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Optimal novel approach for estimating the pavement transition probabilities used in Markovian prediction models

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ABSTRACT

An optimal novel approach is proposed to estimate the transition probabilities associated with both homogeneous and non-homogeneous Markov chains. The approach applies an exhaustive optimisation technique to search for the optimal transition probabilities associated with minimal sum of squared errors (SSE), wherein the error defined as the difference between predicted and observed pavement condition ratings. Three state transitions are allowed in constructing the relevant transition probability matrix (TPM). In the homogenous chain, the approach yields one optimal TPM applicable to an analysis period of (n) years. Whereas, one distinct optimal TPM can be derived for each year if non-homogenous chain is deployed. A sequential iterative optimisation approach is proposed wherein the optimal TPM for a given year becomes the input for the subsequent year. Sample results are presented for two projects (A & B) with superior and inferior performances, respectively. The sample results indicate that the non-homogenous chain provided significant reduction in the SSE compared to the homogeneous one. However, the use of 10 condition states instead of 5 resulted in moderate reduction in the SSE considering both homogeneous and non-homogeneous Markov chains. Also, the results indicate that the use of three state transitions made significant impact when deploying 10 condition states instead of 5 especially in the case of inferior performance.

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1. Introduction

Pavement rehabilitation and management remains to be an important area of research because of the continuous need for preserving the huge national investment in the roadway infrastructure system. Pavement performance prediction is an essential component of any advanced pavement management system. The vast majority of pavement management models developed in the last couple of decades had incorporated some form of performance prediction model (Ferreira *et al.* 2002, Abaza *et al.* 2004, Gao and Zhang 2008, Abaza and Ashur 2009, Jorge and Ferreira 2012, Mathew and Isaac 2014, Cirilovic *et al.* 2015, Saliminejad and Perrone 2015). The function of any performance prediction model is to forecast the future pavement conditions so that appropriate maintenance and rehabilitation strategies can be identified and scheduled while taking into consideration the required financial constraints. Therefore, the pavement management problem is essentially an optimisation problem with a set of pavement performance and cost constraints designed to meet a certain performance outcome at the network level.

Pavement performance prediction models are generally classified into two categories: deterministic and probabilistic, however the most popular are the probabilistic ones (Wang *et al.* 1994, Amin 2015, Abed *et al.* 2019, Fuentes *et al.* 2019). This is because pavement performance has long been recognised to be highly probabilistic in nature. Several probabilistic-based prediction models have been used to model pavement performance, however the most popular are the Markovian-based ones (Li *et al.* 1996, Abaza *et al.* 2004, Yang *et al.* 2006, Kobayashi *et al.* 2010, Zhang and Gao 2012, Lethanh and Adey 2013, Lethanh *et al.* 2015, Meidani and Ghanem 2015, Abaza 2016, Daniel *et al.* 2019). In particular, these researchers have used different versions of the discrete-time Markov model including the homogenous Markov chain, non-homogeneous Markov chain, random Markov chain, exponential hidden Markov chain, Poisson hidden Markov chain, and recurrent Markov chain. However, they all require a main input parameter known as the transition probabilities regardless of the Markov chain type used. The transition probabilities mainly represent the pavement deterioration rates in the absence of any pavement maintenance and rehabilitation works. Reliable estimates of the transition probabilities are vital for effective prediction of pavement deterioration and consequently sound pavement management decisions.

Generally, the transition probabilities are estimated from historical records of pavement condition including pavement distress and ride quality measurements. A few researchers proposed different procedures to estimate the transition probabilities (Ortiz-Garcia *et al.* 2006, Kobayashi *et al.* 2010, Abaza 2016, Abaza 2017). But it seems there isn't yet a well-recognised procedure that has gained widespread acceptance by the scientific community. For example, Ortiz-García *et al.* (2006) developed three approaches to compute the deterioration transition probabilities as derived

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from minimising the sum of squared errors (SSE). The first approach assumes the availability of original pavement condition records, the second one involves the derivation of a regression curve from the pavement condition records, and the third one requires the availability of yearly distributions of pavement condition records. Kobayashi *et al.* (2010) applied the exponential hazard models to define the deterioration transition probabilities between the deployed condition states using non-uniform intervals amongst the inspection time points. Abaza (2016) derived the mathematical models required to compute the transition probabilities as a function of the state probabilities associated with two consecutive transitions. The deployed transition probability matrix (TPM) assumes only two state transitions. Abaza (2017) proposed an empirical model to estimate the non-homogenous transition probabilities. The model accounts for the progressive increase in traffic loading and progressive decrease in pavement strength.

In this paper, it is proposed to estimate the deterioration transition probabilities associated with both homogeneous and non-homogeneous Markov chains using an optimal approach. The optimal approach applies an exhaustive search algorithm to minimise the SSE associated with an analysis period comprised of (n) transitions. The error is defined as the difference between the predicted and observed distress ratings, therefore it is assumed that distress records are available for the analysis period. The required distress records can be in terms of the well-known pavement condition indicators such as the PSI, PCI and IRI. Three state transitions are used in defining the TPM, which include the probability of remaining in the current state, transitioning to the 1st worst state, and transitioning to the 2nd worst state. In the case of homogenous Markov chain, the optimal approach minimises the SSE over the entire analysis period in the search for the optimal transition probabilities. Whereas it deploys a sequential iterative approach in the search for (n) sets of optimal transition probabilities in the case of non-homogenous Markov chain. In the sequential iterative approach, the optimal solution derived for one transition becomes the input for the subsequent one.

2. Overview of discrete-time Markov model

The Markovian prediction model has been extensively used by several researchers to predict the pavement long-term performance with an emphasis on pavement deterioration prediction. As outlined in the introduction section, different forms of the Markov model have generally been used in predicting pavement performance, but the most popular ones used discrete-time and discrete condition states considering both homogeneous and non-homogeneous Markov chains (Li *et al.* 1996, Abaza *et al.* 2004, Kobayashi *et al.* 2010, Abaza 2016, Abaza 2017, Daniel *et al.* 2019). As explained in the subsequent sections, the homogeneous Markov chain deploys the same transition probability matrix (TPM) for each transition (i.e. year) within the analysis period, however the TPM can be different for each transition in the case of non-homogeneous Markov chain.

2.1. Homogeneous Markov chain

Equation (1) presents the mathematical model associated with the discrete-time and discrete states homogenous Markov chain. The state probability row vector, **S(n),** represents the state probabilities, $S_i(n)$, at the end of an analysis period comprised of (n) transitions (i.e. years) with (m) being the number of condition states. The state probability vector, **S(n),** is computed from multiplying the initial state probability row vector**, S(0),** by the transition probability matrix, $\mathbf{P^{(n)}}$, raised to the power (n). The state probabilities represent the pavement proportions that exist in the various deployed condition states at any given time, hence the sum of state probabilities must add up to one. It is also typical to assign all project pavements to the best condition state (i.e. state **1**) when considering a new pavement structure as indicated by Equation (1).

$$S(n) = S(\mathbf{0})\ P^{(n)} \tag{1}$$

where: $S(n) = [S_1(n), S_2(n), S_3(n), \ldots, S_m(n)]$

$S(\mathbf{0}) = [S_1(0), S_2(0), S_3(0), \ldots, S_m(0)]$

$= (1, 0, 0, 0, \ldots, 0)$ for new pavement

$$\sum_{i=1}^{m} S_i(k) = 1.0 \quad (k = 0, 1, 2, 3, \ldots, n)$$

The elements of the transition probability matrix (**P**) used in Equation (1) denote the transition probabilities. The transition probability ($P_{i,j}$) represents the probability of transitioning from state (**i**) to state (**j**) in one transition (i.e. year), therefore the sum of any row in the transition matrix must add up to one. Different forms of the transition probability matrix have been used in modelling pavement deterioration, however the most popular ones only used 2 or 3 state transitions. This means that pavement in state (**i**) can remain in the same state with ($P_{i,i}$) probability, transition to state (**i+1**) with ($P_{i,i+1}$) probability, or transition to state (**i+2**) with ($P_{i,i+2}$) probability as indicated by Equation (2) assuming 3 state transitions.

$$P = \begin{bmatrix} P_{1,1} & P_{1,2} & P_{1,3} & 0 & 0 & 0 & \ldots & 0 \\ 0 & P_{2,2} & P_{2,3} & P_{2,4} & 0 & 0 & \ldots & 0 \\ 0 & 0 & P_{3,3} & P_{3,4} & P_{4,5} & 0 & \ldots & 0 \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ & & & \vdots & & & & \\ 0 & 0 & \ldots & P_{m-1,m-1} & & & P_{m-1,m} & \\ 0 & 0 & 0 & 0 & 0 & & \ldots & 1.0 \end{bmatrix} \tag{2}$$

The matrix entries above the main diagonal represent the pavement deterioration rates whereas the entries below the main diagonal indicate the pavement improvement rates. Entries below the diagonal are assigned zero values in the absence of pavement maintenance and rehabilitation as defined in Equation (2). Therefore, the TPM as outlined by Equation (2) can mainly be used in predicting pavement deterioration.

