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Simplified Markovian-based pavement management model for sustainable long-term rehabilitation planning

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ABSTRACT

A simplified pavement management model for developing a long-term rehabilitation schedule is proposed for flexible pavement. The model deploys the discrete-time Markov model to predict the deterioration of both original and rehabilitated pavement performances. The main objective of the proposed model is to generate optimal annual rehabilitation cycles over a specified analysis period. This objective is achieved by optimising a cost-effectiveness index defined as the ratio of anticipated performance improvement and annual rehabilitation cost. It therefore seeks to find the optimal annual rehabilitation cycle that maximises pavement condition improvement and minimises rehabilitation cost. The corresponding optimum model is subject to a number of constraints that include the non-negativity constraints, upper-limit variable value constraints, and budget constraints. However, the rehabilitation variables are incorporated into the state probabilities rather than the transition matrix when predicting future deterioration of rehabilitated pavement. The optimum model can simply be solved using an exhaustive search approach as it makes use of a limited number of rehabilitation variables. Two case studies are presented to demonstrate the potential uses of the proposed model. The first one examined the relationship between variable budget levels and sustainable long-term pavement performances, whereas the second one investigated the long-term cost-effectiveness of individual rehabilitation treatments. Generally, the sample results re-emphasised the famous theme of 'better roads at lower costs'.

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Flexible pavement; pavement management; performance prediction; rehabilitation strategies; Markovian processes

1. Introduction

Maintaining any nation's infrastructure system in an acceptable condition is considered to be a key requirement for development and prosperity. The road network usually represents the major component of any nation's infrastructure system as it is used on a daily basis by the vast majority of the travelling public especially in countries where road travel is the predominant mode of transportation. The main component of any roadway is the pavement structure which deteriorates over time mainly due to the progressive action of traffic loadings. Therefore, pavements are expected to receive periodical maintenance and rehabilitation (M&R) works to keep them in satisfactory structural capacity and safe driving conditions. However, this task requires substantial resources mainly in terms of time and money which are typically limited when dealing with a large pavement network. Consequently, the concept of pavement management is of great importance to assist pavement engineers in developing optimal M&R schedules to be applied at the network level while taking into consideration available time and money.

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In particular, the pavement management system (PMS) has been proposed as a comprehensive mathematical tool to assist in solving the pavement management problem at the network level (Khattak et al., 2008; Li et al., 2006; Sebaaly et al., 1996; Torres-Machí et al., 2015). The pavement management problem is essentially an optimisation problem recognised to be complex to solve because of the need to consider a huge number of pavement sections, a large number of potential maintenance and rehabilitation strategies, while forecasting a number of years within a study period typically extending up to ten years. Several optimisation methods have been used to solve the pavement management problem including linear and non-linear algorithms, genetic algorithms and efficient search methods, however the main difficulty has been optimal convergence which contributed to the large problem size (Jorge & Ferreira, 2012; Khavandi Khiavi & Mohammadi, 2018; Mathew & Isaac, 2014; Santos et al., 2019; Zhang & Gao, 2012). The PMS typically includes four main components, namely: (1) performance prediction module to predict the pavement future conditions, (2) maintenance and rehabilitation module that specifies the appropriate M&R strategies based on the prevailing pavement conditions, (3) optimal decision-making policy that seeks to optimise the pavement condition at the network level while enforcing M&R variable and budget constraints, and (4) an optimisation method that can effectively solve the pavement management problem and yield reliable optimal solutions.

Several pavement management models have been developed in the last three decades but very few gained international publicity. This is because the majority are found either too unreliable or too complex to use and labelled 'data hungry' as they require extensive data records (Khattak et al., 2008; Li et al., 2006; Sebaaly et al., 1996; Torres-Machí et al., 2015). Generally, there are three key requirements for the successful implementation of any PMS. The first one is the incorporation of an effective performance prediction model that can predict the pavement future conditions, the second one is the ability to incorporate the M&R improvement rates into the performance prediction modelling, and the third one is the integration of a reliable optimisation method that can yield the best M&R schedules. Performance prediction models used in pavement management are either deterministic or probabilistic/stochastic. However, the stochastic-based models have been extensively used because the deterioration of pavements has long been recognised as being probabilistic in nature (Abed et al. 2019; Amin, 2015; Fuentes et al., 2021). The most popularly used stochastic models are the Markovian-based ones with a discrete number of condition states (Lethanh et al., 2015; Lethanh & Adey, 2012; Meidani & Ghanem, 2015). The Markovian-based prediction models have been widely used in several applications related to pavement rehabilitation and management including life-cycle analysis at both project and network levels (Abaza, 2017; Galvis Arce & Zhang, 2021; Pittenger et al., 2012).

In this paper, it is proposed to develop a simplified pavement management model that focuses on major rehabilitation works as applied to small pavement networks and individual large projects with the main research objectives summarised as follows:

(1) To develop a simplified pavement management model that can generate a long-term rehabilitation schedule comprised of a number of annual rehabilitation cycles with each consisting of a limited number of rehabilitation treatments.
(2) The proposed model shall make use of an effective performance prediction model such as the discrete-time Markov model with minimal data requirement to be used in predicting future conditions of both original and rehabilitated pavements.
(3) The proposed model shall include a simple mechanism for incorporating the rehabilitation treatment variables into the long-term performance prediction process. This is achieved by incorporating the rehabilitation variables into the state probabilities rather than the transition matrix. This step represents the main difference compared to other similar models.
(4) The proposed model shall be based on an effective decision-making policy capable of yielding optimal annual rehabilitation cycles that account for both performance and cost of potential rehabilitation treatments. This is accomplished by optimising an appropriate cost-effectiveness indicator. This step is another key difference compared to other models.

(5) The proposed model shall utilise simple and efficient optimisation approach that can easily be implemented by highway agencies to yield reliable optimal solutions. This is made possible due to the limited number of rehabilitation variables involved, thus offering another advantage compared to other models that apply a much larger number of M&R variables.
(6) The proposed model shall help decision makers in estimating the long-term rehabilitation budgets required to sustain a certain level of pavement performance.
(7) To present potential case studies that can demonstrate the simple and effective use of the proposed optimum rehabilitation model.

