[10] S.R. LÓPEZ-PERMOUTH, *Rings characterized by their weakly injective modules*, Glasgow Math. J, 1992, 349-353.

[11] M. SALEH, *A characterization of semisimple rings*, Muátah J., 1996, 265-278.

[12] M. SALEH, *A note on tightness*, Glasgow Math. J.,1999, 43-44.

[13] R. WISBAUER, **Foundations of Module and Ring Theory**, Gordon and Breach, 1991.

[14] Y. ZHOU, *Notes on weakly semisimple rings*, Bull. Aust. Math. Soc., 1996, 517-525.

N-COMPACTNESS AND θ-CLOSED SETS

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Abstract

In this paper we introduce a new generalizations of $\delta-$closed and $\delta-$open sets. Using these sets, we obtain a new characterization of $H-$closed spaces. Among other results, it is shown that an N-compact space over which every one point set is $\theta-$closed is a completely regular normal space.

1. Introduction. The concepts of *$\delta-$closure and $\theta-$closure* operators were first introduced by Veličko [16]. Although $\theta-$interior and $\theta-$closure operators are not idempotents, the collection of all $\delta-$open sets in a topological space (X,Γ) forms a topology Γ_s on X, called the semiregularization topology of Γ weaker than Γ and the class of all regular open sets in Γ forms an open basis for Γ_s, and the collection of all $\theta-$open sets in a topological space (X,Γ) forms a topology Γ_θ on X weaker than Γ_s. So far, numerous applications of such operators have been found in studying different types of continuous like maps, axioms of separation, and above all, to many important types of compact like properties. For a set A in a space X, let us denote by $Int(A)$ or A° and $cls(A)$ or $\overline{A}$ for the interior and the closure of A in X, respectively.

Following Veličko, a point x of a space X is called a *$\delta-$adherent point* of a subset A of X iff $Int(clsU) \cap A \neq \emptyset$, for every open set U containing x.

The set of *all* $\delta-$*adherent points of* A *is called the* $\delta-$*closure of* A, *denoted by* $cls_\delta A$. A subset A of a space X is called $\delta-$*closed* iff $A = cls_\delta A$. The complement of a $\delta-$closed set is called $\delta-$open. A point x of a space X is called a $\theta-$*adherent point* of a subset A of X iff $clsU \cap A \neq \emptyset$, for every open set U containing x. The set of all $\theta-$*adherent points of* A *is called the* $\theta-$*closure of* A, denoted by $cls_\theta A$. A subset A of a space X is called $\theta-$*closed* iff $A = cls_\theta A$. The complement of a $\theta-$closed set is called $\theta-$open. A topological property $\mathcal{P}$ is *semi-regular* provided that a topological space (X,Γ) has property $\mathcal{P}$ iff (X,Γ_s) has property $\mathcal{P}$. A space (X,Γ) is called *semi-regular* iff $\Gamma = \Gamma_s$ and it is called *almost regular* if Γ_s is regular. Clearly every $\delta-closed(\delta-open)$ set is closed (open) but not conversely (see[16]). Over a semi- regular space the converse is also true. A set A in a topological space (X,Γ) is called *semiopen* if $Int_\Gamma A \subseteq A \subseteq cls_\Gamma(Int_\Gamma A)$. A space X is called Urysohn if for every $x \neq y \in X$, there exist open sets U, V containing x, y respectively, such that $\overline{U} \cap \overline{V} = \emptyset$. It is well-known that one of the most weaker forms of compactness is *closure compactness* (QHC). Also, it is well-known that closure compactness and near compactness are semi-regular properties. *A closure compact Hausdorff space is called H−closed*, first defined by Alexandroff and Urysohn.

The motivation of this paper is to derive certain characterizations of QHC spaces by the applications of two types of sets, introduced here in terms of $\delta-$closure and $\delta-$interior operators. We get similar results to some of those contained in [3, 4, 5, 8, 11, 14, 15, 16]. It is well-known that a compact subset of a Hausdorff space is closed but not conversely. It is easy to see that a space X over which, for every $x \neq y \in X$ either $\{x\}$ or $\{y\}$ is $\theta-$closed must be Hausdorff. Moreover, a space X is Hausdorff iff every compact subset is $\theta-$closed. Also, if every one point set of an N-compact space X is $\theta-$closed then X is a completely regular normal space. Also, we show that a space X is Urysohn iff every H-closed subset of X is $\theta-$closed.