2.2. Non-homogeneous Markov chain

Equation (3) provides the mathematical model associated with the discrete-time non-homogeneous Markov chain. It can incorporate a different TPM for each transition within an analysis period comprised of (n) transitions. Therefore, the **P(k)** is the TPM associated with the (k) transition. The non-homogeneous Markov chain can provide improved solutions of pavement deterioration predictions, however it has substantial requirements for historical distress records to be used in developing the required TPMs. Whereas, the homogeneous Markov chain only requires one TPM which can be estimated from the distress records obtained from conducting two consecutive cycles of distress assessment. Equation (3) is similarly used to estimate the future state probabilities, **S(n)**, which are key parameters for pavement deterioration prediction.

$$S(n) = S(0)\prod_{k=1}^{n} P(k) \quad (3)$$

Equation (4) provides an example of the TPM to be used in the non-homogeneous Markov chain. It is similar to the one indicated by Equation (2) as it only allows for three state transitions, but it can have different deterioration transition probabilities for each transition (i.e. year).

$$P(k) = \begin{bmatrix} P(k)_{1,1} & P(k)_{1,2} & P(k)_{1,3} & 0 & 0 & & 0 & \dots & 0 \\ 0 & P(k)_{2,2} & P(k)_{2,3} & P(k)_{2,4} & 0 & & 0 & \dots & 0 \\ 0 & 0 & P(k)_{3,3} & P(k)_{3,4} & P(k)_{4,5} & & 0 & \dots & 0 \\ & & & & \vdots & & & & \\ & & & & \vdots & & & & \\ & & & & \vdots & & & & \\ 0 & 0 & 0 & \dots & P(k)_{m-1,m-1} & & & P(k)_{m-1,m} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1.0 \end{bmatrix} \quad (4)$$

2.3. Pavement deterioration prediction

Pavement deterioration defines how pavement deteriorates over time in terms of a specified pavement condition indicator. Several pavement condition indicators have been used in modelling pavement deterioration including internationally recognised ones such as the present serviceability index (PSI), pavement condition index (PCI), and international roughness index (IRI). Abaza (2016) proposed a pavement condition indicator called distress rating (DR) mainly estimated from cracking and deformation as explained later. The expected DR value associated with the (k) transition for a particular project, $DR_p(k)$, can be computed using Equation (5) as a function of the future state probabilities, $S_i(k)$, and state mean distress ratings ($\overline{DR}_i$). The future state probabilities are to be estimated using either the homogeneous Markov chain or non-homogeneous Markov chain as explained earlier. The (i) state mean distress rating is typically estimated as the average of the lower and upper distress ratings (LDR_i & UDR_i) used to define the (i) condition state. The deployed DR scale has a range of (0–100) with higher ratings indicating better pavements.

$$DR_p(k) = \sum_{i=1}^{m} \overline{DR}_i \times S_i(k) \quad (k = 0, 1, 2, \dots, m) \quad (5)$$

Where: $\overline{DR}_i = (LDR_i + UDR_i)/2$

For example, the $\overline{DR}_i$ are assigned the values of (95, 85, 75, … .., 5) when considering a Markov chain with 10 condition states defined using equal DR ranges (i.e. 100–90, 90–80, 80–70, … ., 10–0). The $\overline{DR}_i$ values become equal to (90, 70, 50, 30, 10) when 5 condition states are used with equal DR ranges (i,e, 100–80, 80–60, … .., 20–0). These two $\overline{DR}_i$ examples are used in the sample problems presented later. The best condition state is defined using DR ranges of (90–100 & 80–100) while the worst state is defined using DR ranges of (0–10 & 0–20) for the previously outlined two examples. Therefore, pavement deterioration can be predicted using Equation (5) provided the required state probabilities, $S_i(k)$, are available. Figure 1 shows two typical performance curves with parabolic deterioration trends (i.e. smooth curvature), which can be developed using the predicted distress ratings, $DR_p(k)$, calculated using Equation (5). Other potential pavement condition indicators such as PSI, PCI and IRI can be used to replace DR in Equation (5).

3. Methodology

The methodology section presents the optimal simplified novel approach proposed to estimate the pavement deterioration transition probabilities considering both homogeneous and non-homogeneous discrete-time Markov chains. The optimal approach for the homogenous Markov chain aims to obtain the optimal deterioration transition probabilities over an analysis period comprised of (n) years. Whereas the optimal approach for the non-homogeneous Markov seeks to yield the optimal transition probabilities associated with each year within the analysis period by applying a sequential iterative approach. In this sequential approach, the optimal transition probabilities derived for a given year becomes the input for the subsequent year. In both cases, the optimal approach is carried out based on the minimisation of sum of squared errors (SSE) wherein the error is defined as the difference between the predicted and observed distress ratings.

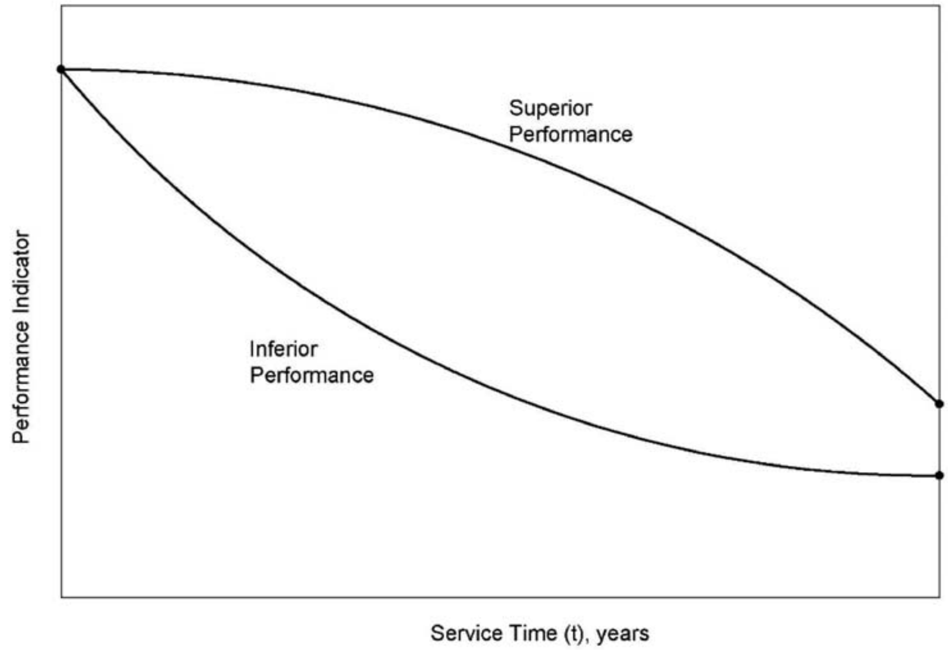

Figure 1. Typical pavement performance curves with parabolic deterioration trends.