2. Overview of Markovian-based performance prediction

This overview section presents the foundation for the proposed optimum rehabilitation model. Pavement performance prediction has typically been investigated using the discrete-time Markov model (Abaza, 2017; Abed et al. 2019; Galvis Arce & Zhang, 2021; Li et al., 2006; Zhang & Gao, 2012). Two popular forms of the Markov model are used to predict future pavement conditions. The first one is the homogeneous Markov model as defined in Equation (1), which assumes that pavement deterioration rates remain unchanged over the analysis period. The second one is the non-homogeneous Markov model which can account for changes in pavement deterioration rates over time. The deterioration rates are represented by the transition probabilities incorporated within the transition matrix (***P***). The discrete-time Markov model requires using equal discrete-time intervals called transitions that make up the analysis period (*n*). It also requires deploying a discrete number of condition states (*m*). The Markov model defined in Equation (1) is used to estimate the state probability row vector, ***S(n)***, after exactly (*n*) transitions as a multiplication product of the initial state probability row vector, ***S(0)***, and transition probability matrix (***P***) raised to power (*n*). The state probabilities are defined as the pavement proportions that exist in the various deployed condition states at a specified time (*k*). The non-homogeneous Markov model is similar to the one defined in Equation (1) but it can incorporate a different transition probability matrix for each time interval (i.e. transition). Logically, the sum of state probabilities at any transition (k) must add up to one.

$$\boldsymbol{S}(\boldsymbol{n}) = \boldsymbol{S}(0)\ \boldsymbol{P}^{\boldsymbol{n}} \tag{1}$$

where

$$\begin{aligned}
\boldsymbol{S}(\boldsymbol{n}) &= [S_1(n), S_2(n), S_3(n), \dots, S_m(n)] \\
\boldsymbol{S}(\mathbf{0}) &= [S_1(0), S_2(0), S_3(0), \dots, S_m(0)] \\
&= (1, 0, 0, 0, \dots, 0) \text{ for new pavement} \\
\sum_{i=1}^{m} S_i(k) &= 1.0 \quad (k = 0, 1, 2, 3, \dots, n)
\end{aligned}$$

The transition probability matrix is $(m \times m)$ square matrix containing the transition probabilities that represent the pavement deterioration rates amongst the various deployed condition states. The matrix entries along the main diagonal ($P_{i,i}$) represent the probabilities of remaining in the same condition state after one transition (i.e. time interval typically taken as one year), and entries above the main diagonal denote the probabilities of transiting to the worst condition states in one transition, thus representing pavement deterioration rates. However, the entries below the main diagonal can define the pavement improvement rates from the worst states to better states in one transition. Different forms of the transition probability matrix have been used by researchers, however the most popular is the one that assumes only two state transitions as indicated by Equation (2) (Abaza, 2022; Abed et al. 2019; Galvis Arce & Zhang, 2021). This means that a pavement section currently in state (**i**) can either remain in the same state (**i**) with probability ($P_{i,i}$) or transit to the next worst state (**i + 1**) after one transition with probability ($P_{i,i+1}$). The transition matrix form indicated by Equation (2) is only used to model pavement deterioration without considering any pavement improvements since all entries below the main diagonal are assigned zero values. This form of transition matrix has been found to be effective in modelling pavement deterioration over time when using a relatively small number of condition states, typically 5 condition states (Abaza, 2021). Logically, the sum of any row in the transition matrix must add up to one.

$$P = \begin{bmatrix} P_{1,1} & P_{1,2} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & P_{2,2} & P_{2,3} & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & P_{3,3} & P_{3,4} & 0 & 0 & \dots & 0 \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ 0 & 0 & \dots & 0 & P_{m-1,m-1} & & & P_{m-1,m} \\ 0 & 0 & & 0 & 0 & & & 1.0 \end{bmatrix} \quad (2)$$

It has been reported that the use of the homogeneous Markov model with a constant transition probability matrix similar to the one defined in Equation (2) provided satisfactory results considering an analysis period comprised of up to five transitions with transition length taken equal to one year (Abaza, 2017; Abed et al. 2019; Galvis Arce & Zhang, 2021). Therefore, the state probabilities, $S_i(k)$, estimated from the homogeneous Markov model at the kth transition can now be used to estimate the corresponding pavement performance rating values, $V(k)$, as indicated by Equation (3). The kth performance rating value, $V(k)$, can be estimated using one of the popular pavement performance indicators such as present serviceability index (PSI), pavement condition index (PCI), and international roughness index (IRI).

$$V(k) = \sum \bar{V}_i \times S_i(k) \quad (k = 0, 1, 2, \dots, n; i = 1, 2, \dots, m) \quad (3)$$

where

$$\bar{V}_i = {(UV_i + LV_i)}/{2}$$

Equation (3) requires multiplying the state probabilities, $S_i(k)$, by the average state performance values, ($\bar{V}_i$), to yield the predicted kth performance rating value, $V(k)$. The average state performance value is computed as the average of state upper and lower performance rating values (UV_i & LV_i). For example, the average state performance values are equal to (90, 70, 50, 30, 10) when using the PCI as the performance indicator for a Markov chain with 5 condition states. In this example, the five condition states are defined using equal PCI ranges (i.e. 100-80, 80-60, 60-40, 40-20, 20-0). The best and worst states are assigned the ranges of (100-80) and (20-0) with (90 & 10) being the average state PCI values, respectively. Therefore, according to Equation (3), the predicted initial PCI value, PCI(0), for new pavement is 90 assuming all pavements are assigned to condition state (**1**).

3. Simplified pavement rehabilitation model development

The proposed simplified pavement rehabilitation model applies the homogeneous Markov chain in predicting the long-term pavement performance in the presence of major rehabilitation works. Typically, pavement improvement rates have been incorporated into the transition matrix as represented by the transition probabilities below the main diagonal as outlined earlier. However, this is applicable when dealing with a large pavement network so that maintenance and rehabilitation works are carried out throughout the entire transition length (typically one year), which is a requirement for using the discrete-time Markov chain. Additionally, incorporating the pavement improvement rates within

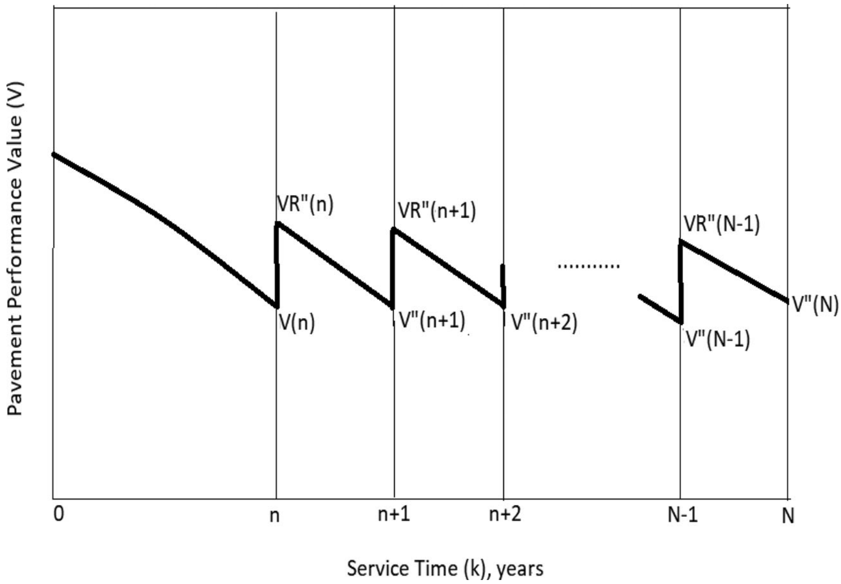

Figure 1. Typical pavement performance curve with long-term rehabilitation schedule.

the transition matrix requires a rational method to adjust the deterioration transition probabilities so that the sum of any matrix row remains equal to one. This rational method can affect the deterioration prediction outcomes. In this paper, it is proposed to incorporate the pavement improvement rates to be part of the state probabilities while keeping the transition matrix to only represent the pavement deterioration mechanism of both original and rehabilitated pavements. This approach becomes of a particular interest when considering major rehabilitation works applied to small pavement networks and large individual projects.

