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Empirical-Markovian approach for estimating the flexible pavement structural capacity: Caltrans method as a case study

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A B S T R A C T

An Empirical-Markovian approach is proposed to estimate the pavement structural capacity as a function of key stochastic and design parameters. The key stochastic parameters are the initial and terminal deterioration transition probabilities typically estimated from pavement distress records. These two transition probabilities have a major impact on the pavement performance trend predicted using Markovian processes. In addition, typical pavement design factors are included as related to traffic loadings, materials properties, and climate conditions. In particular, two distinct Empirical-Markovian models are developed to estimate the pavement structural capacity in terms of relative strength indicators such as the structural number (SN) and gravel equivalent (GE). The first model can estimate the structural capacity based on the initial transition probability and relevant design parameters, while the second one deploys the terminal transition probability along with other design parameters. The recommendation is to use the higher of the two structural capacity values estimated for a particular pavement project. The sample models presented based on the California Department of Transportation (Caltrans) design method have resulted in good model fittings as demonstrated by the various deployed statistics and error analysis, thus indicating the usefulness of the proposed Empirical-Markovian approach in estimating the pavement structural capacity for rehabilitation and design purposes. The model exponents can be obtained from solving a linear system of equations using data from a small project sample or solving a multi-variable linear regression model when a large road sample is available. The latter case provided sample generalized models with statistics indicating their high significance.

Introduction

Structural capacity is a term often used to quantify the overall relative strength associated with a particular pavement structure. There are several strength parameters used to describe the structural adequacy of the various materials used to make up the pavement structure such as the resilient modulus for granular materials, dynamic modulus for hot asphalt mixtures, and modulus of rupture for Portland cement mixtures. However, there has always been a need for a single parameter that can describe the overall relative strength associated with the entire pavement structure. This single parameter has been historically referred to as the pavement structural capacity. The two most popular examples of structural capacity indicators

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are the structural number (SN) used in the former AASHTO design method for flexible pavement (AASHTO, 1993), and the gravel equivalent (GE) that is still being used in the recent Caltrans design manual for flexible pavement design (Caltrans, 2017). These two structural capacity indicators have been used not just for design of new pavement structures, but often used in pavement rehabilitation applications such as the estimation of required overlay thicknesses associated with both plain overlay, and cold milling and overlay (Abaza, 2018a, Abaza and Murad, 2019). Abaza and Murad (2019) proposed an empirical approach to estimate the reduced (i.e. effective) asphaltic strength used in computing the asphaltic overlay thickness. It is estimated based on the reduced asphaltic surface thickness defined as a function of the existing asphaltic surface thickness and asphaltic remaining strength factor derived from the pavement long-term performance. However, most highway agencies use surface deflections to evaluate the in-service pavement structural capacity at the project level (Romanoschi and Metcalf, 1999, Gedafa et al., 2010, Elbagalati et al., 2016, Saleh, 2016, Nasimifar et al., 2019).

A literature review reveals that there has been considerable research efforts devoted for estimating/predicting the pavement structural capacity, but mostly using pavement surface deflections. One study proposed a simple approach to relate the surface deflections obtained from the falling weight deflectometer (FWD) to the performance of pavement structures as expressed by the structural number (SN). The study reported adequate correlation between the (FWD) deflection and (SN) data considering low volume roads (Romanoschi and Metcalf, 1999). Another study was performed to examine the feasibility and reliability of using the rolling weight deflectometer (RWD) to measure surface deflections for the purpose of computing the effective structural numbers (SN_{eff}) at the network level. The study concluded that the (SN_{eff}) obtained from both (RWD) and (FWD) deflections are statistically alike (Gedafa et al., 2010). Another similar study proposed a model to predict the pavement structural capacity (SN) as a function of surface deflections measured using the rolling weight deflectometer (RWD). The model can be used to predict (SN) at an interval length of 0.16 km (0.1 mi) for pavement management applications (Elbagalati et al., 2016). Another relevant study investigated four different performance scales derived from the deflection bowls to evaluate and rank the pavement structural capacity. In particular, the area ratio, deflection ratio and their normalized values were investigated using both field measured and computer simulated deflection data (Saleh, 2016). A recent study deployed the traffic speed deflectometer (TSD) to continuously measure the pavement surface deflections at the network level. The study proposed a practical approach to estimate (SN_{eff}) from (TSD) deflection data for pavement management applications (Nasimifar et al., 2019).

Therefore, it seems that pavement structural capacity is dominantly estimated from deflection testing, which is not readily available in most developing countries. The main objective of estimating the pavement structural capacity is to evaluate the pavement condition for pavement management applications. Predictive models have been developed to estimate the structural capacity over time. Deterministic models such as regression-based ones have been proposed for this purpose. In addition, probabilistic models have been developed to model pavement performance using main structural capacity indicators. The most widely used probabilistic models are the Markovian-based ones. Different forms of the Markov model have been used, however the most popular one is the discrete-time Markov model with both homogeneous and non-homogeneous chains (Zhang and Gao, 2012, Meidani and Ghanem, 2015, Saha et al., 2017, Abaza, 2018a, Daniel et al., 2021, Pérez-Acebo et al., 2019).

In this paper, an Empirical-Markovian model is proposed to estimate the pavement structural capacity at the network level mainly deploying key stochastic and design parameters. The key stochastic parameters used are the initial and terminal deterioration transition probabilities, which can be estimated from historical records of pavement distress. These two stochastic parameters greatly influence the pavement performance trend (Abaza, 2018a, Abaza, 2018b). The pavement structural capacity is to be estimated using popular relative strength indicators such as the structural number (SN) and gravel equivalent (GE) (AASHTO, 1993, Caltrans, 2017). The proposed Empirical-Markovian model can be solved as a system of linear equations with a minimum number of pavement projects or as a multi-variable linear regression model with a larger number of pavement projects.