3.1. Estimation of trial TPM

The main objective of the proposed optimal novel approach is to estimate the deterioration transition probabilities as required by the TPM defined in Equations (2) and (4). The proposed approach requires that pavement deterioration curves similar to the ones shown in Figure 1 are available. Therefore, the proposed optimal approach aims to yield the optimal values of the relevant deterioration transition probabilities that when they used would generate deterioration curves that best match the available ones. In essence, the proposed approach applies a backward solution to an existing forward one. The forward solution assumes the availability of transition probabilities to be used in developing the corresponding deterioration curve as a function of DR values estimated using Equation (5). However, the backward approach seeks to find the deterioration transition probabilities corresponding to a known pavement deterioration curve.

There are three transition probabilities that need to be estimated as defined in Equations (2) and (4), namely the probability of remaining in the same condition state ($P_{i,i}$), probability of transitioning to the first worst state ($P_{i,i+1}$), and probability of transitioning to the second worst state ($P_{i,i+2}$). All transition probabilities required by the TPM defined in Equations (2) and (4) are to be computed from only three main input parameters, namely the initial deterioration transition probability ($P_{1,2}$), terminal transition probability ($P_{m-1,m}$), and transition probability ratio (λ) defined next. The following sections explain the detailed algorithm used in computing the trial transition probabilities for both homogeneous and non-homogeneous Markov chains.

3.1.1. Homogeneous Markov chain

There is only one TPM to be estimated when using the discrete-time homogeneous Markov chain. The first step is to estimate all transition probabilities associated with transitioning to the first worst state (i.e. $P_{i,i+1}$) from the specified initial and terminal transition probabilities ($P_{1,2}$ & $P_{m-1,m}$) as indicated by Equation (6a). The estimation is performed based on the assumption of linear interpolation as defined by the incremental transition probability (Δp). The incremental transition probability is positive when the initial transition probability is smaller than the terminal transition probability, an indication of superior performance as depicted in Figure 1. Superior performance is typically associated with increasingly higher deterioration rates (i.e. $P_{1,2} < P_{2,3} < P_{3,4} \ldots . < P_{m-1,m}$). Similarly, the incremental transition probability is negative when the initial transition probability is larger than the terminal transition probability, an indication of inferior performance as depicted in Figure 1. Inferior performance is typically associated with decreasingly lower deterioration rates (i.e. $P_{1,2} > P_{2,3} > P_{3,4} \ldots . > P_{m-1,m}$).

$$P_{i,i+1} = P_{i-1,i} + \Delta P \quad (i = 2, 3, \ldots., m-2) \tag{6a}$$

Where:

$$\Delta P = \frac{P_{m-1,m} - P_{1,2}}{m-2}$$

The second step is to estimate the deterioration transition probabilities associated with transitioning to the second worst state (i.e. $P_{i,i+2}$). They are simply estimated as a proportion of the deterioration transition probabilities ($P_{i,i+1}$) as defined in Equation (6b). The transition probability ratio (λ) is expected to be larger when using a larger number of condition states. This is because the probability of transitioning to the second worst state in one transition (i.e. time interval) becomes higher when a larger number of condition states is used.

$$P_{i,i+2} = \lambda \times P_{i,i+1} \quad (\lambda < 1) \tag{6b}$$

The third and last step is to compute the transition probabilities associated with remaining in the same condition state (i.e. $P_{i,i}$) as indicated by Equation (6c). Equation (6c) simply enforces the sum of any row in the TPM must add up to one.

$$P_{i,i} = 1 - P_{i,i+1} - P_{i,i+2} \tag{6c}$$

The previously outlined algorithm for computing the trial transition probabilities as required by the TPM defined in Equation (2) is to be used in the optimal approach that searches for the optimal TPM. Therefore, the optimal approach will essentially yield the optimal values of the three main input parameters (i.e. λ, $P_{1,2}$, $P_{m-1,m}$) to be used in defining the TPM outlined in Equation (2).

3.1.2. Non-homogeneous Markov chain

Equation (7) provides the mathematical algorithm to be used in estimating the required trial TPMs as defined in Equation (4), which is similar to the algorithm previously outlined for homogenous Markov chain. This algorithm is to be used in estimating a number of TPMs that is equal to the number of transitions used in the analysis period (n). The trial TPM for the (k) transition is to be estimated from the three main input parameters, namely $\lambda(k)$, $P(k)_{1,2}$ and $P(k)_{m-1,m}$.

$$P(k)_{i,i+1} = P(k)_{i-1,i} + \Delta P(k) \quad (i = 2, 3, \ldots, m-2;\ k = 1, 2, \ldots, n) \tag{7a}$$

Where:

$$\Delta P(k) = \frac{P(k)_{m-1,m} - P(k)_{1,2}}{m-2}$$

$$P(k)_{i,i+2} = \lambda(k) \times P(k)_{i,i+1} \quad \lambda(k) < 1 \tag{7b}$$

$$P(k)_{i,i} = 1 - P(k)_{i,i+1} - P(k)_{i,i+2} \tag{7c}$$

The algorithms defined in Equations (6) and (7) are mainly introduced to simplify and facilitate the calculations of the remaining transition probabilities from only the initial and terminal ones (i.e. $P_{1,2}$ & $P_{m-1,m}$). Therefore, modifications to these algorithms can be made if deemed necessary.

3.2. Estimation of optimal TPM

In the search for the optimal TPM, an exhaustive search procedure is to be applied as a function of the three previously outlined main parameters, namely the transition probability ratio (λ), initial transition probability ($P_{1,2}$), and terminal transition probability ($P_{m-1,m}$). The optimal TPM is the one that generates a pavement deterioration curve that provides the best-fit with the known deterioration curve. The optimisation procedure mainly relies on the minimisation of sum squared errors (SSE) as outlined next.

3.2.1. Homogeneous Markov chain

The application of the homogenous Markov chain only requires the estimation of one TPM as presented in Equation (1). The minimisation of sum of squared errors (SSE) is defined as indicated by Equation (8) wherein the (k) error is computed as the difference between the predicted and observed DR values, namely $DR_p(k)$ & $DR_o(k)$. The $DR_p(k)$ is obtained from Equation (5) based on a trial TPM generated using the algorithm outlined in Equation (6). The $DR_o(k)$ is to be obtained from the known deterioration curve generated from historical records of pavement distress for a given pavement project.

$$\text{Minimise:} \quad \text{SSE} = \sum_{k=1}^{n} [\text{DR}_p(k) - \text{DR}_o(k)]^2 \tag{8}$$

The proposed optimal approach for homogenous chain as defined in Equation (8) accounts for pavement deterioration over the entire analysis period (n), and not just using two consecutive distress assessments as occasionally done. The (k) transition is associated with a time interval that is the same for successive transitions, and it is typically assumed to be equal to one year when considering an analysis period comprised of (n) transitions (i.e. years).

3.2.2. Non-Homogeneous Markov chain

Application of the non-homogenous Markov chain as outlined in Equation (3) requires the estimation of (n) optimal TPMs. The proposed optimal approach applies a sequential iterative procedure to yield an optimal TPM for each transition. In each iteration, the squared error (SE) is minimised as defined in Equation (9). The optimal TPM obtained from the kth iteration is used as an input in solving the (k+1) iteration. The corresponding optimal state probabilities are also computed as a function of the state probabilities associated with the (k-1) transition and optimal TPM for the (k) transition, thus indicating an iterative application of Equation (3).

$$\text{Minimise:} \quad \text{SE}(k) = [\text{DR}_p(k) - \text{DR}_o(k)]^2 \quad (k = 1, 2, \ldots, n) \tag{9}$$

The first optimisation iteration is carried out based on the assumed initial state probabilities, **S(0)**, and variable trial TPM, **P(1)**, as indicated by Equation (10a). Then, the corresponding optimal state probabilities, **S″(1),** are computed using Equation (10b) based on the derived optimal TPM, **P″(1)**. The optimal TPM is mainly defined in terms of the optimal $\lambda''(1)$, $P''(1)_{1,2}$, $P''(1)_{m-1,m}$.