3.1. Estimation of rehabilitation state probabilities

Figure 1 shows a typical pavement long-term performance curve wherein the first rehabilitation cycle is applied at the nth transition while subsequent rehabilitation cycles are annually applied for an analysis period comprised of (N) years. The prediction of the original part of the long-term performance curve (i.e. up to the nth transition) in the absence of any rehabilitation work can be estimated using the homogeneous Markov model defined in Equation (4). This Markov model form is equivalent to the one presented in Equation (1) but applied in a successive mode ($n = 1$). The original state probabilities predicted from Equation (4) can then be used to estimate the corresponding pavement performance rating values, $V(k)$, as defined in Equation (3).

$$\boldsymbol{S}(\boldsymbol{k}) = \boldsymbol{S}(\boldsymbol{k}-\mathbf{1})\boldsymbol{P} \quad (k = 1, 2, \dots, n) \tag{4}$$

In the proposed simplified rehabilitation model, it is assumed that there are ($m - 1$) treatment strategies applicable to pavements in condition states (**2, 3, ... , m − 1, m**) with state (**m**) denotes the state with worst pavement condition. The proposed model only focuses on major rehabilitation actions and doesn't consider low-cost maintenance work. The annual rehabilitation cycle may consist of ($m - 1$) treatments with each being represented by one rehabilitation variable, $X_i(k)$. Therefore, all states can receive major rehabilitation work with the exception of state (**1**), which represents the best pavements (i.e. new pavements). For example, four distinct treatment strategies can be implemented when using a Markov chain with 5 condition states to be applied to states (**2, 3, 4, 5**). It is further assumed that the applied treatment strategies will transfer the pavements from the current states to condition state (**1**). The rehabilitation state probabilities, $SR_i(k)$, at the kth transition are obtained using Equation (5). The rehabilitation state probabilities, $SR_i(k)$, are estimated as the differences between the deterioration state probabilities, $S_i(k)$, and rehabilitation variables, $X_i(k)$, as defined in Equation (5a) considering all states except state (**1**). Therefore, the deterioration state probabilities represent the maximum proportions available for rehabilitation while the rehabilitation state probabilities denote the new pavement proportions existing immediately after rehabilitation.

The rehabilitation variables, $X_i(k)$, representing the improvement rates are simply to be subtracted from the deterioration state probabilities associated with states receiving major rehabilitation actions as indicated by Equation (5a). However, the same variables, $X_i(k)$, are to be added to the state probability associated with state (**1**) being the new state after rehabilitation as defined in Equation (5b). Therefore, the rehabilitation variables are proportions to be subtracted from the deterioration state probabilities, $S_i(k)$, and added to the state probability of state (**1**). The improvement rates as probabilities are computed by dividing the rehabilitation variables, $X_i(k)$, by the corresponding state probabilities, $S_i(k)$. Logically, the sum of rehabilitation state probabilities must remain equal to one as indicated by Equation (5c). Once, the rehabilitation state probabilities are estimated from Equation (5), the corresponding rehabilitation performance rating value, $VR(k)$, can be computed using Equation (3).

$$SR_i(k) = S_i(k) - X_i(k) \quad (i = 2, 3, \ldots, m; k = n, n+1, n+2, \ldots, N-1) \tag{5a}$$

$$SR_1(k) = S_1(k) + \sum X_i(k) \quad (i = 2, 3, \ldots, m;\ k = n,\ n+1,\ n+2, \ldots, N-1) \tag{5b}$$

$$\sum_{i=1}^{m} SR_i(k) = 1.0 \tag{5c}$$

Application of Equation (5) implies instantaneous pavement improvements as indicated by the vertical rises in the long-term performance curve as shown in Figure 1. As per definition of Markovian processes, this vertical rise requires that the major rehabilitation work takes place instantaneously at the kth transition; however a short rehabilitation duration can practically be used so that any pavement deterioration taking place during this duration can be neglected. A duration of about 1–2 months should be small enough when considering one-year transition length, and it can be equally split before and after the kth year so that its effect on the unaccounted for pavement deterioration can be minimised. In practice, this would be applicable to small pavement networks and individual large projects wherein rehabilitation works can be completed within 1–2 months. The deterioration state probabilities associated with rehabilitated pavement are determined using the homogeneous discrete-time Markov model but with a modified transition probability matrix (***MP***) to reflect the deterioration rates of rehabilitated pavement as indicated by Equation (6).

$$\boldsymbol{S}(\boldsymbol{k}+\mathbf{1}) = \boldsymbol{SR}(\boldsymbol{k})\boldsymbol{MP} \quad (k = n,\ n+1,\ n+2, \ldots, N-1) \tag{6}$$

The modified transition matrix (***MP***) can be estimated from field assessment of rehabilitated pavement performance, an approach similar to the estimation of original transition matrix (***P***) (Abaza, 2021, 2022). However, a simplified approach based on experience and engineering judgement is presented in the sample presentation section to estimate the matrix (***MP***) from matrix (***P***).

3.2. Optimum pavement rehabilitation model

It is proposed to generate an optimum long-term rehabilitation schedule that consists of $(N-n)$ annual rehabilitation cycles as shown in Figure 1. An annual rehabilitation cycle can include up to $(m-1)$ treatment strategies with each represented by one rehabilitation variable, $X_i(k)$. A simplified but yet effective optimum rehabilitation model is proposed in Equation (7) with the aim of generating a long-term rehabilitation schedule. The objective function of this model seeks to maximise a cost-effectiveness index, $I_{CE}(k)$, defined as the ratio of rehabilitation performance improvement, $\Delta V(k)$, and total cost, $TC(k)$, associated with the kth rehabilitation cycle. The performance improvement, $\Delta V(k)$, is defined as the difference between the performance rating values, $VR(k)$ & $V(k)$. Therefore, the performance improvement change, $\Delta V(k)$, is simply equal to the vertical rise in the long-term performance curve shown in Figure 1. The rehabilitation cost, $TC(k)$, for the kth cycle is computed from the multiplication of pavement surface area (A_p), unit costs of treatment strategies, $UC_i(k)$, and rehabilitation variable values, $X_i(k)$.