Overview of discrete-time Markov model

The discrete-time Markov model is the most widely used stochastic model in predicting pavement performance (Zhang and Gao, 2012, Meidani and Ghanem, 2015, Saha et al., 2017, Abaza, 2018a, Pérez-Acebo et al., 2018, Daniel et al., 2021). The main model components are: 1) discrete number of condition states (m), 2) discrete number of state transitions (n) representing the length of analysis period, 3) discrete-time interval indicating the transition length, 4) state probabilities denoting pavement proportions in the various condition states at any given transition, and 5) transition probabilities defining the probabilities of transiting from one state to the other states during one transition. The transition probabilities make up what is known as the transition probability matrix (**P**). There are two general forms of the discrete-time Markov model. The first one applies a homogeneous (**P**) wherein the transition probabilities remain constant over the analysis period as indicated by Eq. (1).

$$\mathbf{S}(\mathbf{n}) = \mathbf{S}(0)\mathbf{P}^{(\mathbf{n})} \tag{1}$$

where: $\mathbf{S}(\mathbf{n}) = [S_1(n), S_2(n), S_3(n), ..., S_m(n)]$

$\mathbf{S}(0) = [S_1(0), S_2(0), S_3(0), ..., S_m(0)]$

$$\sum_{i=1}^{m} S_i(k) = 1.0 \quad (k =, 0, 1, 2, 3, \cdots., n)$$

The second model incorporates a non-homogeneous, **P(k)**, in which the transition probabilities can be different for each transition as defined in Eq. (2). The main objective of the Markov model as presented in Eqs. (1) and (2) is to estimate the state probabilities, **S(n)**, at the end of an analysis period comprised of (n) transitions. They are estimated from simply multiplying the initial state probability vector, **S(0)**, by the matrix (**P**) raised to the power (n) for the homogeneous case indicated by Eq. (1), or by the multiplication product of (n) distinct, **P(k)**, matrices for the non-homogeneous case defined in Eq. (2).

$$\mathbf{S}(\mathbf{n}) = \mathbf{S}(\mathbf{0}) \prod_{\mathbf{k}=1}^{\mathbf{n}} \mathbf{P}(\mathbf{k}) \tag{2}$$

State probabilities estimated at different service times can then be used to estimate the pavement performance in terms of a specified pavement condition index such as the present serviceability index (PSI), pavement condition index (PCI), and international roughness index (IRI). Researchers have used different forms of the transition matrix, however the most popular one is as presented in Eq. (3) (Abaza, 2016, Abaza, 2018a, Abaza, 2018b). The $P(k)_{i,i}$ probability along the main diagonal represents the probability of remaining in the same condition state after one transition. The $P(k)_{i,i+1}$ probability denotes the probability of transiting to the next worst state after one transition. The $P(k)_{i,i+1}$ probabilities define the pavement deterioration rates. All transition probabilities below the main diagonal represent pavement improvement rates which vanish in the absence of pavement maintenance and rehabilitation. The sum of any row in the matrix, **P(k)**, must add up to one.

$$\mathbf{P}(\mathbf{k}) = \begin{bmatrix} P(k)_{1,1} & P(k)_{1,2} & 0 & 0 & 0 & \cdots. & 0 \\ 0 & P(k)_{2,2} & P(k)_{2,3} & 0 & 0 & \cdots. & 0 \\ 0 & 0 & P(k)_{3,3} & P(k)_{3,4} & 0 & \cdots. & 0 \\ \vdots & & & & & & \\ \vdots & & & & & & \\ \vdots & & & & & & \\ 0 & 0 & \cdots. & P(k)_{m-1,m-1} & P(k)_{m-1,m} & & \\ 0 & 0 & 0 & 0 & 0 & \cdots. & 1.0 \end{bmatrix} \tag{3}$$

Therefore, only two transition probabilities [$P(k)_{i,i}$ & $P(k)_{i,i+1}$] are assumed for each condition state according to Eq. (3). This assumption mainly requires a maximum number of condition states to be valid considering the same transition length. The typical maximum number is around 10 condition states assuming one-year transition length. It was found that two transition probabilities were considered to be the critical ones in defining the shape of the pavement performance curve, namely the initial and terminal transition probabilities [$P(k)_{1,2}$ & $P(k)_{m-1,m}$] (Abaza, 2018a, Abaza, 2018b). Once these two critical ones are known for a given transition, the remaining transition probabilities can be estimated from linear interpolation. Former researches indicated that linear interpolation of remaining transition probabilities had resulted in satisfactory prediction of future pavement conditions especially in the case of parabolic performance trends as demonstrated by Fig. 1 (Abaza, 2018a, Abaza, 2018b). Different methods were proposed by researchers to estimate the deterioration transition probabilities mainly relying on pavement distress records (Mishalani and Madanat, 2002, Ortiz-García et al., 2006, Kobayashi et al., 2010, Abaza, 2016, Pérez-Acebo et al., 2018). Generally, distress records obtained from two consecutive cycles of distress assessment are required to estimate the deterioration transition probabilities, $P(k)_{i,i+1}$. The two cycles are typically separated by a transition length of one year. Abaza (2016) derived the equations for estimating the deterioration transition probabilities in the case of homogeneous Markov chain using the outcomes of two cycles of pavement distress assessment.