First iteration

$$S(\mathbf{1}) = S(\mathbf{0})\ P(\mathbf{1}) \tag{10a}$$

$$S''(\mathbf{1}) = S(\mathbf{0})\ P''(\mathbf{1}) \tag{10b}$$

Similarly, the second optimisation iteration is executed as presented in Equation (11). The optimal state probabilities, **S″(1)**, associated with the 1st iteration become the initial state probabilities used in Equation (11a) for the purpose of yielding the optimal TPM. Then, Equation (11b) is used to compute the optimal state probabilities for the 2^{nd} iteration based on the optimal TPM. The optimal TPM, **P″(2)**, is a function of the optimal $\lambda''(2)$, $P''(2)_{1,2}$ and $P''(2)_{m-1,m}$.

Second iteration

$$S(\mathbf{2}) = S''(\mathbf{1})\ P(\mathbf{2}) \tag{11a}$$

$$S''(\mathbf{2}) = \mathbf{S}''(\mathbf{1})\ P''(\mathbf{2}) \tag{11b}$$

Equation (12) provides the general model to be used in carrying out the outlined sequential iterative procedure considering (n) transitions. The general model is applicable to all iterations with the exception of the 1st one to be executed using Equation (10). The (k) iteration will yield the corresponding optimal TPM, $\mathbf{P}''(\mathbf{k})$, in terms of the optimal $\lambda''(k)$, $P''(k)_{1,2}$ and $P''(k)_{\mathrm{m-1,m}}$.

General model

$$S(k) = S''(k-\mathbf{1})\ P(k) \quad (k = 2, 3, \ldots, n) \tag{12a}$$

$$S''(k) = S''(k-\mathbf{1})\ P''(k) \tag{12b}$$

4. Pavement deterioration assessment

Pavement distress assessment and ride quality measurements are typically used to evaluate pavement deterioration over time. Several internationally recognised pavement condition indicators have been used to model pavement deterioration including the PSI, PCI and IRI. The PSI is a function of the slope variance associated with the roadway longitudinal profile, and pavement cracking and deformation (AASHTO 1993, Shah *et al.* 2013, Fuentes *et al.* 2019). However, the PSI has been found to be highly correlated to the IRI in (m/km). For examples, Equations (13) and (14) were developed by Paterson (1986), and Al-Omari and Darter (1994), respectively, for asphalt concrete pavement.

$$\mathrm{PSI} = 5\mathrm{e}^{(-0.18\mathrm{IRI})} \tag{13}$$

$$\mathrm{PSI} = 5\mathrm{e}^{(-0.24\mathrm{IRI})} \tag{14}$$

Currently, several highway agencies around the world use the IRI to measure profile roughness. The development of the IRI was sponsored by the World Bank to provide a common basis for conducting and comparing roughness measurements. The IRI provides a summary measure of the longitudinal surface profile obtained from the surface elevation data collected using a mechanical profilometer. The World Bank published guidelines for conducting and calibrating roughness measurements as reported by Sayers *et al.* (1986). The IRI gets larger in value as pavement deterioration progresses whereas the corresponding PSI value gets smaller.

In addition, researchers developed regression models to estimate the PCI from the IRI data. For example, Park *et al.* (2007) proposed the model presented in Equation (15) to estimate the PCI from the IRI in (m/km). The PCI has a scale of 100 points with higher ratings indicating better pavements. It is estimated based on visual inspection of pavement defects and simple related measurements. ASTM (2007) outlined the procedure to be followed in estimating the PCI for a particular pavement segment.

$$\mathrm{PCI} = 87.098\ (\mathrm{IRI})^{-0.481} \tag{15}$$

Abaza (2016) proposed simple models to calculate a pavement condition indicator for flexible pavement called distress rating (DR). It is estimated as a function of the two most significant load-related pavement defects, namely cracking and deformation. Equation (16) presents an example of such models wherein the DR is estimated for a lane segment using the localised cracked and deformed areas (A_C & A_D) multiplied by their corresponding severity factors (SF_C & SF_D), which are assigned the values of 1, 2, and 3 for low, medium and high severity, respectively. The average DR value associated with all pavement segments is then computed to represent the distress condition of the entire highway project. Distress assessment is typically performed annually or biennially. (A_S) represents the total surface area of a lane segment.

$$\mathrm{DR} = \left(\frac{3A_S - \sum_i \mathrm{SF}_{C_i}A_{C_i} - \sum_i \mathrm{SF}_{D_i}A_{D_i}}{3A_S}\right) \times 100 \tag{16}$$

where: $\sum \mathrm{SF}_{C_i}A_{C_i} + \sum_i \mathrm{SF}_{D_i}A_{D_i} \leq 3A_S$ and $\sum_i A_{C_i} + \sum_i A_{D_i} \leq A_s$

5. Sample presentation

This section provides sample optimal TPMs derived using the outlined optimal approach considering both homogenous and non-homogeneous Markov chains. Two pavement projects (A & B) have been investigated with parabolic deterioration trends similar to the ones shown in Figure 1. Table 1 provides the observed DR values for the two sample projects over an analysis period of 10 years. An exhaustive optimisation approach has been used with function evaluations made at one hundredth point (0.01) in the search for minimal SSE. The values of the three main input parameters (λ, $P_{1,2}$, $P_{\mathrm{m-1,m}}$) can theoretically be varied over the (0.0–1.0) range resulting in a total of $(101)^3$ combinational function evaluations which can easily be handled by all computers. The Markov chain size (m) has been varied to include 5 and 10 condition states.

5.1. Sample optimal homogeneous TPMs

The sample optimal TPMs associated with the homogeneous Markov chain have been derived using the previously outlined optimal approach as presented by Equations (1), (2), (5), (6) and (8). Table 2 provides the optimal TPM associated with 5 condition states for both projects (A and B). The optimal transition probability ratio ($\lambda'' = 0.0$) has no impact on Project A (superior performance) compared to Project B (inferior performance). Therefore, the second transition probabilities ($P_{\mathrm{i,i+2}}$) have vanished from the optimal TPM in the case of Project A. However, the impact of optimal transition probability ratio ($\lambda'' = 0.06$) has been limited in the case of Project B. Also, the optimal initial transition probability ($P''_{1,2} = 0.10$) is smaller than the optimal terminal transition probability ($P''_{4,5} = 0.53$)

Table 1. Observed distress ratings ($\mathrm{DR_o}$) estimated from distress assessment.

Year	0	1	2	3	4	5	6	7	8	9	10
Project A	95.0	92.7	90.5	87.9	85.5	83.1	80.2	77.6	73.4	70.7	66.1
Project B	95.0	90.1	80.8	74.5	69.3	64.9	59.8	55.2	50.6	43.7	39.8

Table 2. Sample optimal homogeneous TPM with 5 condition states for 10-year analysis period.

	Project A[a] ($\lambda'' = 0.0$, $P''_{1,2} = 0.10$, $P''_{4,5} = 0.53$)					Project B[b] ($\lambda'' = 0.06$, $P''_{1,2} = 0.30$, $P''_{4,5} = 0.19$)				
State	1	2	3	4	5	1	2	3	4	5
1	0.900	0.100	0.000	0.000	0.000	0.682	0.300	0.018	0.000	0.000
2	0.000	0.757	0.243	0.000	0.000	0.000	0.721	0.263	0.016	0.000
3	0.000	0.000	0.613	0.387	0.000	0.000	0.000	0.760	0.226	0.014
4	0.000	0.000	0.000	0.470	0.530	0.000	0.000	0.000	0.810	0.190
5	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000

[a]Minimal SSE = 5.34.
[b]Minimal SSE = 19.68.

for Project A, an indication of superior performance. Similarly, the optimal initial transition probability ($P''_{1,2} = 0.30$) is larger than the optimal terminal transition probability ($P''_{4,5} = 0.19$) for Project B, an indication of inferior performance. Figures 2 and 4 show a good agreement between the predicted and observed DR values associated with Projects A and B, respectively, using 5 condition states and 10 years analysis period. The predicted DR values are computed using the two optimal TPMs provided in Table 2. The optimal TPMs provided in Table 2 are computed using Equation (6) as a function of optimal values (λ'', $P''_{1,2}$, $P''_{m-1,m}$).