Therefore, the proposed cost-effectiveness index accounts for both performance and cost associated with each rehabilitation cycle as represented by the corresponding rehabilitation variables, $X_i(k)$. In essence, the optimal rehabilitation cycle is the one associated with the highest performance and lowest cost. The computation of the rehabilitation performance rating values, $VR(k)$ & $V(k)$, is also dependent on the rehabilitation variable values as explained earlier. As per Figure 1, the 1st rehabilitation cycle takes place at the nth year and the last one at year (N-1). Equation (7) is multiplied by the constant (10^4) so that the value of $I_{CE}(k)$ is larger than one as determined based on the sample results presented later.

$$\text{Maximize: } I_{CE}(k) = \left(\frac{\Delta V(k)}{TC(k)}\right) \times 10^4 \quad (i = 2, 3, \ldots, m;\ k = n,\ n+1,\ n+2, \ldots, N-1) \tag{7}$$

where

$$\Delta V(k) = VR(k) - V(k)$$

$$TC(k) = A_p \times \sum [UC_i(k) \times X_i(k)]$$

Subject to the following constraints:

(1) $X_i(k) \geq 0.0$
(2) $X_i(k) \leq S_i(k)$
(3) $X_i(k) \leq B(k)/[A_P \times UC_i(k)]$
(4) $TC(k) \leq B(k)$

The proposed optimum rehabilitation model is subject to four sets of constraints. The first set is the non-negativity constraints, and the second set requires the rehabilitation variable values to be less than or equal to the corresponding deterioration state probabilities, $S_i(k)$. The third set enforces the rehabilitation variable values to be less than or equal to the maximum rehabilitation work that can be done considering the *i*th treatment strategy and allocated annual budget, $B(k)$. Therefore, constraint sets 2 and 3 are essentially used to establish the variable upper-limit values which are equal to the lower values obtained from the two sets, thus reducing the optimisation efforts. The fourth set requires the rehabilitation cycle cost to be less than or equal to the *k*th allocated annual budget. Appropriate optimisation techniques can be used to sequentially solve the optimum rehabilitation model outlined in Equation (7) with the outcome being the generation of $(N - n)$ optimal annual rehabilitation cycles scheduled starting at the nth year as depicted in Figure 1.

3.3. Optimal long-term rehabilitation schedule

Once the optimal rehabilitation variable values, $X_i''(k)$, are obtained for the *k*th year as derived from the optimum model outlined in Equation (7), the corresponding optimal rehabilitation state probabilities, $SR_i''(k)$, can be computed as outlined in this section. The optimal rehabilitation state probabilities, $SR_i''(n)$, for the nth rehabilitation cycle ($k = n$) are determined using Equation (8). Equation (8a) computes the optimal rehabilitation state probabilities for all states except condition state (**1**) with its corresponding rehabilitation state probability, $SR_1''(n)$, computed from Equation (8b). The original state probabilities, $S_i(n)$, are determined as outlined earlier using Equation (1) based on the transition matrix

(***P***) associated with original pavement.

$$SR_i''(n) = S_i(n) - X_i''(n) \tag{8a}$$

$$SR_1''(n) = S_1(n) + \sum X_i''(n) \tag{8b}$$

Similarly, the remaining optimal rehabilitation state probabilities for ($k > n$) are determined using Equation (9). These optimal rehabilitation state probabilities are used to estimate the corresponding optimal rehabilitation performance values, $VR''(k)$, using Equation (3), as shown in Figure 1.

$$SR_i''(k) = S_i''(k) - X_i''(k) \tag{9a}$$

$$SR_1''(k) = S_1''(k) + \sum X_i''(k) \tag{9b}$$

The optimal deterioration state probabilities, $S_i''(k)$, used in Equation (9) are to be determined from Equation (6) for ($k \geq n$) using a modified transition probability matrix (***MP***) as outlined earlier to account for rehabilitated pavement with different deterioration rates compared to the original pavement. The optimal deterioration state probabilities, $S_i''(k)$, are used to estimate the optimal performance values, $V''(k)$, as shown in Figure 1. As a summary, the optimal rehabilitation variables, $X_i''(k)$, are used to estimate the optimal rehabilitation state probabilities, $SR_i''(k)$, which are then used to compute the optimal deterioration state probabilities, $S_i''(k+1)$, using Equation (6). The only exception are the $SR_i''(n)$ estimated using $S_i(n)$ as indicated by Equation (8).

4. Simulated examples

Two case studies are presented in this section to demonstrate the potential use of the proposed simplified Markovian-based pavement rehabilitation model. The first one has sought to derive sample optimal long-term rehabilitation schedules for a new pavement structure considering an analysis period of 12 years (N). The second case study has investigated the long-term cost-effectiveness of using three distinct rehabilitation strategies with each consisting of a specific treatment option. A transition probability matrix with 5 condition states and one-year transition length are deployed when predicting the long-term pavement performance using the discrete-time Markov model. The well-known pavement condition index (PCI) has been used to represent the performance rating value (V). Equal PCI ranges are used to define the 5 condition states as outlined earlier at the end of section 2.

The initial state probabilities, $S_i(0)$, are assumed equal to (1, 0, 0, 0, 0) for new pavement structure. The elements of the transition probability matrix (***P***) are typically estimated using distress records obtained from conducting two consecutive field surveys. It was reported that two elements of the transition matrix greatly affect the performance prediction reliability when considering the matrix form presented in Equation (2), namely the initial transition probability ($P_{1,2}$), and terminal transition probability ($P_{m-1,m}$) (Abaza, 2017, 2021). A simple optimum approach was also proposed to estimate the transition matrix from historical annual performance records (Abaza, 2022).