2. Basic results.

Clearly $clsA \subset cls_\delta A$, but not equal even over a Urysohn space as it is shown in the next example. Over a semi- regular space, it is clear that $clsA = cls_\delta A$.

Example 2.1. Let $X = \Re$ with the topology $\Im$ generated by a basis with members of the form (a, b) and $(a, b) - K$, where $K = \{\frac{1}{n} : n \in Z^+\}$. Then K is closed but not $\delta-$closed.

Definition 2.2. *A set* A *in a space* X *is called*
(a) *a* $\delta^o -$ *set iff* $A = Int_\delta B$, *for some* $B \subseteq X$,
(b) *a* $\delta^c -$ *set iff* $A = cls_\delta B$, *for some* $B \subseteq X$.

Remarks 2.3.
(I) *It is easy to see that for a set* A *in a space* X, $cls_\delta(X-A) = X - Int_\delta A$. *Thus a set* A *in* X *is a* $\delta^o -$ *set iff* $X - A$ *is a* $\delta^c -$ *set.*
(II) *It is well-known that* $cls_\delta(A \cup B) = cls_\delta A \cup cls_\delta B$, *for any subsets* A, B *of* X. *Thus* $Int_\delta(A \cap B) = Int_\delta A \cap Int_\delta B$, *for any subsets* A, B *of* X.
(III) *Let* $B_\delta = \{Int_\delta A : A \subseteq X\}$, *then* B_δ *forms a basis for some topology* $\Gamma_{(B_\delta)}$ *on* X.
(IV) *For an open set* U, $cls_\delta U = cls_\theta U = clsU$. *Moreover,* $clsU$ *is* $\delta-closed$.

It is well-known that every regularly open set is $\delta - open$ and thus δ^o- set. Consequently, the above remarks lead to the following.

Proposition 2.4. *For a space* (X,Γ), $\Gamma_{(B_\delta)} = \Gamma_s$, *and thus* B_δ *forms an open basis for the semiregularization topology* Γ_s *on* X.

Recall that $A \subset X$ is called N-closed relative to X or *nearly compact* iff every open cover of X has a finite subcollection whose interiors of their closures cover X. Equivalently, A is nearly compact iff every cover of regularly open sets of X has a finite subcover of A. A space X is called *an S-closed subset* iff for every semi-open cover $\{U_\alpha | \alpha \in \Lambda\}$ of X there exists a finite subcollection $\{U_{\alpha_i} | i = 1, ..., n\}$ such that the unions of their closures cover X.

The next two results follow immediately from the well-known characterizations of nearly compact and S-closed spaces in terms of regularly open sets. The proof follows directly from [14, Theorem 2.1] and [15, Theorem 2].

Theorem 2.5. *For a space* X, *the following are equivalent:*
(a) X *is nearly compact,*
(b) *Every cover of* X *by* $\delta^o -$ *sets has a finite subcover,*

(c) *Every family of* $\delta^c -$ *sets with the finite intersection property has non-void intersection.*

Theorem 2.6. *For a space* X*, the following are equivalent:*
(a) X *is S-closed,*
(b) *Every cover of* X *by* $\delta^c -$ *sets has a finite subcover,*
(c) *Every family of* $\delta^o -$ *sets with the finite intersection property has non-void intersection.*

Corollary 2.7. *For a space* X*, the following are equivalent:*
(a) X *is nearly compact,*
(b) *Every cover of* X *by* $\delta -$ *open sets has a finite subcover,*
(c) *Every family of* $\delta -$ *closed sets with the finite intersection property has non-void intersection.*

Corollary 2.8. *For a space* X*, the following are equivalent:*
(a) X *is S-closed,*
(b) *Every cover of* X *by* $\delta -$ *closed sets has a finite subcover,*
(c) *Every family of* $\delta -$ *open sets with the finite intersection property has non-void intersection.*