Table 3 provides the optimal TPM associated with Project A using 10 condition states while Table 4 provides similar results for Project B. It can be noted that the use of 10 condition states has reduced the SSE from (5.34) to (2.01) in the case of Project A, and from (19.68) to (11.98) in the case of Project B. Another observation can be made in relation to the influence of the optimal transition probability ratio which is lower in the case of Project A ($\lambda'' = 0.20$) compared to ($\lambda'' = 0.30$) in the case of Project B. Therefore, the impact of the optimal second transition probabilities ($P_{i,i+2}$) is somewhat more significant in the case of inferior performance (Project B) because of higher (λ''). Generally, the overall contribution of the second transition probabilities ($P_{i,i+2}$) is more significant when using 10 condition states as would be expected. Also, Table 3 indicates that the optimal initial transition probability ($P''_{1,2} = 0.16$) is lower than the optimal terminal transition probability ($P''_{9,10} = 0.45$), an indication of superior performance. Similarly, Table 4 indicates that the optimal initial transition probability ($P''_{1,2} = 0.41$) is higher than the optimal terminal transition probability ($P''_{9,10} = 0.21$), an indication of inferior performance. Figures 3 and 5 show an improved agreement between the predicted and observed DR values when using 10 condition states compared to 5 states considering Projects A & B, respectively.

5.2. Sample optimal non-homogeneous TPMs

The sample optimal TPMs associated with the non-homogeneous Markov chain have been derived using the previously outlined optimal approach as indicated by Equations (3)–(5), (7), (9), (10) and (12). Table 5 provides the optimal non-homogeneous parameters, $\lambda''(k)$, $P''(k)_{1,2}$, $P''(k)_{4,5}$, associated with 5 condition states for Projects A & B. Table 5 also provides the optimal squared error, SE″ (k), with the error defined as the difference between the predicted and observed DR values as defined in Equation (9). Observations similar to the ones indicated in case of

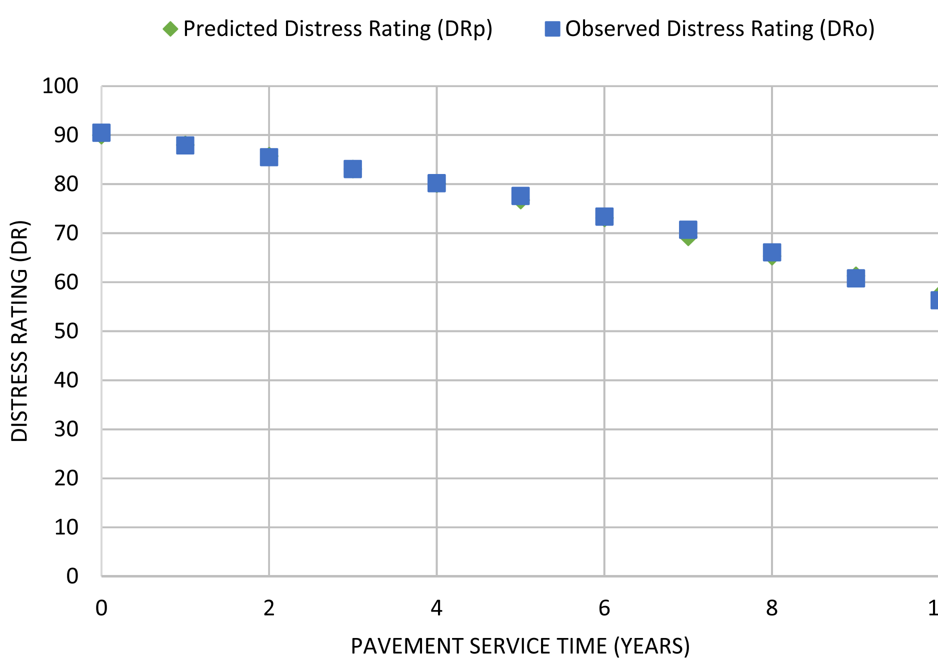

Figure 2. Sample distress ratings predicted using homogeneous Markov chain with 5 condition states (Project A, SSE = 5.34).

Table 3. Sample optimal homogeneous TPM with 10 condition states for Project A.

	$(\lambda'' = 0.20,\ P''_{1,2} = 0.16,\ P''_{9,10} = 0.45)^{a}$									
State	1	2	3	4	5	6	7	8	9	10
1	0.808	0.160	0.032	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.765	0.196	0.039	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.722	0.232	0.046	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.677	0.269	0.054	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.634	0.305	0.061	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.591	0.341	0.068	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.547	0.377	0.076	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.503	0.414	0.083
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.550	0.450
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

[a]Minimal SSE = 2.01.

Table 4. Sample optimal homogeneous TPM with 10 condition states for Project B.

	$(\lambda'' = 0.30,\ P''_{1,2} = 0.41,\ P''_{9,10} = 0.21)^{a}$									
State	1	2	3	4	5	6	7	8	9	10
1	0.467	0.410	0.123	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.499	0.385	0.116	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.532	0.360	0.108	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.564	0.335	0.101	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.597	0.310	0.093	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.630	0.285	0.085	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.662	0.260	0.078	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.695	0.234	0.071
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.790	0.210
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

[a]Minimal SSE = 11.98.

homogeneous Markov chain can be made regarding the optimal transition probability ratios, $\lambda''(k)$, which are smaller in the case of superior performance (Project A) compared to inferior performance (Project B). Also, the optimal initial transition probabilities, $P''(k)_{1,2}$, are lower than the optimal terminal transition probabilities, $P''(k)_{4,5}$, in the case of Project A, but they are generally larger in the case of Project B. The SSE has substantially been reduced from (5.34) to (0.0022) in the case of Project A, and from (19.68) to (0.0035) in the case of Project B.

Similarly, Table 6 provides the optimal non-homogeneous parameters, $\lambda''(k)$, $P''(k)_{1,2}$, $P''(k)_{9,10}$, associated with 10 condition states for Projects A & B. The same earlier observations can be made regarding the optimal transition probability ratios, $\lambda''(k)$, optimal initial transition probabilities, $P''(k)_{1,2}$, and optimal terminal transition probabilities, $P''(k)_{9,10}$, considering Projects A &

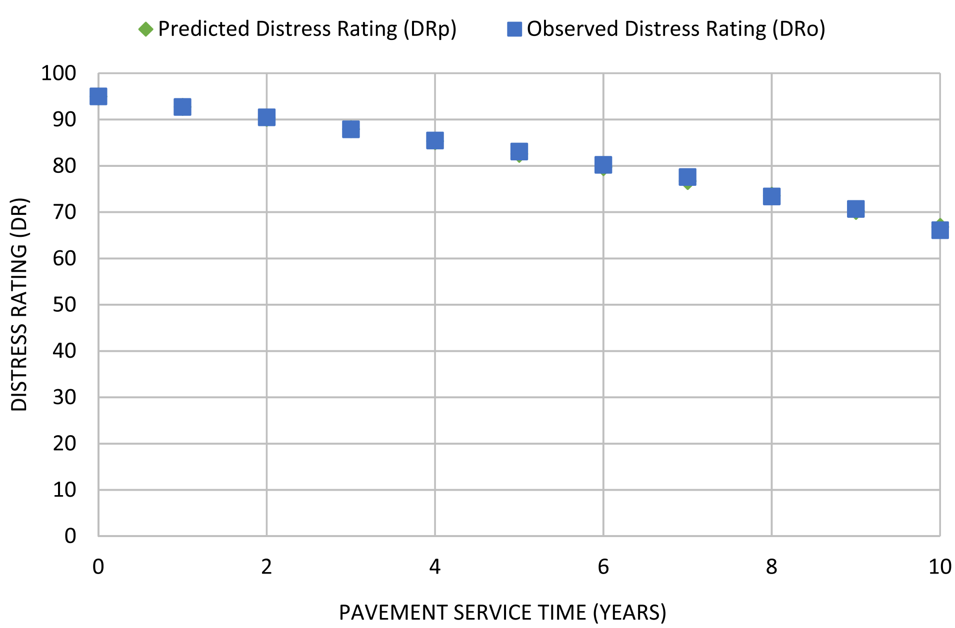

Figure 3. Sample distress ratings predicted using homogeneous Markov chain with 10 condition states (Project A, SSE = 2.01).