Prior Empirical-Markovian modelling at project level

Abaza (2018a), Abaza (2018b) deployed a general Empirical-Markovian model to predict the non-homogeneous deterioration transition probabilities, $P(k+1)_{i,i+1}$, as a function of the first-year deterioration transition probabilities, $P(k=1)_{i,i+1}$, traffic load factor (F_L), and pavement structural capacity factor (F_S) as defined in Eq. (4). The load factor accounts for pavement deterioration resulting from the progressive increase in traffic loading over time, and the structural capacity factor accounts for the deterioration caused by the progressive decrease in pavement strength over time. The two factors as defined in Eq. (4) have values greater than one. The Empirical-Markovian model presented in Eq. (4) has two exponents (A and B) to be determined from a calibration procedure based on minimization of sum of squared errors (Abaza, 2018a & Abaza, 2018b). Eq. (4) mainly estimates the non-homogeneous deterioration transition probabilities for a particular project.

$$P(k+1)_{i,i+1} = P(k)_{i,i+1} (F_L)^A (F_S)^B \quad (k = 1, 2, 3, \cdots.., n) \tag{4}$$

Where: $F_L = \frac{W(k+1)}{W(k)}$ and $F_S = \frac{S(k)}{S(k+1)}$

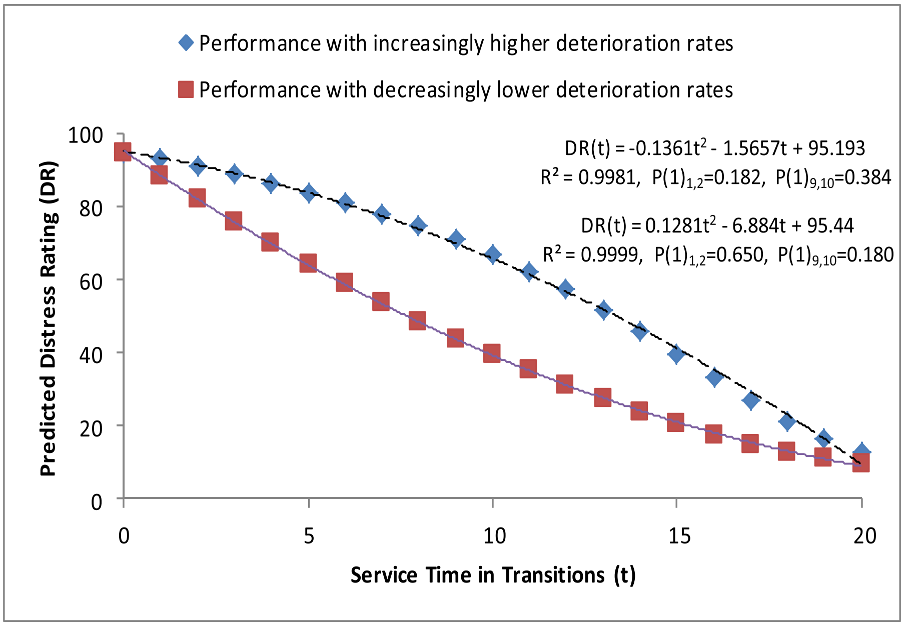

Fig. 1. Sample pavement performance curves predicted using non-homogenous Markov chain.

$P(k+1)_{i,i+1}$ = transition probability associated with the (k + 1)th transition representing the deterioration rate of transiting from state (**i**) to the next worst state (**i + 1**),

$W(k+1)$ = 80kN equivalent single axle load applications expected to travel the pavement during the $(k+1)^{th}$ transition, and

$S(k+1)$ = pavement structural capacity at the $(k+1)^{th}$ transition.

The above model was mainly used to estimate the non-homogeneous deterioration transition probabilities for the purpose of predicting pavement performance at the project level. Pavement performance as represented by a relevant deterioration curve, Fig. 1, was used in several pavement management applications such as estimating future overlay thicknesses, and developing optimal rehabilitation strategies (Abaza, 2018a, Abaza, 2018b, Abaza and Murad, 2019).

Abaza (2018a) proposed a model similar to Eq. (4) but for the purpose of estimating the heterogeneous transition probabilities associated with potential rehabilitation strategies. The model is a function of the transition probabilities associated with original pavement, future traffic loadings, and existing structural capacity. The model is developed under the assumption that the performances associated with both original and rehabilitated pavements are similar. Then, potential rehabilitation strategies have been evaluated based on their costs and performances to yield the optimal rehabilitation strategy at the project level. Abaza (2018b) proposed a model, Eq. (5), based on Eq. (4) to estimate the structural capacity of overlaid pavement, $SN_1(t)$, as a function of the structural capacity associated with original pavement, $SN_1(0)$, annual traffic growth rate (r), deterioration transition probabilities of original pavement for the 1st transition, $P(1)_{i,i+1}$, deterioration transition probabilities of overlaid pavement for the (t + 1) transition, $P(t+1)_{i,i+1}$, rehabilitation scheduling time (t), and two calibration constants (A &B). The structural capacity is defined in terms of the structural number associated with the asphaltic layer (SN_1) as defined in Eq. (5). Two solutions of Eq. (5) can be derived based on the initial transition probabilities, $P(1)_{1,2}$ & $P(t+1)_{1,2}$, and terminal transition probabilities, $P(1)_{m-1,m}$ & $P(t+1)_{m-1,m}$. Eq. (5) is mainly used for rehabilitation purposes at the project level. Calibration of Eq. (5) was sought by minimizing sum of squared errors using trial and error approach.

$$SN_1(t) = SN_1(0)(1+r)^{t(A/B)}\left(\frac{P(1)_{i,i+1}}{P(t+1)_{i,i+1}}\right)^{1/B} \qquad (i = 1, 2, \ldots, m-1) \tag{5}$$

In this paper, it is proposed to deploy an approach similar to the one used in developing Eq. (5) but to be used for both rehabilitation and design purposes at the network level. It can estimate the structural capacity associated with the entire pavement structure, and it can incorporate as many pavement design factors as needed. Therefore, the proposed approach has the potential to be incorporated to the current methods of empirical pavement design. In essence, any empirical design method can be upgraded to become an Empirical-Markovian method, thus accounting for the probabilistic nature of pavement performance. Calibration of the proposed model is to be attempted using either matrix algebra for a roadway network with limited size or multi-variable linear regression analysis for a roadway network with unlimited size.

