 | 0.3316 | 0.3580 | 0.1932 | 0.0401 | 0.0771 | 9.25 |
| 5 | 0.2125 | 0.0467 | 0.0090 | 0.3242 | 0.3613 | 0.2125 | 0.0467 | 0.0553 | 9.46 |
| 6 | 0.2149 | 0.0476 | 0.0074 | 0.3289 | 0.3623 | 0.2149 | 0.0476 | 0.0463 | 9.50 |
| 7 | 0.2154 | 0.0476 | 0.0072 | 0.3330 | 0.3652 | 0.2154 | 0.0476 | 0.0388 | 9.48 |

Interpretation of optimal solutions is outlined in relation to the sample results provided in Table 4. The optimal solution associated with the first rehabilitation cycle ($k = 1$) consists of only $X_{3,1} = 0.3053$, which represents the proportion of pavements in condition state (**3**) to be rehabilitated during the first-year out of the available $\bar{S}_3(k) = 0.3230$ proportion at the beginning of the 1st year. This is to be accomplished using the first rehabilitation treatment (i.e. removal and replacement of 3 cm asphalt surface). It is equivalent to $(0.3053/0.3230) \times 100\% = 94.52\%$ of pavement available in state (**3**). The optimal solution associated with the 2nd rehabilitation cycle ($k = 2$) is comprised of $X_{3,1} = 0.1423$, $X_{4,1} = 0.0799$, and $X_{5,1} = 0.0174$. This means all pavements (100%) in condition states (**3**) & (**4**) have to be rehabilitated during the 2nd year using the outlined rehabilitation treatments 1 & 2, respectively. In addition, $(0.0174/0.1602) \times 100\% = 10.86\%$ of pavements in condition state (**5**) is to be rehabilitated using the 3rd rehabilitation treatment (i.e. complete removal and replacement), and to be selected based on worst-first criterion.

### *5.3. Sample prediction uncertainty impact*

The previously presented sample results have been estimated using the network mean transition probabilities. Statistically, these sample results are associated with only 50% occurrence chance. The impact of prediction uncertainty on the derived optimal solutions can be investigated by placing lower and upper limits on the network mean transition probabilities. In particular, lower and upper limits can be established for the network initial and terminal transition probabilities ($A_{1,2}$ & $A_{4,5}$) previously estimated as averages of the highway transition probabilities provided in Table 1. Equation (16a) is used to compute the upper-limit value ($UA_{i,i+1}$), and Equation (16b) calculates the lower-limit value ($LA_{i,i+1}$).

**Table 5.** Sample optimal lower, mean and upper-limit performance ratings (RB) with different annual budget (AB) and 95% confidence level.

| | $0.5 Million (AB) | | | $0.75 Million (AB) | | | $1.00 Million (AB) | | |
|---|---|---|---|---|---|---|---|---|---|
| Year ($k$) | $RB_L$ [a] | $RB_M$ [b] | $RB_U$ [c] | $RB_L$ | $RB_M$ | $RB_U$ | $RB_L$ | $RB_M$ | $RB_U$ |
| 1 | 57.70 | 57.17 | 56.67 | 61.62 | 61.35 | 61.07 | 65.58 | 65.52 | 65.52 |
| 2 | 57.51 | 56.49 | 55.43 | 64.43 | 63.76 | 63.05 | 70.44 | 69.69 | 68.91 |
| 3 | 57.10 | 55.71 | 54.39 | 65.47 | 64.04 | 62.59 | 72.21 | 70.84 | 69.43 |
| 4 | 56.45 | 54.92 | 53.66 | 65.14 | 63.16 | 61.78 | 73.47 | 71.48 | 69.46 |
| 5 | 56.02 | 54.55 | 53.28 | 64.93 | 63.16 | 61.90 | 74.63 | 72.07 | 69.53 |
| 6 | 55.88 | 54.39 | 53.06 | 65.07 | 63.35 | 62.08 | 75.20 | 72.62 | 69.65 |
| 7 | 55.83 | 54.31 | 52.89 | 65.13 | 63.37 | 62.07 | 75.02 | 73.09 | 69.76 |
| Average | 56.64 | 55.36 | 54.20 | 64.54 | 63.17 | 62.08 | 72.37 | 70.76 | 68.90 |
| % Diff. | +2.31 | – [d] | −2.10 | +2.17 | – | −1.76 | +2.28 | – | −2.63 |