Table 5. Sample optimal non-homogeneous transition probabilities using 5 condition states.

Year (k)	Project A $\lambda''(k)$	$P''(k)_{1,2}$	$P''(k)_{4,5}$	$SE''(k)$[a]	Project B $\lambda''(k)$	$P''(k)_{1,2}$	$P''(k)_{4,5}$	$SE''(k)$[b]
1	0.08	0.09	0.53	1.44E-4	0.29	0.29	0.10	1.30E-3
2	0.08	0.09	0.49	1.12E-6	0.31	0.21	0.12	5.12E-4
3	0.08	0.09	0.27	2.61E-5	0.32	0.19	0.07	4.48E-5
4	0.08	0.09	0.41	1.23E-6	0.32	0.17	0.07	3.31E-4
5	0.08	0.06	0.41	1.38E-4	0.33	0.22	0.07	5.16E-8
6	0.08	0.11	0.54	6.64E-5	0.32	0.22	0.07	1.43E-4
7	0.08	0.11	0.17	1.02E-4	0.33	0.22	0.10	2.95E-4
8	0.08	0.11	0.52	6.74E-4	0.32	0.22	0.29	5.57E-4
9	0.08	0.16	0.49	6.73E-4	0.35	0.21	0.13	1.58E-4
10	0.08	0.13	0.44	3.84E-4	0.34	0.21	0.19	1.98E-4

[a]Minimal SSE = $\sum SE''(k) = 0.0022$.
[b]Minimal SSE = $\sum SE''(k) = 0.0035$.

Table 7. Sample optimal errors associated with predicted DR values using 10 condition states for Project A.

Year (k)	Homogeneous Markov chain $DR''_P(k)$	$DR_o(k)$	$Error''(k)$[a]	Non-homogeneous Markov chain $DR''_P(k)$	$DR_o(k)$	$Error''(k)$[b]
1	92.760	92.7	0.060	92.696	92.7	−0.004
2	90.406	90.5	−0.094	90.500	90.5	0.000
3	87.933	87.9	0.033	87.900	87.9	0.000
4	85.334	85.5	−0.166	85.504	85.5	0.004
5	82.604	83.1	−0.496	83.095	83.1	−0.005
6	79.735	80.2	−0.465	80.191	80.2	−0.009
7	76.723	77.6	−0.877	77.602	77.6	0.002
8	73.563	73.4	0.163	73.398	73.4	−0.002
9	70.259	70.7	−0.441	70.685	70.7	−0.015
10	66.819	66.1	0.719	66.090	66.1	−0.010

[a]Average absolute DR error = 0.351.
[b]Average absolute DR error = 0.005.

B. In particular, the optimal transition probability ratios, $\lambda''(k)$, and optimal initial transition probabilities, $P''(k)_{1,2}$, are generally higher than the corresponding values provided in Table 5 with 5 condition states as would be expected. This also indicates the significance contribution of the second transition probabilities ($P_{i,i+2}$) when using 10 condition states compared to 5 states especially for Project B. However, there are only minor improvements in the SSE values when using 10 condition states compared to 5 states. The SSE is reduced from (0.0022) to (0.0005) in the case of Project A, and from (0.0035) to (0.0014) in the case of Project B. This may indicate that using 5 condition states is adequate when the non-homogenous Markov chain is deployed.

Table 7 provides the predicted and observed DR values along with their differences (i.e. errors) for both homogeneous and non-homogeneous Markov chains for Project A. The predicted DR values are as obtained using the optimal TPMs derived from both the homogeneous and non-homogeneous Markov chains. The average absolute DR error is reduced from (0.351) to (0.005) when using the non-homogeneous Markov chain compared to the homogeneous one. Table 8 provides similar results but for Project B. It can be noticed that the average absolute DR error is reduced from (0.987) to (0.009) when using the non-homogeneous Markov chain. Therefore, it can be concluded that the non-homogeneous Markov chain has provided substantially improved solutions compared to the homogenous one. Also, the use of 10 condition states has resulted in slightly improved solutions compared to 5 states in the case of non-homogenous chain.

Generally, increasing the number of condition states (m) would yield improved results, however it might require the incorporation of additional state transitions (i.e. more than three state transitions), thus resulting in higher function evaluations and computation time. The optimal number of condition states can be derived by specifying an absolute average DR error as a threshold value. For example, a number of 10 condition states is adequate if a maximum of (0.1) absolute average DR error is specified considering the presented sample non-homogeneous Markov chains, but it is inadequate in the case of sample homogeneous Markov chains based on the results provided in Tables 7 and 8. Alternatively, a maximum SSE value can be specified as a threshold value for yielding the optimal number of condition states.

The sample results presented have mainly dealt with pavement condition data that exhibits parabolic deterioration

Table 8. Sample optimal errors associated with predicted DR values using 10 condition states for Project B.

Year (k)	Homogeneous Markov chain $DR''(k)$	$DR_o(k)$	$Error''(k)$[a]	Non-homogeneous Markov chain $DR''_P(k)$	$DR_o(k)$	$Error''(k)$[b]
1	88.440	90.1	−1.660	90.100	90.1	0.000
2	82.142	80.8	1.342	80.794	80.8	−0.006
3	76.097	74.5	1.597	74.506	74.5	0.006
4	70.293	69.3	0.993	69.308	69.3	0.008
5	64.721	64.9	−0.179	64.919	64.9	0.019
6	59.374	59.8	−0.426	59.796	59.8	−0.004
7	54.249	55.2	−0.951	55.210	55.2	0.010
8	49.350	50.6	−1.250	50.587	50.6	−0.013
9	44.688	43.7	0.988	43.723	43.7	0.023
10	40.280	39.8	0.480	39.805	39.8	0.005

[a]Average absolute DR error = 0.987.
[b]Average absolute DR error = 0.009.

Table 6. Sample optimal non-homogeneous transition probabilities using 10 condition states.

Year (k)	Project A $\lambda''(k)$	$P''(k)_{1,2}$	$P''(k)_{9,10}$	$SE''(k)$[a]	Project B $\lambda''(k)$	$P''(k)_{1,2}$	$P''(k)_{9,10}$	$SE''(k)$[b]
1	0.22	0.16	0.45	1.60E-5	0.20	0.35	0.20	2.02E-28
2	0.15	0.16	0.47	4.10E-9	0.70	0.40	0.20	3.60E-5
3	0.21	0.16	0.57	1.97E-7	0.60	0.30	0.22	4.27E-5
4	0.11	0.16	0.57	2.06E-5	0.43	0.30	0.22	5.95E-5
5	0.11	0.15	0.55	2.18E-5	0.30	0.30	0.22	3.67E-4
6	0.11	0.18	0.57	7.24E-5	0.45	0.30	0.22	1.40E-5
7	0.09	0.14	0.57	2.78E-6	0.37	0.30	0.22	1.07E-4
8	0.15	0.25	0.59	2.40E-6	0.40	0.30	0.22	1.77E-4
9	0.12	0.15	0.41	2.23E-4	0.89	0.30	0.22	5.49E-4
10	0.15	0.35	0.37	1.02E-4	0.34	0.30	0.22	2.09E-5

[a]Minimal SSE = $\sum SE''(k) = 0.0005$.
[b]Minimal SSE = $\sum SE''(k) = 0.0014$.