Proposed Empirical-Markovian modelling at network level

Structural capacity as used in pavement rehabilitation and design applications is typically a function of several factors. However, the most widely used factors in pavement design include the traffic load factor, climate factor, subgrade strength, and pavement performance. Eq. (6) presents an Empirical-Markovian model similar to the one presented in Eq. (4), but it deploys the four mentioned factors affecting pavement structural capacity. The pavement performance factor is stochastically defined using the initial and terminal transition probabilities, $P(j)_{1,2}$ and $P(j)_{m-1,m}$. According to Eq. (6), the initial transition probability, $P(j)_{1,2}$, associated with the jth project is to be estimated as a function of the initial transition probability, $P(b)_{1,2}$, associated with the base project multiplied by the four outlined design factors. The traffic load factor (W_j/W_b), raised to the (A_1) power, implies that $P(j)_{1,2}$ increases compared to $P(b)_{1,2}$ as the traffic loading (W_j) becomes larger than the loading associated with the base project (W_b). The pavement strength factor (S_b/S_j), raised to the (B_1) power, indicates that $P(j)_{1,2}$ decreases compared to $P(b)_{1,2}$ as the structural capacity associated with the jth project (S_j) gets larger than the structural capacity associated with the base project (S_b). Likewise, the factors associated with climate condition (M_b/M_j) and subgrade strength (R_b/R_j) are introduced in a way similar to the strength factor with higher values of (M) and (R) indicating improved climate and subgrade conditions. The base project is practically selected as a representative project that exhibits standard design and performance characteristics, but mathematically it can be any project without affecting the overall solution outcomes as demonstrated later.

$$P(j)_{1,2} = P(b)_{1,2}\left(\frac{W_j}{W_b}\right)^{A_1}\left(\frac{S_b}{S_j}\right)^{B_1}\left(\frac{M_b}{M_j}\right)^{C_1}\left(\frac{R_b}{R_j}\right)^{D_1} \tag{6}$$

Eq. (7) is constructed in a similar way as Eq. (6) but using the terminal transition probability, $P(j)_{m-1,m}$, with four different exponents. Both Eqs. (6) and (7) are to be solved for the purpose of estimating the pavement structural capacity as a function of all other involved factors and model exponents. The initial and terminal transition probabilities used in Eqs. (6) and (7) can be estimated for the current/present transition ($k = 1$) assuming homogeneous Markov chain. Abaza (2016) derived the backward solutions from Eqs. (1) & (3) to compute these transition probabilities. Two consecutive cycles of pavement distress assessments are required to estimate the homogeneous transition probabilities.

$$P(j)_{m-1,m} = P(b)_{m-1,m}\left(\frac{W_j}{W_b}\right)^{A_2}\left(\frac{S_b}{S_j}\right)^{B_2}\left(\frac{M_b}{M_j}\right)^{C_2}\left(\frac{R_b}{R_j}\right)^{D_2} \tag{7}$$

The estimation of the model exponents requires the availability of relevant performance and design data for a project sample with adequate size. The required size of project sample to solve Eqs. (6) and (7) is 5 projects. One project is to be used as a base project, and the other four are used to solve four equations with four unknowns, namely the model four exponents. The solution of Eq. (6) can be obtained from linear algebra by solving a system of linear equations. Eq. (6) is rearranged as indicated by Eq. (8) for the purpose of constructing the relevant system of linear equations. An equation similar to Eq. (8) can also be constructed using the terminal transition probabilities and different set of model exponents (A_2, B_2, C_2, D_2).

$$\left(\frac{W(j)}{W_b}\right)^{A_1}\left(\frac{S_b}{S(j)}\right)^{B_1}\left(\frac{M_b}{M(j)}\right)^{C_1}\left(\frac{R_b}{R(j)}\right)^{D_1} = \frac{P(j)_{1,2}}{P(b)_{1,2}} \tag{8}$$

Solving Eq. (8) requires converting the corresponding non-linear equation to a linear one by applying the logarithmic function to both sides of the equation. Then, a linear system of four equations can be constructed as defined in Eq. (9) which can provide exact solutions to Eq. (8). Again, a matrix equation similar to Eq. (9) can be constructed for the model presented in Eq. (7) based on the terminal transition probabilities ($\mathbf{FX_2 = E_2}$).