For sample presentation purposes, a sample transition probability matrix is constructed assuming (0.25) initial transition probability ($P_{1,2}$) for an urban arterial with flexible pavement comprised of (10 cm) asphaltic surface and (40 cm) aggregate base. It was reported that for a good pavement performance (i.e. $P_{1,2} < P_{m,m-1}$), the expected range for the initial transition probability is (0.10–0.40), so the mid-range value has been used (Abaza, 2022). It was also proposed that the terminal transition probability can be estimated from multiplying the initial transition probability by a deterioration rate factor (F_d) as indicated by Equation (10). For good pavement performance with degradation curve being concave downward ($P_{1,2} < P_{m,m-1}$), the terminal transition probability ($P_{m-1,m}$) was reported to be (2–3) times greater than the initial one ($P_{1,2}$) (Abaza, 2022). Therefore, the mid-range value of

(2.5) has been used which results in (0.625) terminal transition probability ($P_{4,5}$).

$$P_{m-1,m} = F_d \times P_{1,2} \tag{10}$$

The remaining transition probabilities ($P_{i,i+1}$) can be estimated from the initial and terminal transition probabilities ($P_{1,2}$ & $P_{m-1,m}$) using linear interpolation as indicated by Equation (11) (Abaza, 2017, 2021).

$$P_{i,i+1} = P_{1,2} + \frac{P_{m-1,m} - P_{1,2}}{m-2} \quad (i = 2, 3, \ldots, m-2) \tag{11}$$

Now, all elements of the sample transition probability matrix ($\boldsymbol{P}$) are defined as provided in Equation (12). This sample transition matrix is used in both case studies to predict the deterioration of original pavement structure from time zero to the nth year as shown in Figure 1 using Equations (3) and (4).

$$\boldsymbol{P} = \begin{bmatrix} 0.750 & 0.250 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.625 & 0.375 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.500 & 0.500 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.375 & 0.625 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.000 \end{bmatrix} \tag{12}$$

A simplified procedure is proposed to estimate the modified initial transition probability ($MP_{1,2}$) by multiplying the original initial transition probability ($P_{1,2}$) by a modification factor (F_m) as outlined in Equation (13). This factor value is greater than one if the rehabilitated pavement is expected to be weaker than original pavement, and it is less than one if expected to be stronger. The remaining elements of the modified transition matrix (***MP***) can then be estimated as done in constructing the sample transition matrix (***P***) using Equations (10) and (11).

$$MP_{1,2} = F_m \times P_{1,2} \tag{13}$$

The modification factor (F_m) is assumed equal to one in both case studies, which means both matrices (***P*** & ***MP***) are the same. Generally, the (***PM***) matrix should be estimated from distress records collected on rehabilitated pavement. However, it can be estimated from the (***P***) matrix as an approximation. The (F_m) value can be one if it is expected that the structural capacity and loading conditions associated with original and rehabilitated pavements are similar, which means both pavements are expected to experience similar deterioration rates. The modified transition matrix is used to estimate the deterioration of rehabilitated pavement as indicated by Equation (6).

According to the proposed pavement rehabilitation model, a transition matrix with size ($m = 5$) allows the inclusion of four rehabilitation variables with each represented by a single treatment strategy. However, only three potential treatment strategies are used in both case studies as defined in Table 1. The table provides details about the three strategies including the local unit costs (UC_i), and expected average improvement outcomes (ΔPCI). The treatment unit costs (UC_i) represent the present values in (\$/m^2) but are assumed to remain constant over the analysis period. Rehab Plan I is applied to pavements in state (**3**) representing average condition, Rehab Plan II applied to state (**4**) denoting poor condition, and Rehab Plan III applied to state (**5**) indicating bad condition. The rehabilitation strategies provided in Table 1 are applicable to the sample arterial under investigation, so different strategies are required for different projects, or the same strategies can be applied to a pavement network with similar pavement structures. The proposed sample model allows to add a fourth treatment strategy to be applied to state (**2**) with good pavement condition if so desired. The outcome in all cases is pavement upgrade to condition state (**1**) with very good pavements. The expected average improvements (ΔPCI) are computed as the differences of state average performance values, ($\bar{V}_i$), provided at the end of Section 2.

Table 1. Details of sample rehabilitation treatments.

Plan name	Treatment strategy	Rehab variable	Unit cost ($/m^2)	PCI improvement (ΔPCI)
Rehab Plan I	3 cm surface cold milling replaced with 3 cm new hot asphalt mix applied to condition state (3)	X_3	13	40
Rehab Plan II	5 cm surface cold milling replaced with 5 cm new hot asphalt mix applied to condition state (4)	X_4	20	60
Rehab Plan III	Removal of existing asphaltic surface and placement of 10 cm new hot asphalt mix applied to condition state (5)	X_5	30	80

Table 2. Sample optimal long-term solutions for $10,000 annual budget.

	Optimal variables[a]			Optimal state probabilities[b]					
Year (k)	$X_3''(k)$	$X_4''(k)$	$X_5''(k)$	$S_1''(k)SR_1''(k)$	$S_2''(k)SR_2''(k)$	$S_3''(k)SR_3''(k)$	$S_4''(k)SR_4''(k)$	$S_5''(k)SR_5''(k)$	$I_{CE}''(k)$[c]
5	0.1099	0.0000	0.0000	0.2373	0.2839	0.2335	0.1501	0.0952	4.396
				0.3472	0.2839	0.1236	0.1501	0.0952	
6	0.1099	0.0000	0.0000	0.2604	0.2642	0.1682	0.1181	0.1891	4.396
				0.3703	0.2642	0.0583	0.1181	0.1891	
7	0.1099	0.0000	0.0000	0.2777	0.2577	0.1283	0.0734	0.2629	4.396
				0.3876	0.2577	0.0184	0.0734	0.2629	
8	0.1058	0.0026	0.0000	0.2907	0.2580	0.1058	0.0367	0.3088	4.392
				0.3991	0.2580	0.0000	0.0341	0.3088	
9	0.0967	0.0085	0.0000	0.2994	0.2610	0.0967	0.0128	0.3301	4.382
				0.4046	0.2610	0.0000	0.0043	0.3301	
10	0.0979	0.0016	0.0041	0.3034	0.2643	0.0979	0.0016	0.3328	4.342
				0.4070	0.2643	0.0000	0.0000	0.3287	
11	0.0991	0.0000	0.0047	0.3053	0.2669	0.0991	0.0000	0.3287	4.338
				0.4091	0.2669	0.0000	0.0000	0.3240	
12	–[d]	–	–	0.3068	0.2691	0.1001	0.0000	0.3240	–
				–	–	–	–	–	

[a] $X_i''(k)$ = optimal rehabilitation variables as applied to states (3, 4 & 5).
[b] $S_i''(k)$ = optimal deterioration state probabilities with the exception of the 1st row being the original state probabilities, $S_i(5)$. $SR_i''(k)$ = optimal rehabilitation state probabilities.
[c] $I_{CE}''(k)$ = optimal cost-effectiveness index.
[d] Not Applicable.