3. H-closed spaces

Recall that $A \subset X$ is called *closure compact(QHC or quasi H-closed)* iff every open cover of X has a finite proximate subcover (every open cover has a finite subfamily whose closures cover the space X). *A closure compact Hausdorff space is called H-closed.*

Theorem 3.1. *A Hausdorff space* (X,Γ) *is H-closed iff whenever* $\mathcal{U}$ *is a cover of* X *by* $\delta^c -$ *sets, such that for each* x *of* X *some member of* $\mathcal{U}$ *is a neighborhood of* x*, then* $\mathcal{U}$ *has a finite subcover.*

Proof. Let X be H-closed and $\mathcal{U}$ be a cover of X by $\delta^c -$ *sets* with the given property. For each $x \in X$, there exists some $U_x \in \mathcal{U}$ and an open set V_x such that $x \in V_x \subseteq U_x$. Then $\{V_x : x \in X\}$ is an open cover of X. Since X is H-closed, there is a finite subset $\{x_1, ..., x_n\}$ of X such that

$X = \cup_{i=1}^{n} clsV_{x_i} \subseteq \cup_{i=1}^{n} clsU_{x_i} = \cup_{i=1}^{n} U_{x_i}$, since U_{x_i} 's are closed sets. Conversely, let the given condition holds for a space X, and $\mathcal{U}$ be an open cover of X. For each $x \in X$, there exists $U_x \in \mathcal{U}$ such that $x \in U_x$. Since U_x is open, $clsU_x = cls_\delta U_x$ and thus $clsU_x$ is a $\delta^c -$ *set, say,* V_x for each $x \in X$. Then $\{V_x : x \in X\}$ is a cover of X by $\delta^c -$ *sets* with the stated property. By our hypothesis, $X = \cup_{i=1}^{n} V_{x_i} = \cup_{i=1}^{n} clsU_{x_i}$, for a finite subset $\{x_1, ..., x_n\}$ of X, proving that X H-closed.

Definition 3.2. *A family* $\mathcal{F}$ *of sets in a space* (X,Γ) *is said to have* δ^o-*FIP iff every finite intersections of members of* $\mathcal{F}$ *has non-void* $\delta -$ *interior.*

Theorem 3.3. *A Hausdorff space* (X,Γ) *is H-closed iff every family of* $\delta^c -$ *sets with* δ^o-*FIP has non-null intersections.*

Proof. Let X be H-closed and $\{F_\alpha : \alpha \in \Lambda\}$ be a family of $\delta^c -$ *sets* in X with δ^o-FIP. If $\bigcap_{\alpha\in\Lambda} F_\alpha = \emptyset$, then $\mathcal{U} = \{X - F_\alpha : \alpha \in \Lambda\}$ is a cover of X by $\delta^o -$ *sets*. Since X is H-closed, $\mathcal{U}$ has a finite subcollection, say, $\{X - F_i : i = 1, ..., n\}$ and $X = \cup_{i=1}^{n} cls(X - F_i)$. Thus $\emptyset = X - \cup_{i=1}^{n} cls(X - F_i) = \cap_{i=1}^{n} IntF_i = \cap_{i=1}^{n} Int_\delta F_i = Int_\delta(\cap_{i=1}^{n} F_i)$, contradicting the hypothesis that $\{F_\alpha : \alpha \in \Lambda\}$ has δ^o-FIP. Conversely, let the given condition holds for a space X, we first show that (X, Γ_s) is H-closed. Since $\delta^o -$ *sets* form an open basis for Γ_s, it is enough to show that every cover of X by $\delta^o -$ *sets* has a finite $\Gamma_s -$ *proximate cover*. So let $\mathcal{V} = \{V_\alpha : \alpha \in \Lambda\}$ be a cover of X by $\delta^o -$ *sets*. Then $\{X - V_\alpha : \alpha \in \Lambda\}$ is a family of $\delta^c -$ *sets* with $\cap\{(X - V_\alpha) : \alpha \in \Lambda\} = \emptyset$, and thus this family cannot have δ^o-FIP. Thus there exists finitely many sets $(X - V_{\alpha_i}) : i = 1, ..., n$ such that $Int_\delta(\cap_{i=1}^{n}(X - V_{\alpha_i})) = \emptyset$. Hence $X = X - Int_\delta(\cap_{i=1}^{n}(X - V_{\alpha_i})) = cls_\delta(\cup_{i=1}^{n}(V_{\alpha_i}) = \cup_{i=1}^{n}(cls_\delta V_{\alpha_i}) = \cup_{i=1}^{n}(cls_{\Gamma_s} V_{\alpha_i})$, proving that (X, Γ_s) is H-closed. Since H-closeness is a semi-regular property, it follows that (X,Γ) is H-closed.