$$\mathbf{FX}_1 = \mathbf{E}_1 \tag{9}$$

Where:

$$E_1 = \begin{pmatrix} \Delta\log P(1)_{1,2} \\ \Delta\log P(2)_{1,2} \\ \Delta\log P(3)_{1,2} \\ \Delta\log P(4)_{1,2} \end{pmatrix}, X_1 = \begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix}$$

$$F = \begin{pmatrix} \Delta\log W_1 & \Delta\log S_1 & \Delta\log M_1 & \Delta\log R_1 \\ \Delta\log W_2 & \Delta\log S_2 & \Delta\log M_2 & \Delta\log R_2 \\ \Delta\log W_3 & \Delta\log S_3 & \Delta\log M_3 & \Delta\log R_3 \\ \Delta\log W_4 & \Delta\log S_4 & \Delta\log M_4 & \Delta\log R_4 \end{pmatrix}$$

$$\Delta\log P(j)_{1,2} = \log P(j)_{1,2} - \log P(b)_{1,2} \quad (j = 1, 2, 3, 4)$$

$$\Delta logW_j = logW(j) - logW_b \quad (j = 1,2,3,4)$$

$$\Delta logS_j = logS_b - log\,S(j) \quad (j = 1,2,3,4)$$

$$\Delta logM_j = logM_b - logM(j) \quad (j = 1,2,3,4)$$

$$\Delta logR_j = logR_b - log\,R(j) \quad (j = 1,2,3,4)$$

Once the relevant matrices (**E** and **F**) are developed from available stochastic and design data associated with a sample of five projects with one being used as a base project, then a linear system of four equations can be solved to obtain the model exponent values (A_1, B_1, C_1, D_1) as represented by the vector ($\mathbf{X_1}$). Therefore, the solution vector ($\mathbf{X_1}$) is obtained from multiplying the matrix inverse ($\mathbf{F^{-1}}$) by the vector ($\mathbf{E_1}$) as defined in Eq. (10) based on the initial transition probabilities. Similarly, the solution vector ($\mathbf{X_2}$) can be obtained from multiplying the matrix inverse ($\mathbf{F^{-1}}$) by the vector ($\mathbf{E_2}$) in the case of terminal transition probabilities.

$$\mathbf{X}_1 = \mathbf{F}^{-1}\mathbf{E}_1 \quad (10)$$

Once the model exponents are determined, the two models similar to the ones presented in Eqs. (6) and (7) can be solved to yield the pavement structural capacity, S(j), in terms of the other relevant factors. The higher of the two obtained S(j) values shall be used in pavement rehabilitation and design applications. In practice, the traffic loading, W(j), is typically represented by the 80kN equivalent single axle load (ESAL) applications, and subgrade strength, R(j), is normally defined using the resilient modulus. Whereas the pavement structural capacity, S(j), is commonly classified using the structural number (SN) as implemented in the AASHTO empirical design method (AASHTO, 1993), and gravel equivalent (GE) as deployed in the Caltrans design method (Caltrans, 2017). However, the climate condition has been characterized using factors such as average annual air temperature, annual rainfall amount, and quality of provided roadway drainage system.

Application to California design method (Caltrans)

The previously outlined Empirical-Markovian models can be applied to the design method of flexible pavement deployed by the California Department of Transportation (Caltrans, 2017). The Caltrans method is essentially an empirical one that has been in use for many years, and it is still being used at least to supplement the new Mechanistic-Empirical (ME) approach proposed by Caltrans. The empirical method can be used to yield a trial design needed to initiate the ME design procedure (Caltrans, 2017). According to the Caltrans method for flexible pavement design, the structural capacity associated with a pavement structure is represented by the gravel equivalent (GE). The gravel equivalent has mainly been related to two design factors, namely the traffic index (TI) and subgrade strength defined using the resistance value (R value). The traffic index (TI) is determined based on the design 80kN ESAL using Eq. (11).

$$TI = 9.0 \times \left(\frac{ESAL}{10^6}\right)^{0.119} \quad (11)$$

The design gravel equivalent (GE) can then be computed as a function of traffic index (TI) and subgrade resistance value (R) as indicated by Eq. (12). Therefore, there are only two simple equations needed to determine the structural capacity (GE) of any pavement structure according to the Caltrans design method (Caltrans, 2017). The (R) value is estimated on a scale of 100 points similar to the California Bearing Ratio (CBR). However, the test procedure for estimating the (R) value is totally different from the one used in obtaining the (CBR) value.

$$GE = 0.0032 \times TI \times (100 - R) \quad (12)$$

The earlier outlined Empirical-Markovian approach can now be applied to the Caltrans design method as presented in Eq. (13). The Empirical-Markovian approach essentially converts the Caltrans design method from an empirical method into an Empirical-Markovian one by incorporating the two most influential stochastic parameters, namely the initial and terminal deterioration transition probabilities, $P(j)_{1,2}$ and $P(j)_{m-1,m}$, which have been found to greatly affect the pavement performance trend (Abaza, 2018a, Abaza, 2018b). A similar equation can also be presented based on the terminal transition probabilities and different set of model exponents.

$$\left(\frac{TI(j)}{TI_b}\right)^{A_1}\left(\frac{GE_b}{GE(j)}\right)^{B_1}\left(\frac{R_b}{R(j)}\right)^{C_1} = \frac{P(j)_{1,2}}{P(b)_{1,2}} \quad (13)$$

Similar to Eq. (8), the solution of Eq. (13) can be derived from converting it into an equivalent linear system of equations. The corresponding linear system of equations can be solved to yield the values of the model exponents as defined in Eq. (14). There is only a total of three exponents (A_1, B_1, C_1) to be computed, therefore the stochastic and design data for a sample of four projects are required to formulate and solve two linear systems. The first system is as presented in Eq. (14) while the second one is based on the terminal transition probabilities yielding the exponents (A_2, B_2, C_2). One of the four projects is to be used as a base project, which can be any project without affecting the overall solution outcomes.