[a] $RB_L$: Lower-limit rehabilitation performance rating.
[b] $RB_M$: Mean rehabilitation performance rating.
[c] $RB_U$: Upper-limit rehabilitation performance rating.
[d] Not-applicable.

The use of Equation (16) requires the sample size (N) to be equal or greater than (30) wherein the standard normal variable ($Z_{\propto/2}$) is used with (1-$\alpha$) confidence level. The normal variable ($Z_{\propto/2}$) is equal to (1.96) for (95%) confidence level with (S) being the sample standard deviation.

$$UA_{i,i+1} = A_{i,i+1} + Z_{\propto/2}\frac{S}{\sqrt{N}} \tag{16a}$$

$$LA_{i,i+1} = A_{i,i+1} - Z_{\propto/2}\frac{S}{\sqrt{N}} \tag{16b}$$

In case the sample size is less than (30), then the normal statistic ($Z_{\propto/2}$) is replaced by the t-statistic ($t_{\propto/2}$), which is the case in the previously presented case study wherein ($N = 12$). The t-statistic is equal to (2.201) for a sample size with (11) degree of freedom (N-1) and 95% confidence level. The mean and standard deviation associated with the initial transition probabilities provided in Table 1 are computed to be equal to ($A_{1,2} = 0.386$) and ($S_{1,2} = 0.141$), respectively. Similarly, the mean and standard deviation for the terminal transition probabilities are calculated to be ($A_{4,5} = 0.549$) and ($S_{4,5} = 0.171$), respectively. Equation (16) results in (0.296) and (0.476) as the lower and upper-limit values, respectively, for the mean initial transition probability ($A_{1,2}$). It also yields (0.439) and (0.659) as the limits for the mean terminal transition probability ($A_{4,5}$).

Now, two additional sets of optimal rehabilitation schedules have been generated similar to the schedules provided in Tables 2–4. One set has applied the estimated lower-limits (0.296 & 0.439) of mean initial and terminal transition probabilities, respectively, and the other set deployed the upper-limits (0.476 & 0.659) to be compared with the initial solutions associated with the mean values of (0.386 & 0.549). Table 5 provides a summary of the corresponding optimal performance ratings, RB(k), considering the three different probability sets (i.e. lower-limits, mean-values, upper-limits), and the same three different annual budgets. Also, the results associated with the lower and upper-limits have been displayed in Figures 2–4. It can be concluded from the figures that both lower and upper-limits have resulted in a fairly symmetrical impact with respect to the mean values. According to Figures 2–4, this impact in terms of $\Delta$RB is smaller for the first two rehabilitation cycles compared to the other cycles, and it becomes relatively steady afterward. The lower-limit solutions have provided higher performance ratings, thus representing the optimistic case. However, the higher-limit solutions have yielded inferior performance ratings, an indication of the pessimistic case.

It can also be concluded that the three RB averages provided at the bottom of Table 5 are relatively close in values for the same annual budget, an indication of slight uncertainty impact for 95% confidence level. The percent difference (%Diff) provided in the last row of Table 4 has been calculated from the RB averages using Equation (17). The percent difference has a maximum absolute value of only 2.63%, an indication of mild impact.

$$\%Diff = \frac{\overline{RB}_L \text{or} \overline{RB}_U - \overline{RB}_M}{\overline{RB}_M} \times 100\% \quad (17)$$