4.1. Case study I: optimal long-term rehabilitation schedules

The proposed optimum rehabilitation model has been used to generate sample optimal long-term rehabilitation schedules considering the data outlined earlier and three different annual budget values for the sample arterial with 7000 m^2 pavement surface area (A_p). An exhaustive optimisation search approach has been used wherein the values of the three rehabilitation variables are simultaneously varied until the maximum cost-effectiveness index value (I_{CE}) is reached as presented in Equation (7). An analysis period of 12 years (N) has been used with first rehabilitation cycle applied at the 5th year (n), thus resulting in a total of 7 rehabilitation cycles ($N - n$). Table 2 provides the optimal annual rehabilitation solutions for $10,000 annual budget. It can be noted that rehabilitation cycles for years (5–7) include only Rehab Plan I, rehabilitation cycles for years (8–9) contain Rehab Plans I & II, and the 10th year cycle applies the three treatment strategies. Therefore, it can be concluded that Rehab Plan I (X_3'') has dominated the solutions, followed by Rehab Plan II (X_4''), and then Rehab Plan III (X_5''). This is expected based on their unit costs (UC_i) and improvement outcomes (ΔPCI) with Rehab Plan I having the highest ($\Delta PCI/UC_i$) ratio, followed by Rehab Plan II as provided in Table 1.

It is be noticed from Table 2 that the 1st cycle optimal rehabilitation state probabilities, $SR_i''(5)$, are obtained from the original state probabilities, $S_i(5)$, using Equation (8). Then, the optimal $SR_i''(5)$ are used to predict the subsequent optimal deterioration state probabilities, $S_i''(6)$, using Equation (6). The remaining optimal rehabilitation state probabilities, $SR_i''(k)$, are computed using Equation (9) with the subsequent optimal deterioration state probabilities, $S_i''(k+1)$, are also determined from

Table 3. Sample optimal long-term solutions for \$20,000 annual budget.

	Optimal variables[a]			Optimal state probabilities[b]					
Year (k)	$X''_3(k)$	$X''_4(k)$	$X''_5(k)$	$S''_1(k)SR''_1(k)$	$S''_2(k)SR''_2(k)$	$S''_3(k)SR''_3(k)$	$S''_4(k)SR''_4(k)$	$S''_5(k)SR''_5(k)$	$I''_{CE}(k)$[c]
5	0.2198	0.0000	0.0000	0.2373	0.2839	0.2335	0.1501	0.0952	4.396
				0.4571	0.2839	0.0137	0.1501	0.0952	
6	0.1133	0.0631	0.0040	0.3428	0.2917	0.1133	0.0631	0.1891	4.322
				0.5232	0.2917	0.0000	0.0000	0.1851	
7	0.1094	0.0000	0.0478	0.3924	0.3131	0.1094	0.0000	0.1851	4.102
				0.5496	0.3131	0.0000	0.0000	0.1373	
8	0.1174	0.0000	0.0443	0.4122	0.3331	0.1174	0.0000	0.1373	4.124
				0.5739	0.3331	0.0000	0.0000	0.0930	
9	0.1249	0.0000	0.0411	0.4304	0.3517	0.1249	0.0000	0.0930	4.144
				0.5864	0.3517	0.0000	0.0000	0.0519	
10	0.1319	0.0000	0.0381	0.4473	0.3689	0.1319	0.0000	0.0519	4.161
				0.6173	0.3689	0.0000	0.0000	0.0138	
11[d]	0.1383	0.0000	0.0138	0.4630	0.3849	0.1383	0.0000	0.0138	4.286
				0.6151	0.3849	0.0000	0.0000	0.0000	
12	–[e]	–	–	0.4614	0.3943	0.1443	0.0000	0.0000	–
				–	–	–	–	–	

[a]$X''_i(k)$ = optimal rehabilitation variables as applied to states (3, 4 & 5).
[b]$S''_i(k)$ = optimal deterioration state probabilities with the exception of 1st row being the original state probabilities, $S_i(k)$. $SR''_i(k)$ = optimal rehabilitation state probabilities.
[c]$I''_{CE}(k)$ = optimal cost-effectiveness index.
[d]Rehabilitation cost of last cycle = \$15,478.
[e]Not Applicable.

Equation (6). The optimal variable, $X''_i(k)$, represents the pavement proportion to be deducted from the proportion in the relevant state (**i**). For example, Table 2, the optimal solution for the 5th year is only ($X''_3(5) = 0.1099$), which means that (0.1099) is to be subtracted from the 3rd state probability ($SR''_3(k) = 0.2335$) and sent to state (**1**), which is equivalent to an improvement rate (i.e. probability) of (0.1099/0.2335 = 0.4707). However, identifying specific road segments for rehabilitation is to be performed in the field by selecting the ones in worst pavement condition.

Similarly, Table 3 provides the optimal solutions for \$20,000 annual budget using the same analysis period of 12 years with 7 rehabilitation cycles, however the cost of last cycle is only \$15,478 covering the rehabilitation of all pavement proportions available in states (**3**) and (**5**). Figure 2 displays three sample long-term performance curves generated for three different annual budgets. The predicted performance rating values shown in Figure 2 are computed from Equation (3) using the corresponding state probabilities [i.e. $S_i(k = 0, 1, 2, \dots, n)$; $SR''_i\ (k = n, n+1, \dots, N-1)$; & $S''_i\ (k = n+1, n+2, \dots, N)$]. Figure 2 provides the average long-term PCI value computed as the arithmetic average of all data points used in constructing the long-term performance curve. It also displays the optimal total long-term rehabilitation cost. It can be noticed from Figure 2 that an annual budget of \$10,000 is adequate to sustain a steady state condition in terms of the PCI value. This would probably be the practical case for most highway agencies. However, the other two cases with \$15,000 and \$20,000 annual budgets have resulted in a gradual increase in the PCI value over time. Of course, the more money spent on rehabilitation, the higher is the average long-term PCI value. An annual budget of less than \$10,000 is expected to result in a gradual decrease in PCI value over time. These sample optimal presentations are obtained with the 1th rehabilitation cycle applied at the 5th year (n), a decision to be made by policy makers. Technically, different potential start years (n) can be investigated and the one with the best optimal long-term rehabilitation schedule is to be selected.

4.2. Case study II: individual treatment strategy cost-effectiveness

The second case study investigates the long-term cost-effectiveness associated with the rehabilitation policy of implementing a distinct treatment strategy as typically adopted by several highway agencies.