$$\mathbf{X}_1 = \mathbf{F}^{-1}\mathbf{E}_1 \tag{14}$$

Where:

$$\mathbf{E}_1 = \begin{pmatrix} \Delta\log P(1)_{1,2} \\ \Delta\log P(2)_{1,2} \\ \Delta\log P(3)_{1,2} \end{pmatrix}, \; \mathbf{X}_1 = \begin{pmatrix} A_1 \\ B_1 \\ C_1 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} \Delta\log TI_1 & \Delta\log GE_1 & \Delta\log R_1 \\ \Delta\log TI_2 & \Delta\log GE_2 & \Delta\log R_2 \\ \Delta\log TI_3 & \Delta\log GE_3 & \Delta\log R_3 \end{pmatrix}$$

$$\Delta\log P(j)_{1,2} = \log P(j)_{1,2} - \log P(b)_{1,2} \quad (j = 1, 2, 3)$$

$$\Delta\log TI_j = \log TI(j) - \log TI_b \quad (j = 1, 2, 3)$$

$$\Delta\log GE_j = \log GE_b - \log GE(j) \quad (j = 1, 2, 3)$$

$$\Delta\log R_j = \log R_b - \log R(j) \quad (j = 1, 2, 3)$$

Once the model exponents (A_1, B_1, C_1) are computed from solving the linear system presented in Eq. (14), the model outlined in Eq. (15) can be used to estimate the pavement structural capacity (GE) as a function of the other involved stochastic and design parameters defined in Eq. (13). A model similar to Eq. (15) can also be developed to estimate the (GE) based on the terminal transition probabilities. Therefore, two different models can provide two different estimates of the required structural capacity, $GE(j)_{1,2}$ and $GE(j)_{m-1,m}$, for the j^{th} project with the maximum of the two values should be used in any rehabilitation and design applications. The design parameters associated with the base project are labeled as (GE_b), (TI_b) and (R_b) for gravel equivalent, traffic index, and subgrade resistance value, respectively.

$$GE(j)_{1,2} = GE_b \left(\frac{P(b)_{1,2}}{P(j)_{1,2}}\right)^{(1/B_1)} \left(\frac{TI(j)}{TI_b}\right)^{(A_1/B_1)} \left(\frac{R_b}{R(j)}\right)^{(C_1/B_1)} \tag{15}$$

The Empirical-Markovian models presented in Eqs. (10) and (14) yield exact solutions based on a small project sample, therefore the practical use of these equations is somewhat limited. A more practical approach is to use a larger project sample, but the solutions in this case are to be obtained from the minimization of sum of squared errors. In the latter case, the solutions are to be essentially derived from the multi-variable linear regression analysis. Eq. (16) is proposed to be equivalent to the one presented in Eq. (15), however the stochastic and design parameters associated with the base project (i.e. $P(b)_{1,2}$, GE_b, R_b, TI_b) are replaced by their corresponding average parameter values obtained from the project sample [i.e. $\overline{P}(j)_{1,2}, \overline{GE}(j), \overline{R}(j), \overline{TI}(j)$], respectively. Again, a model similar to Eq. (16) can be constructed based on the terminal transition probabilities and model exponents (B_1, B_2, B_3).

$$GE(j)_{1,2} = \overline{GE}(j) \left(\frac{\overline{P}(j)_{1,2}}{P(j)_{1,2}}\right)^{A_1} \left(\frac{\overline{R}(j)}{R(j)}\right)^{A_2} \left(\frac{TI(j)}{\overline{TI}(j)}\right)^{A_3} \tag{16}$$

The application of the multi-variable linear regression analysis requires transforming the non-linear Eq. (16) into an equivalent linear one by applying the logarithmic function to both sides of the equation. Then, appropriate statistical software can be used to solve the multi-variable linear regression problem. The solution provides estimates of the model exponents along with several statistics that describe the significance of the model derived as demonstrated in the sample presentation.

Sample presentation

The proposed Empirical-Markovian approach has been investigated using a road sample consisting of 12 minor rural highways. These minor highways connect villages in the Nablus District, West Bank, Palestine, with the nearby major highways. Table 1 provides the required stochastic and design parameters for the investigated road sample. The pavement structures associated with these roads mainly consist of two layers made up of asphalt concrete surface placed over aggregate base layer. The roads are mainly classified as low volume roads as evidenced from the associated design ESAL. The same road sample had been used before in a former publication (Abaza, 2011).

The roads provided in Table 1 are listed in an ascending order with respect to the initial transition probability, $P(j)_{1,2}$. The initial and terminal transition probabilities [$P(j)_{1,2}$ and $P(j)_{9,10}$] were estimated from conducting two consecutive cycles of pavement distress assessment separated by one-year time interval, and using a homogeneous Markov chain with 10 condition states (m). These transition probabilities are mainly corresponding to the current/present transition (i.e. k = 1). The rel-

Table 1
Key stochastic and pavement design parameters for project sample.

Road No. (j)	$P(j)_{1,2}$	$P(j)_{9,10}$	R(j)	ESAL ($x10^6$)	TI(j)	GE(j) (ft)
1	0.18	0.38	46	470	8.23	1.422
2	0.25	0.36	41	320	7.86	1.484
3	0.26	0.69	30	350	7.94	1.779
4	0.29	0.51	27	180	7.34	1.715
5	0.32	0.74	24	250	7.63	1.856
6	0.35	0.31	40	520	8.33	1.599
7	0.38	0.49	28	650	8.55	1.970
8	0.39	0.75	21	700	8.63	2.182
9	0.47	0.85	10	620	8.50	2.448
10	0.52	0.49	26	580	8.44	1.999
11	0.58	0.52	16	660	8.57	2.304
12	0.64	0.50	13	750	8.70	2.422
Average	0.386	0.549	26.83	NA	8.23	1.932

evant distress measurements and detailed calculations of the sample initial and terminal transition probabilities were provided in a former publication (Abaza, 2011). The traffic index (TI) and gravel equivalent (GE) for each road have been computed from the design ESAL and resistance value (R) using Eqs. (11) and (12), respectively.