Definition 3.4. *Let* $\{U_\alpha : \alpha \in \mathcal{D}\}$ *be a net of* $\delta^o -$ *sets in a space* X *with the directed set* $(\mathcal{D}, \geq)$ *as its domain. A point* x *of* X *is said to be a* δ^o-*adherent point of the net iff for each* $\alpha \in \mathcal{D}$, and *for every open neighborhood* U *of* x*, there is a* $\beta \in \mathcal{D}$ *with* $\beta \geq \alpha$ *such that* $U_\beta \cap clsU \neq \emptyset$.

Theorem 3.5. *A Hausdorff space (X, Γ) is H-closed iff every net of $\delta^o - sets$ has a δ^o – adherent point.*

Proof. Let $\{U_\alpha : \alpha \in \mathcal{D}\}$ be a net of $\delta^o - sets$ in H-closed X. For each $\alpha \in \mathcal{D}$, let $F_\alpha = cls_\delta(\{\cup U_\beta : \beta \in \Lambda \text{ and } \beta \geq \alpha\})$. Then $\mathcal{F} = \{F_\alpha : \alpha \in \mathcal{D}\}$ is a family of $\delta^c - sets$ with δ^o–FIP. By Theorem 3.3, there exists an $x \in \bigcap_{\alpha \in \mathcal{D}} F_\alpha$. Then for any open nbhd U of x and any $\alpha \in \mathcal{D}$, $clsU \cap (\{\cup U_\beta : \beta \in \Lambda \text{ and } \beta \geq \alpha\}) \neq \emptyset$. Thus there is a $\beta \in \mathcal{D}$ with $\beta \geq \alpha$ such that $clsU \cap U_\beta \neq \emptyset$. Hence the net $\{U_\alpha : \alpha \in D\}$ of $\delta^o - sets$ in X has a $\delta^o - adherent$ point in X. Conversely, let $\mathcal{F}$ be a collection of $\delta^c - sets$ in X with δ^o–FIP. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of $\mathcal{F}$ directed by the relation $\geq$ where $F_1 \geq F_2$ iff $F_1 \subseteq F_2$ $(F_1, F_2 \in \mathcal{F}^*)$. For each $F \in \mathcal{F}^*$, we assign the set $Int_\delta F$, which is none empty, as $\mathcal{F}$ has δ^o–FIP. Then $\{Int_\delta F : F \in (F^*, \geq)\}$ is a net of $\delta^o - sets$ in X. By our hypothesis, some x *of* X is a $\delta^o - adherent$ point of this net. In view of Theorem 3.3, it is enough to show that $x \in \bigcap_{F \in \mathcal{F}} F$. In fact, let $F \in \mathcal{F}$ and V be an open nbhd of x. Since $F \in \mathcal{F}^*$, there exists $G \in \mathcal{F}^*$ with $G \geq F$ such that $Int_\delta G \cap clsV \neq \emptyset$, and thus $Int_\delta F \cap cls\ V \neq \emptyset$. Thus $x \in cls_\delta(Int_\delta F) = cls(IntF) \subseteq F$, since F, being a $\delta^c - set$, is closed. Thus $x \in F$, for each $F \in \mathcal{F}$, and consequently, $x \in \bigcap_{F \in \mathcal{F}} F$.