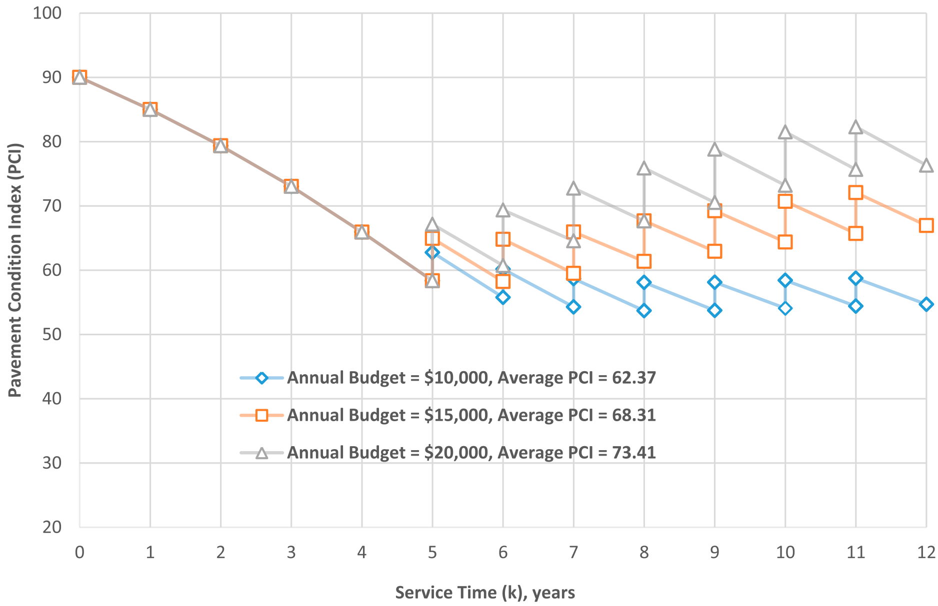

Figure 2. Sample optimal long-term performance curves for different annual budgets.

This policy aims to annually rehabilitate all pavement proportion reaching a specific condition state using applicable treatment strategy. For example, the most popular treatment strategy in many countries around the World for flexible pavement involves cold milling and overlay similar to Rehab Plans I and II used in this sample presentation. Therefore, case study II applies the proposed approach to estimate the rehabilitation state probabilities, $SR_i(k)$, and subsequent deterioration state probabilities, $S_i(k+1)$, which are then used to compute the corresponding performance rating values, $VR(k)$ & $V(k+1)$, respectively. Case study II applies the same sample input data used in case study I.

Table 4 provides sample long-term solutions using only Rehab Plan I as applied to state (**3**). The objective is to rehabilitate the entire pavement proportion, $S_3(k)$, reaching state (**3**) for each rehabilitation cycle. Table 4 indicates the first pavement proportion reaching state (**3**) is at the end of 2nd year, thus the 1st rehabilitation cycle is applied at the 2nd year (n). This policy continues to annually rehabilitate the whole pavement proportion reaching state (**3**) as provided in Table 4. Similarly, Table 5 provides sample solutions for annual implementation of Rehab Plan III as applied to state (**5**). The first rehabilitation cycle takes place at the 4th year because it is the year at which some pavement proportion has reached state (**5**) for the first time. Likewise, Rehab Plan II as applied to state (**4**) results in the first rehabilitation cycle being applied at the 3rd year. According to Tables 4 and 5, it can be noted that the annual rehabilitation cost has reached steady state condition when considering 12-year analysis period. It is also noted that the cost-effectiveness index (I_{CE}) is constant for all rehabilitation cycles with the same treatment strategy, an indication of linear relationship between performance improvement, $\Delta V(k)$, and corresponding annual rehabilitation cost, $TC(k)$.

Figure 3 depicts the sample long-term performance curves associated with the individual application of the three treatment strategies. The corresponding average PCI value and total rehabilitation cost are also displayed in Figure 3. It can be concluded from Figure 3 that both Rehab Plans I and II have reached steady state condition in terms of PCI earlier than Rehab Plan III. It can also be noted that the vertical improvement, $\Delta V(k)$, is directly proportional to the annual rehabilitation cost, $TC(k)$, as the cost-effectiveness index (I_{CE}) is constant. It is clear that Rehab Plan I is superior to Rehab Plans II & III because it is associated with the highest average PCI value and lowest total rehabilitation cost, whereas Rehab Plan II is superior to Plan III. This latter conclusion re-emphasises the famous theme

Table 4. Sample long-term solutions using Rehab Plan I as applied to state (3).

	Rehab. variable	State probabilities[a]						
Year (k)	X_3 (k)	$S_1(k)SR_1(k)$	$S_2(k)SR_2(k)$	$S_3(k)SR_3(k)$	$S_4(k)SR_4(k)$	$S_5(k)SR_5(k)$	$TC(k)^b$($)	$I_{CE}(k)^c$
2	0.0937	0.5625	0.3438	0.0937	0.0000	0.0000	8531	4.396
		0.6562	0.3438	0.0000	0.0000	0.0000		
3	0.1289	0.4922	0.3789	0.1289	0.0000	0.0000	11,730	4.396
		0.6211	0.3789	0.0000	0.0000	0.0000		
4	0.1421	0.4658	0.3921	0.1421	0.0000	0.0000	12,930	4.396
		0.6079	0.3921	0.0000	0.0000	0.0000		
5	0.1470	0.4560	0.3970	0.1470	0.0000	0.0000	13,380	4.396
		0.6030	0.3970	0.0000	0.0000	0.0000		
6	0.1489	0.4522	0.3989	0.1489	0.0000	0.0000	13,549	4.396
		0.6011	0.3989	0.0000	0.0000	0.0000		
7	0.1496	0.4508	0.3996	0.1496	0.0000	0.0000	13,612	4.396
		0.6004	0.3996	0.0000	0.0000	0.0000		
8	0.1498	0.4504	0.3998	0.1498	0.0000	0.0000	13,636	4.396
		0.6002	0.3998	0.0000	0.0000	0.0000		
9	0.1499	0.4502	0.3999	0.1499	0.0000	0.0000	13,645	4.396
		0.6001	0.3999	0.0000	0.0000	0.0000		
10	0.1500	0.4500	0.4000	0.1500	0.0000	0.0000	13,648	4.396
		0.6000	0.4000	0.0000	0.0000	0.0000		
11	0.1500	0.4500	0.4000	0.1500	0.0000	0.0000	13649	4.396
		0.6000	0.4000	0.0000	0.0000	0.0000		
12	–[d]	0.4500	0.4000	0.1500	0.0000	0.0000	–	–
		–	–	–	–	–		

[a] $S_i(k)$ = deterioration state probabilities with the exception of $S_i(2)$ being the original state probabilities. $SR_i(k)$ = rehabilitation state probabilities.
[b] $TC(k)$ = rehabilitation cycle cost.
[c] $I_{CE}(k)$ = cycle cost-effectiveness index.
[d] Not Applicable.

Table 5. Sample long-term solutions using Rehab Plan III as applied to state (5).