The solutions for Eq. (13) have been obtained from solving a linear system of 3 equations as outlined in Eq. (14). Each linear system requires four projects to be formulated and solved to yield the model three exponents (A_1, B_1, C_1). One project has been used as a base project. The same results are obtained regardless of which project is used as a base project. Therefore, the road sample provided in Table 1 has been divided into three project sets with each consisting of four projects as provided in Table 2. It can be noted from Table 2 that the values associated with the three exponents (A_1, B_1, C_1) are highly variable, however the use of any model should be based on the value associated with the initial transition probability. For example, the first model is applicable for initial transition probability in the range of (0.18 – 0.29), while the second one is appropriate for $P(j)_{1,2}$ value in the range of (0.32 – 0.39), and so on. Results similar to the ones provided in Table 2 can be obtained when replacing the initial transition probabilities by the terminal transition probabilities.

As outlined earlier, a more practical approach is to use multi-variable linear regression modelling wherein the entire project sample is used in estimating the model exponents indicated by Eq. (16). In this approach, the average values associated with key stochastic and design parameters are used in lieu of the corresponding values associated with a base project. The multi-variable regression model associated with the initial transition probability is provided in Eq. (17), while the one associated with the terminal transition probability is presented in Eq. (18). It can be noticed that the first two exponents are very

Table 2
Exact solutions for sample project sets based on initial transition probabilities.

Project Set No. 1 [$P(j)_{1,2}$ = 0.18 – 0.29]				
	Project No. (j)			
Parameter	1	2	3	4
$P_{1,2}$	0.18	0.25	0.26	0.29
TI	8.23	7.86	7.94	7.34
GE	1.42	1.48	1.78	1.72
R	46	41	30	27
Solution:	**A_1 = -9.839**	**B_1 = -7.268**	**C_1 = -3.774**	
Project Set No. 2 [$P(j)_{1,2}$ = 0.32 – 0.39]				
	Project No. (j)			
Parameter	5	6	7	8
$P_{1,2}$	0.32	0.35	0.38	0.39
TI	7.63	8.33	8.55	8.63
GE	1.86	1.60	1.97	2.18
R	24	40	28	21
Solution:	**A_1 = 1.190**	**B_1 = -0.397**	**C_1 = -0.088**	
Project Set No. 3 [$P(j)_{1,2}$ = 0.47 – 0.64]				
	Project No. (j)			
Parameter	9	10	11	12
$P_{1,2}$	0.47	0.52	0.58	**0.64**
TI	8.5	8.44	8.57	**8.7**
GE	2.45	2.00	2.3	**2.42**
R	10	26	16	**13**
Solution:	**A_1 = 7.992**	**B_1 = –1.833**	**C_1 = –0.554**	

similar in both models. However, the 3rd exponent (B_3 = 1.198) in Eq. (18) is substantially larger than the corresponding one (A_3 = 0.505) in Eq. (17). This is essentially contributed to the higher values associated with the terminal transition probabilities compared to the values associated with the initial transition probabilities, an indication of superior performance [i.e. $P(j)_{1,2} < P(j)_{2,3} < \ldots < P(j)_{9,10}$] (Abaza, 2018a, Abaza, 2018b). The other constants provided in both models represent the average values associated with the stochastic and design parameters as provided at the bottom of Table 1.

$$GE(j)_{1,2} = 1.932\left(\frac{0.386}{P(j)_{1,2}}\right)^{-0.158}\left(\frac{26.83}{R(j)}\right)^{0.168}\left(\frac{TI(j)}{8.23}\right)^{0.505} \tag{17}$$

$$GE(j)_{9,10} = 1.932\left(\frac{0.549}{P(j)_{9,10}}\right)^{-0.154}\left(\frac{26.83}{R(j)}\right)^{0.166}\left(\frac{TI(j)}{8.23}\right)^{1.198} \tag{18}$$

In addition, the statistics obtained from the commercially used regression software indicate that both models presented in Eqs. (17) and (18) are highly significant. The coefficient of determination (R-squared) associated with Eq. (17) is 93.9% with 0.053 standard error of estimate. The model F-statistic is 41.11 which makes it significant at more than 99.99% confidence level. The t-statistics associated with the model three exponents are 3.04, 4.88 and 1.41, which make them significant at 98.4%, 99.88% and 80.30%, respectively. Similarly, Eq. (18) is associated with 91.8% R-squared and 0.061 standard error of estimate. The model F-statistic is 29.97; thus, making it significant at more than 99.99%. The t-statistics associated with the model three exponents are 2.20, 3.89 and 2.92 making them significant at 94.14%, 99.54% and 98.06% confidence levels, respectively.

The predictive strength of Eqs. (17) and (18) has also been investigated by analyzing the associated residuals as provided in Tables 3 and 4, respectively. Both models have been used to compute the estimated values of gravel equivalent, GE″(j), using the data provided in Table 1. Table 3 provides the estimated GE″(j) values computed using Eq. (17) based on the initial transition probabilities and relevant design parameters. The errors (residuals) are calculated as the difference between the

Table 3
Error analysis associated with regression solution considering initial transition probabilities.