4. N-compactness and θ–closed sets.

It is well-known that a compact subset of a Hausdorff space is closed but not conversely. It is easy to see that a space X is a Hausdorff space iff for every $x \in X$, $\{x\}$ is θ–closed. Moreover, a closure compact subset of a Hausdorff space is θ–closed. The next result is similar for nearly compact subsets.

Theorem 4.1. *A nearly compact subset of a Hausdorff space X is θ–closed.*

Proof. Let A be a nearly compact subset of a Hausdorff space X and let $x \notin A$. Then for each $a \in A$ there exist $U_{x,a}$ and V_a such that $\overline{U}_{x,a} \cap \overline{V}^o_a = \emptyset$. The collection $\{\overline{V}^o_a : a \in A\}$ is a cover by regularly open sets of A. Therefore, there exists a finite subcollection $\overline{V}^o_1, ..., \overline{V}^o_n$ that cover A. Let $U = \overline{U}_1 \cap ... \cap \overline{U}_n$. Then $\overline{U} \cap A = \emptyset$. Thus $x \notin cls_\theta A$, proving that A is θ–closed.

Indeed over a compact space a stronger result is obtained if we assume all one point sets are θ–closed as it is shown in the next theorem.

Definition 4.2. *A space X is said to be N-compact if every closed subset of X is N-closed relative to X.*

Clearly every compact space is N-compact but not conversely as the space of reals with the cocountable topology is N-compact but not compact.

Theorem 4.3. *An N-compact space X is a compact completely regular normal space if every one point set is θ–closed.*

Proof. First of all we will show that X is regular. Let A be a closed subset of X and let $x \notin A$. Since $\{x\}$ is θ–closed, for every $a \in A, a \notin cls_\theta\{x\}$. Hence for every $a \in A$ there exists an open neighborhood V_a of a such that $\overline{V}_a \cap \{x\} = \emptyset$. This implies that $x \notin \overline{V}_a$. Let $C = \{\overline{V}^o_a : a \in A\}$. Then C is a cover of A by regularly open sets. Since A is closed, A is N-closed relative to X. Therefore, there exists a finite subcover of A, say, $\overline{V}^o_{a_1}, ..., \overline{V}^o_{a_n}$. Hence, $x \notin \overline{V}_{a_1} \cup ... \cup \overline{V}_{a_n}$. Therefore, there exists U an open nbhd of x such that $U \cap (\overline{V}^o_{a_1} \cup ... \cup \overline{V}^o_{a_n}) = \emptyset$, proving that X is a regular space. But since one point sets are θ–closed, then X is T_2. Therefore, X is a T_1–regular space. Thus X is a compact Hausdorff space. It follows by Theorem 5.4.6 in [17] that X is a completely regular normal space.

As consequences of Theorem 4.3 we get the following corollaries.

Corollary 4.4. *Let X be a Hausdorff space such that every closed subset of X is N-closed relative to X. Then X is a compact and completely regular normal space.*

Corollary 4.5. *A Hausdorff N-compact space X is compact.*

Corollary 4.6. *A compact space X is normal and completely regular space iff every one point set is θ–closed.*

Corollary 4.7. *If X is compact and every one point set is θ–closed, then* *a subset A of X is $\theta-$closed iff it is closed.*

It is natural to ask if Corollary 4.7 still holds over a closure compact space X, the answer is no, Example 4.8 (d) in [13] is such an example.

In [4, 2.4] it is pointed out that a space X is regular iff for every $A \subseteq X, \overline{A} = cl_\theta A$. The next result is a sharper one.

Theorem 4.8. *Let X be a space such that every closed subset is $\theta-$closed. Then X is a regular space.*

Proof. Let A be a closed subset of X and let $x \notin A$. Since A is $\theta-$closed, there exists a nbhd U of x such that $\overline{U} \cap A = \emptyset$. Hence, for every $a \in A$ there exists a neighborhood V_a of a such that $V_a \cap U = \emptyset$. Let $V = \cup_{a \in A} V_a$. Then $U \cap V = \emptyset$, proving that X is a regular space.