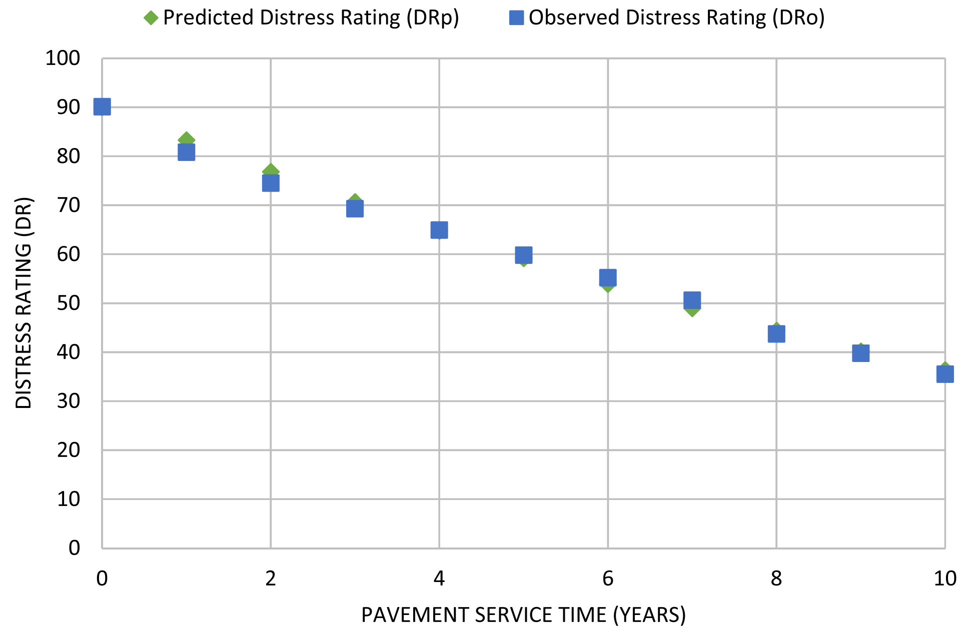

Figure 4. Sample distress ratings predicted using homogeneous Markov chain with 5 condition states (Project B, SSE = 19.68).

trends (i.e. smooth curvature) similar to the ones depicted in Figure 1. However, the proposed non-homogeneous approach can still provide reliable results when applied to non-parabolic deterioration trends (i.e. variable concavity). This is because estimation of the transition probabilities for a particular transition mainly depends on the state probabilities associated with the previous transition and the drop in pavement condition. Therefore, the non-homogeneous approach can yield reliable results, but this may not necessarily be true in the case of the homogeneous approach wherein the SSE is cumulatively minimised over the entire analysis period. A hypothetical non-parabolic sample problem has been presented in Figure 6 wherein the non-homogeneous transition probabilities are estimated using 10 condition states, which are then used to compute the predicted distress ratings (DRp) for 7 transitions (i.e. 7 years). The corresponding optimal non-homogeneous transition probabilities along with the predicted and observed distress ratings are provided in Table 9. It is clear from Figure 6 that there is a strong agreement between the predicted and observed DR values for the investigated deterioration problem with non-parabolic trend.

6. Conclusions and recommendations

The proposed optimal novel approach for estimating the transition probabilities from pavement deterioration curves has

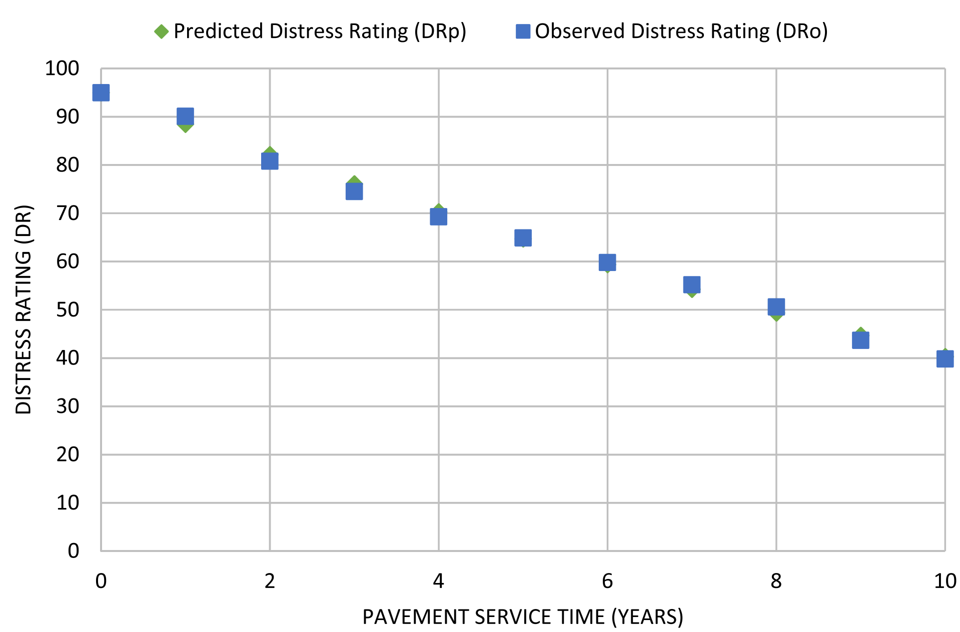

Figure 5. Sample distress ratings predicted using homogeneous Markov chain with 10 condition states (Project B, SSE = 11.09).

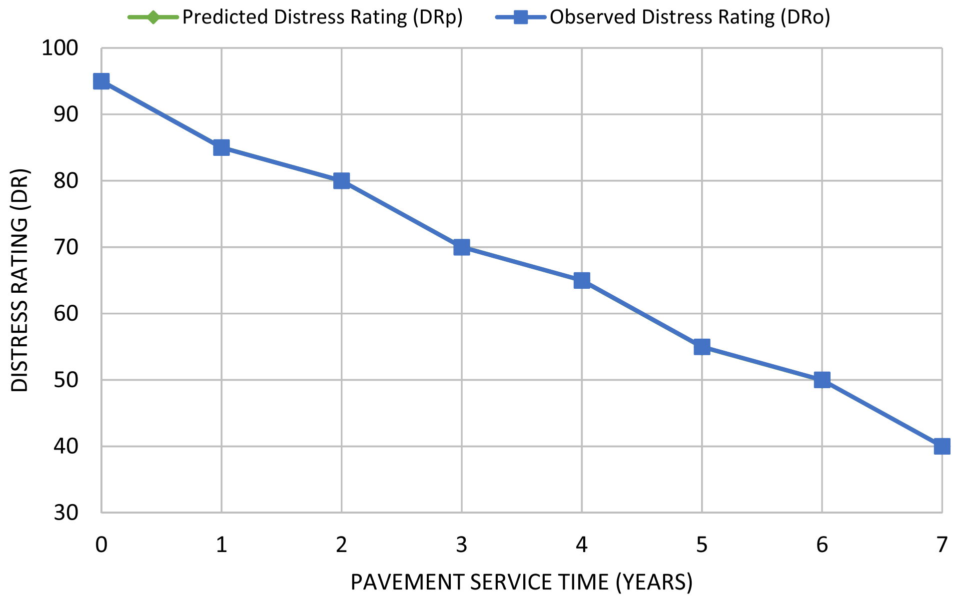

Figure 6. Sample distress ratings predicted using non-homogeneous Markov chain with 10 condition states for non-parabolic deterioration trend (SSE = 0.0044).

been investigated using two sample projects (A & B). The sample results derived for both homogeneous and non-homogeneous Markov chains have indicated the effectiveness of the proposed approach. A few conclusions can be drawn from the sample results obtained using a Markov chain with 5 and 10 condition states. The use of 10 condition states compared to 5 states has resulted in moderate reductions in the SSE considering both homogeneous and non-homogeneous Markov chains. However, substantial reductions in the SSE have been reached when using a non-homogeneous Markov chain compared to a homogeneous one. The use of 3 state transitions has made significant impact when deploying 10 condition states especially in the case of inferior performance (Project B). The superior performance (Project A) has been associated with initial transition probabilities that are lower than the terminal transition probabilities as would be expected. Similarly, the inferior performance (Project B) has been associated with initial transition probabilities that are higher than the terminal transition probabilities as would again be expected considering all investigated cases.