Statistically, the uncertainty impact can be reduced by increasing the sample size. The uncertainty impact can also be minimised by applying the proposed network-level rehabilitation model to highway systems with similar traffic loadings and pavement characteristics. It can further be reduced by analyzing two subsystems instead of one when considering the same highway system. The first subsystem is to be associated with initial transition probability ($P_{1,2}$) lower than the terminal transition probability ($P_{4,5}$), an indication of good pavement performance. The second one is recognised by the initial transition probability being higher than the terminal one, an indication of poor performance. According to Table 1, only four highways belong to the second case, namely highway numbers (6, 10-12). The available annual budget can be divided amongst the various pavement systems/subsystems in proportion to the vehicle-kilometre travelled (VKT) or design equivalent single axle load (ESAL) repetitions.

## 6. Conclusions and recommendations

The previously demonstrated optimum network-level rehabilitation model has the main advantage of incorporating the performances of both original and rehabilitated pavements through using three different transition probability matrices. It has also the advantage of considering both pavement long-term performance and rehabilitation cost in the optimised decision-making policy. The sample results presented have indicated the efficiency of the proposed network-level rehabilitation model in yielding practical long-term rehabilitation schedules. In particular, the sample results have emphasised the importance of allocating adequate rehabilitation funding to minimise the overall life-cycle cost. This requires selecting an optimal long-term rehabilitation schedule that can gradually eliminate the 'very poor' pavements as they impose the greatest negative impact on the life-cycle cost. The sample results have been obtained using a Markov chain with 5 condition states, thus allowing for the inclusion of up to 4 major rehabilitation treatments. This is usually adequate considering the number of potential rehabilitation options available in practice worldwide. However, a higher number of condition states (m) can easily be deployed to incorporate more treatment choices if so desired. The rehabilitation variables represent pavement proportions, however the pavement segments to be rehabilitated in the relevant condition states shall be field identified using 'worst-first' selection criterion.

The uncertainty impact on the predicted long-term performance has been investigated using 95% confidence level. The relevant sample results have indicated relatively low impact considering both lower and upper-limit values. Because the main source of uncertainty is the transition probabilities, it is recommended to apply efficient methods for estimating the individual highway transition probabilities as they are crucial parameters for yielding reliable optimal rehabilitation schedules. It is further recommended to deploy effective procedures in collecting pavement distress data used as the main input for estimating the transition probabilities. Furthermore, it is recommended to use weighted statistics (i.e. mean and standard deviation) rather than arithmetic ones to compute the statistics associated with the highway initial and terminal transition probabilities in case the highways are associated with substantially different lengths. Arithmetic statistics have been used in the highway sample provided in Table 1 because of approximately equal highway lengths. A good practice is to estimate the required transition probabilities for a standardised highway length such as (1-2) kilometres. The remaining input data required for implementing the proposed rehabilitation model are readily available to highway agencies.

The simplifications made are mainly related to the estimation of the relevant deterioration transition matrices, namely (**A** & **C**), used for sample presentation purposes only. However, matrix (**A**) should practically be estimated from performance records associated with original pavement, which is only used in solving the first-year rehabilitation cycle. Similarly, matrix (**C**) should be estimated from performance records corresponding to rehabilitated pavement, which is then used for solving the subsequent rehabilitation cycles. It is recommended that matrix (**C**) be annually updated based on newly collected condition data. Therefore, matrices (**A** & **C**) form the main input for using the proposed optimum rehabilitation model, which can both be estimated from the corresponding observed performances. However, rehabilitation matrix (**B**) is simply estimated from matrices (**A** & **C**) under the reasonable assumption that the ratios of ($A_{i,i+1}/A_{i,i}$ & $C_{i,i+1}/C_{i,i}$) remain the same as the ($B_{i,i+1}/B_{i,i}$) ratio in the presence of the rehabilitation improvement rates ($Q_{i,1}$). Also, it is recommended the proposed approach be used for road network/sub-network with pavement structures having similar performance trends, which is usually the case when roads are associated with similar loading conditions and material characteristics.

## Disclosure statement

No potential conflict of interest was reported by the author(s).

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