	Rehab. variable	State probabilities[a]						
Year (k)	$X_5(k)$	$S_1(k)SR_1(k)$	$S_2(k)SR_2(k)$	$S_3(k)SR_3(k)$	$S_4(k)SR_4(k)$	$S_5(k)SR_5(k)$	$TC(k)^b$($)	$I_{CE}(k)^c$
4	0.0293	0.3164	0.3276	0.2212	0.1055	0.0293	6152	3.810
		0.3457	0.3276	0.2212	0.1055	0.0000		
5	0.0659	0.2593	0.2912	0.2335	0.1501	0.0659	13,843	3.810
		0.3252	0.2912	0.2335	0.1501	0.0000		
6	0.0938	0.2439	0.2633	0.2260	0.1730	0.0938	19,707	3.810
		0.3377	0.2633	0.2260	0.1730	0.0000		
7	0.1081	0.2533	0.2490	0.2117	0.1779	0.1081	22,711	3.810
		0.3614	0.2490	0.2117	0.1779	0.0000		
8	0.1112	0.2711	0.2460	0.1992	0.1725	0.1112	23,343	3.810
		0.3823	0.2460	0.1992	0.1725	0.0000		
9	0.1078	0.2867	0.2493	0.1919	0.1643	0.1078	22,647	3.810
		0.3945	0.2493	0.1919	0.1643	0.0000		
10	0.1027	0.2959	0.2544	0.1894	0.1576	0.1027	21,567	3.810
		0.3986	0.2544	0.1894	0.1576	0.1027		
11	0.0985	0.2989	0.2587	0.1901	0.1538	0.0985	20,678	3.810
		0.3974	0.2587	0.1901	0.1538	0.0985		
12	–[d]	0.2981	0.2610	0.1921	0.1527	0.0961	–	–
		–	–	–	–	–		

[a] $S_i(k)$ = deterioration state probabilities with the exception of $S_i(4)$ being the original state probabilities. $SR_i(k)$ = rehabilitation state probabilities.
[b] $TC(k)$ = rehabilitation cycle cost.
[c] $I_{CE}(k)$ = cycle cost-effectiveness index.
[d] Not Applicable.

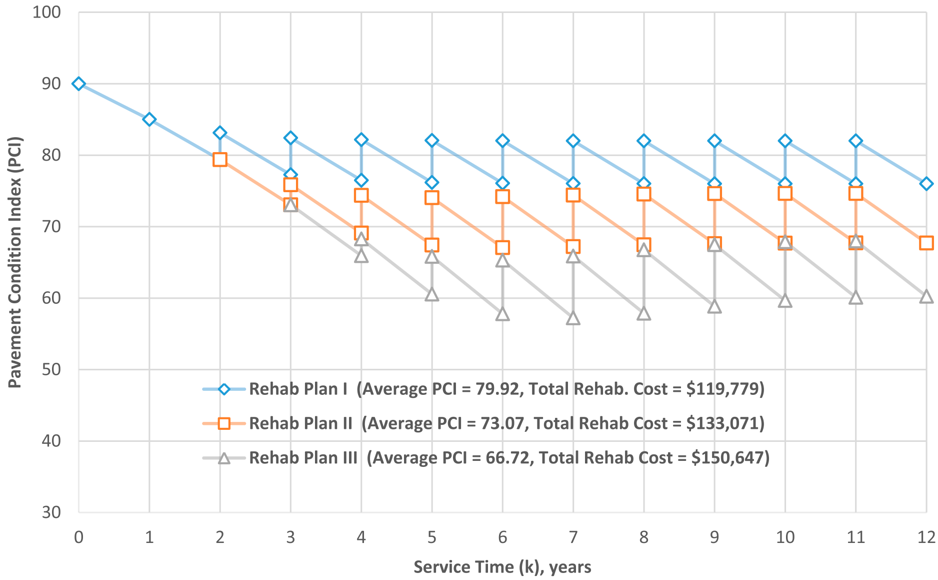

Figure 3. Sample long-term performance curves using individual treatment strategies.

of 'better roads at lower costs'. The sample results presented in case studies I & II only included major rehabilitation cost, however the corresponding life-cycle cost is expected to exponentially increase as pavement life-cycle performance decreases when considering routine maintenance and added user costs (Abaza, 2017).

5. Conclusions and recommendations

The sample results presented in both case studies have indicated the simplicity and efficacy of the proposed pavement rehabilitation model in yielding optimal long-term schedules. The derived optimal schedules simply identify the types and amounts of rehabilitation treatments to be annually carried out over the analysis period. However, the specific pavement sections to be rehabilitated in the relevant condition states need to be identified from field inspection. The sample results also indicated the significance of selecting the appropriate annual budget that best meets the requirement of any highway agency. Typically, highway agencies would like to sustain a steady state pavement condition over time, therefore the proposed model can help identify the annual budget needed to achieve a specified average long-term PCI value. It can also help highway agencies in investigating the long-term cost-effectiveness of applying individual rehabilitation treatments as demonstrated in case study II.

The sample results have been obtained using a Markov chain with 5 condition states (m), and a transition probability matrix with only two state transitions. This allowed for the inclusion of at least 4 potential major rehabilitation treatments applied to states (**2**–**5**) with an improvement outcome to state (**1**). This represents a key simplification because it incorporates a minimum number of rehabilitation variables while simulating the actual rehabilitation practices implemented by most countries around the World. The use of reliable transition matrices ($\boldsymbol{P}$ & $\boldsymbol{MP}$) is essential for obtaining dependable optimal solutions. For sample presentation purposes, a simple procedure has been proposed to estimate the original and modified transition matrices ($\boldsymbol{P}$ & $\boldsymbol{MP}$) mainly as a function of the initial transition probability ($P_{1,2}$), but generally these matrices should be estimated from historical pavement performance records or based on experience and engineering judgment. The other input data are readily available to highway agencies. However, the proposed rehabilitation model is recommended for small pavement networks and individual large projects wherein the rehabilitation work can be carried out within 1–2 months to be equally split around the *k*th year as outlined earlier to minimise any unaccounted for pavement deterioration.

The presented performance curves are developed based on a single ($P_{1,2}$) value. However, stochastic uncertainty should be investigated to establish upper, mean and lower-limit performance curves using upper, mean and lower-limit ($P_{1,2}$) values, respectively, for a specified confidence level. This requires the initial transition probability ($P_{1,2}$) to be obtained for a sample of pavement projects with similar material characteristics and loading conditions. Therefore, uncertainty analysis can provide probabilistic-based outcomes rather than a single deterministic solution.

The proposed model has three main advantages compared to other similar models. Firstly, it provides reliable long-term performance curves with vertical improvement rises that accurately reflect the amount of rehabilitation work performed. Secondly, it applies an effective decision-making policy that maximises the annual vertical improvement while minimises relevant rehabilitation cost. Thirdly, the model's simplicity in relation to the computations involved and data requirements as it deals with pavement proportions rather than individual road segments, thus making it equally useful for teaching in a pavement engineering course at the undergraduate/master level.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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