Therefore, it is recommended to use 3 state transitions when deploying a Markov chain with 10 condition states. It is also recommended to use the non-homogeneous Markov chain over the homogenous one, however 5 condition states with 2 state transitions would be adequate. It is worthy to emphasise that the conclusions drawn are based on sample projects with both parabolic and non-parabolic deterioration trends. Also, the programming and computations associated with the non-homogeneous chain are straightforward and much simpler. An analysis period of 5–10 years would generally be sufficient for most pavement management applications with the sample results presented for 10-year period. The data requirements are minimal which mainly include pavement condition records collected over the specified analysis period for a particular project. The collected condition records can be presented in the form of an appropriate indicator such as the PSI and PCI, thus facilitating the use of the proposed optimal approach. Both PSI and PCI can be estimated from the IRI data using correlation models similar to the ones cited in this paper. The deterioration transition probabilities can be estimated for individual projects or project groups with similar traffic conditions and material properties. The relevant mathematical computations are straightforward and can easily be programmed using computer software such as 'Excel' and 'Matlab'. Also, the computer time required to solve a particular problem is very minimal.

Table 9. Sample optimal non-homogeneous transition probabilities obtained using 10 condition states for non-parabolic deterioration trend.

Year (k)	$\lambda''(k)$	$P''(k)_{1,2}$	$P''(k)_{9,10}$	$DR''_P(k)$	$DR_o(k)$	$SE''(k)^a$
1	0.54	0.48	0.40	85.02	85	0.0004
2	0.37	0.26	0.49	79.99	80	0.0001
3	0.65	0.40	0.58	70.02	70	0.0004
4	0.40	0.22	0.41	64.99	65	0.0001
5	0.50	0.55	0.42	54.97	55	0.0009
6	0.20	0.35	0.37	49.97	50	0.0009
7	0.64	0.39	0.50	40.04	40	0.0016

[a]Minimal $SSE = \sum SE''(k) = 0.0044$.

Disclosure statement

No potential conflict of interest was reported by the author(s).

References

Abaza, K.A., 2016. Back-calculation of transition probabilities for Markovian-based pavement performance prediction models. *International Journal of Pavement Engineering*, 17 (3), 253–264.

Abaza, K., 2017. Empirical approach for estimating the pavement transition probabilities used in non-homogenous Markov chains. *International Journal of Pavement Engineering*, 18 (2), 128–137. doi:10.1080/10298436.2015.1039006.

Abaza, K.A. and Ashur, S.A., 2009. Optimum microscopic pavement management model using constrained integer linear programming. *International Journal of Pavement Engineering*, 10 (3), 149–160.

Abaza, K.A., Ashur, S.A., and Al-Khatib, I., 2004. Integrated pavement management system with a Markovian prediction model. *Journal of Transportation Engineering*, 130 (1), 24–33.
Abed, A., Thom, N., and Neves, L., 2019. Probabilistic prediction of asphalt pavement performance. *Road Materials and Pavement Design*, doi:10.1080/14680629.2019.1593229.
Al-Omari, B. and Darter, M.I., 1994. Relationships between international roughness index and present serviceability rating. *Transportation Research Record: Journal of the Transportation Research Board*, 1435, 130–136.
American Association of State Highway and Transportation Officials (AASHTO), 1993. *AASHTO guide for design of pavement structures*. Washington, DC: AASHTO.
American Standard Testing Manual, 2007. Standard practice for roads and parking lots pavement condition index surveys, D6433-07, Philadelphia. Available from: http://www.astm.org/Standards/D6433.htm.
Amin, Md Shohel Reza, 2015. The pavement performance modeling: deterministic vs. stochastic approaches. In: S. Kadry and A. El Hami, eds. *Numerical methods for reliability and safety assessment*. Cham, Switzerland: Springer International, 179–196.
Cirilovic, J., Mladenovic, G., and Queiroz, C., 2015. Implementation of preventive maintenance in network-level optimization. *Transportation Research Record: Journal of the Transportation Research Board*, 2473, 49–55.
Daniel, O., Arce, G., and Zhang, Z., 2019. Skid resistance deterioration model at the network level using Markov chains. *International Journal of Pavement Engineering*, doi:10.1080/10298436.2019.1578882.
Ferreira, A., Antunes, A., and Picado-Santos, L., 2002. Probabilistic segment-linked pavement management optimization model. *Journal of Transportation Engineering*, 128 (6), 568–577.
Fuentes, L., *et al.*, 2019. Modelling pavement serviceability of urban roads using deterministic and probabilistic approaches. *International Journal of Pavement Engineering*, doi:10.1080/10298436.2019.1577422.
Gao, L. and Zhang, Z., 2008. Robust optimization for managing pavement maintenance and rehabilitation. *Transportation Research Record: Journal of the Transportation Research Board*, 2084, 55–61.
Jorge, D. and Ferreira, A., 2012. Road network pavement maintenance optimization using the HDM-4 pavement performance prediction models. *International Journal of Pavement Engineering*, 13, 39–51.
Kobayashi, K., Do, M., and Han, D., 2010. Estimation of Markovian transition probabilities for pavement deterioration forecasting. *KSCE Journal of Civil Engineering*, 14 (3), 343–351.
Lethanh, N. and Adey, B., 2013. Use of exponential hidden Markov models for modelling pavement deterioration. *International Journal of Pavement Engineering*, 14 (7), 645–654.
Lethanh, N., Kaito, K., and Kobayashi, K., 2015. Infrastructure deterioration prediction with a poisson hidden Markov model on time series data. *Journal of Infrastructure Systems*, 21 (3), 04014051.
Li, N., Xie, W.C., and Haas, R., 1996. Reliability-based processing of Markov chains for modeling pavement network deterioration. *Transportation Research Record: Journal of the Transportation Research Board*, 1524, 203–213.
Mathew, B.S. and Isaac, K.P., 2014. Optimization of maintenance strategy for rural road network using genetic algorithm. *International Journal of Pavement Engineering*, 15, 352–360.
Meidani, H. and Ghanem, R., 2015. Random Markov decision processes for sustainable infrastructure systems. *Structure and Infrastructure Engineering*, 11 (5), 655–667.
Ortiz-Garcia, J., Costello, S., and Snaith, M., 2006. Derivation of transition probability matrices for pavement deterioration modeling. *Journal of Transportation Engineering*, 132 (2), 141–161.
Park, K., Thomas, N., and Wayne Lee, K., 2007. Applicability of the international roughness index as a predictor of asphalt pavement condition. *Journal of Transportation Engineering*, 133 (12), 706–709.
Paterson, W., 1986. International roughness index: relationship to other measures of roughness and riding quality. *Transportation Research Record: Journal of the Transportation Research Board*, 1084, 49–58.
Saliminejad, S. and Perrone, E., 2015. Optimal programming of pavement maintenance and rehabilitation activities for large-scale networks. In: *Transportation research Board 94th Annual Meeting (No. 15-0422)*, Washington, DC.
Sayers, M.W., Gillispie, T.D., and Queiroz, C., 1986. *The international road roughness experiment: establishing correlation and a calibration standard for measurements*. Washington, DC: The World Bank.
Shah, Y., *et al.*, 2013. Modeling the pavement serviceability index for urban roads in Noida. *International Journal of Pavement Research and Technology*, 6 (1), 66–72.
Wang, K.C.P., Zaniewski, J., and Way, G., 1994. Probabilistic behavior of pavements. *Journal of Transportation Engineering*, 120 (3), 358–375.
Yang, J., *et al.*, 2006. Modeling crack deterioration of flexible pavements: comparison of recurrent Markov chains and artificial neural networks. *Transportation Research Record: Journal of the Transportation Research Board*, 1974, 18–25.
Zhang, X. and Gao, H., 2012. Road maintenance optimization through a discrete-time semi-Markov decision process. *Reliability Engineering and System Safety*, 103, 110–119.