Project No. (j)	GE″(j)[a]	GE(j)	E(j)[b]	E(j)[c] (%)
1	1.564	1.422	0.142	9.99
2	1.641	1.484	0.157	10.58
3	1.749	1.779	−0.030	1.69
4	1.741	1.715	0.026	1.52
5	1.839	1.856	−0.017	0.92
6	1.790	1.599	0.191	11.94
7	1.951	1.970	−0.019	0.96
8	2.065	2.182	−0.117	5.36
9	2.391	2.448	−0.057	2.33
10	2.062	1.999	0.063	3.15
11	2.294	2.304	−0.010	0.43
12	2.431	2.422	0.009	0.37

[a] Estimated GE value using Eq. (17).
[b] Error in GE value estimated as [GE″(j) – GE(j)].
[c] Error percentage estimated as [E(j)/GE(j)] × 100, with average absolute error % equals to 4.1%.

Table 4
Error analysis associated with regression solution considering terminal transition probabilities.

Project No. (j)	GE″(j)[a]	GE(j)	E(j)[b]	E(j)[c] (%)
1	1.670	1.422	0.248	17.44
2	1.597	1.484	0.113	7.61
3	1.882	1.779	0.103	5.79
4	1.664	1.715	−0.051	2.97
5	1.882	1.856	0.026	1.40
6	1.680	1.599	0.081	5.07
7	1.974	1.970	0.004	0.20
8	2.235	2.182	0.053	2.43
9	2.531	2.448	0.083	3.39
10	1.967	1.999	−0.032	1.61
11	2.192	2.304	−0.112	4.86
12	2.296	2.422	0.126	5.20

[a] Estimated GE value using Eq. (18).
[b] Error in GE value estimated as [GE″(j) – GE(j)].
[c] Error percentage estimated as [E(j)/GE(j)] × 100, with average absolute error % quals to 4.83%.

estimated GE″(j) values and actual (GE) values. The absolute errors have been calculated as a percentage from the actual (GE) values. It can be noted that the error values and percentages are relatively low with 4.1% average absolute error. Table 4 provides similar error data determined using Eq. (18) based on the terminal transition probabilities. Again, it can be noticed from Table 4 that the error values and percentages are moderately low with 4.83% average absolute error. A linear regression model has been derived correlating the estimated and actual GE values provided in Table 3. The linear model is associated with 95.2% R-squared and 198.51F-statistic indicating more than 99.99% confidence level. A similar linear model obtained for the GE values presented in Table 4 has resulted in 92.6% R-squared and 124.8F-statistic indicating a confidence level of more than 99.99%. These statistics simply imply that the estimated and actual GE values are very significantly correlated.

An important application of Eqs. (17) and (18) concerning pavement rehabilitation is the estimation of the required overlay thickness. The values of initial and terminal transition probabilities are expected to increase over time due to the progressive increase in traffic loading and consequent progressive decrease in structural capacity. Therefore, the values of initial and terminal transition probabilities at a given service time (t) can be used to estimate the corresponding gravel equivalent, GE(j, t), from Eqs. (17) and (18) for the j^{th} project. The required overlay thickness, h(j, t), can then be estimated from Eq. (19) based on the difference between the present gravel equivalent, GE(j, t), and original GE(j) as listed in Table 1. The difference is then divided by the gravel equivalent factor (G_f) to convert the result into thickness. The GE unit is feet, therefore Eq. (19) is multiplied by (30) to change the unit to centimeters. Caltrans recommends (1.9) as a value of (G_f) for hot mix asphalt overlays (Caltrans, 2017).

$$h(j,t) = 30\left(\frac{GE(j,t) - GE(j)}{G_f}\right) \quad (19)$$

For example, assume the initial transition probability associated with the 1st project listed in Table 1 has increased by 50% after 10 years of service life. The present $P(1)_{1,2}$ value becomes 0.27 which when applied in Eq. (17) results in 1.668-ft gravel equivalent. Therefore, the required overlay thickness is 3.88-cm according to Eq. (19). The Caltrans Highway Design Manual applies a model similar to the one presented in Eq. (19) for estimating overlay thickness, however the use of 1.9 as (G_f) value is considered as an approximation since (G_f) varies with traffic index (TI), and existing asphalt layer thickness (Caltrans, 2017).

Conclusions and recommendations

Overall, it can be concluded that the two Empirical-Markovian models presented in Eqs. (17) and (18) for estimating the flexible pavement structural capacity in terms of gravel equivalent are highly significant. In practice, two gravel equivalent estimates can be obtained from both equations with the higher value is to be selected for rehabilitation and design applications in order to guard against the two deterioration trends represented by the initial and terminal transition probabilities. It is also to be reminded that the two models presented in Eqs. (17) and (18) have mainly been developed for stochastic and design data that are relevant to a road sample with low traffic loadings (i.e. low volume roads). Therefore, it is recommended that different sets of equations be developed for different traffic loading levels. It is also recommended, regardless of the road sample size, that the stochastic and design parameters associated with the base project be replaced by the corresponding average parameter values when solving by multi-variable linear regression modelling.

The proposed Empirical-Markovian approach has primarily been applied to the Caltrans method for flexible pavement design with only three main design factors. However, it can easily be applied to other design methods regardless of the number of involved design factors. The overall procedure remains simple and straightforward while incorporating the stochastic impact of pavement performance. In essence, any current pavement design method can be upgraded to become stochastic-based design procedure through the inclusion of the two key stochastic parameters, namely the initial and terminal deterioration transition probabilities. The only additional data needed are the two transition probabilities which can be estimated from the periodical assessment of pavement distress. This makes the proposed Empirical-Markovian approach accessible to all interested parties including local governments and developing countries that cannot afford carrying out expensive pavement testing such as the deflection testing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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