In [16] it is shown that over a Urysohn H-closed space the class of H-closed sets coincides with the class of $\theta-$closed sets. The next two results are a generalization of [16, Theorem 4].

Theorem 4.9. *A $\delta-$closed subset of nearly compact is nearly compact.*

Proof. Let A be a $\delta-$closed subset of a nearly compact space X and let $\mathcal{C}$ be a cover of A by regularly open sets. Since $X \backslash A$ is $\delta-$open, for each $x \in X \backslash A$ there exists an open set U_x such that $\overline{U}_x^\circ \subseteq X \backslash A$. Thus $\mathcal{V} = \mathcal{C} \cup \{\overline{U}_x^\circ : x \in X \backslash A\}$ is a cover of X by regularly open sets. Since X is nearly compact, there exists a finite subcollection $\mathcal{U}$ of $\mathcal{V}$ that cover X. Hence, $\mathcal{U} \cap \mathcal{C}$ is a finite subcollection of $\mathcal{C}$ that cover A, proving that A is nearly compact.

Corollary 4.10. *Every clopen subset of a nearly compact space is nearly compact.*

Theorem 4.11. *A quasi-H-closed space X is Urysohn iff every quasi H-closed subset is $\theta-$closed.*

Proof. Let A be a quasi H-closed subset of a Urysohn space X and let $x \notin A$. Then for each $a \in A$ there exist $U_{x,a}$ and V_a such that $\overline{U}_{x,a} \cap \overline{V}_a = \emptyset$. The collection $\{V_a : a \in A\}$ is an open cover of A. Therefore, there exists a finite subcollection $V_1, ..., V_n$ whose closures cover A. Let $U = U_1 \cap ... \cap U_n$. Since $\overline{U} \cap (\cup_{i=1}^n \overline{V_i}) \subseteq \cup_{i=1}^n (\overline{U_i} \cap \overline{V_i}) = \emptyset$, $\overline{U} \cap A = \emptyset$. Thus $x \notin cls_\theta A$, proving that A is $\theta-$closed. Conversely, since one point sets are compact it follows that X is Hausdorff. Let $x \neq y \in X$. Then there exists an open set containing x such that $y \notin \overline{U}$. By [13, 4.8(e)] it follows that $\overline{U}$ is quasi H-closed and thus $\theta - closed$. Thus there exists an open set V containing y such that $\overline{U} \cap \overline{V} = \emptyset$, proving that X is Urysohn.

Recall that a function $f : X \to Y$ is *weakly continuous (resp.,* closure *continuous)* if for every open set V of Y there exists an open set U of X such that $f(U) \subseteq \overline{V}$ (resp., $f(\overline{U}) \subseteq \overline{V}$).

Theorem 4.12. *Let $f : X \to Y$ be weakly continuous $1-1$, onto. If X is compact, Y Urysohn then the image of every open set is $\theta-$open.*

Proof. Let U be an open subset of X, and thus $X \backslash U$ is a closed subset of X. Hence, $X \backslash U$ is compact. Since f is weakly continuous, $f(X \backslash U)$ is closure compact. Therefore, $f(X \backslash U) = Y \backslash f(U)$ is $\theta-$closed, and thus $f(U)$ is $\theta-$open.

Following as in the proof of Theorem 4.13 we get the following results.

Theorem 4.13. *Let $f : X \to Y$ be weakly continuous. If X is compact, Y Urysohn then the image of every closed set is $\theta-$closed.*

Theorem 4.14. *Let $f : X \to Y$ be closure continuous $1-1$, onto. If X is closure compact, Y Urysohn then the image of every $\theta-$open set is $\theta-$open.*

Theorem 4.15. *Let $f : X \to Y$ be closure continuous. If X is closure compact, Y Urysohn then the image of every $\theta-$closed set is $\theta-$closed.*

Theorem 4.16. *Let $f : X \to Y$ be closure continuous $1-1$, onto. If X is closure compact, Y Hausdorff then the image of every $\theta-$open set is open.*

Theorem 4.17. *Let $f : X \to Y$ be closure continuous. If X is closure compact, Y Hausdorff then the image of every θ−closed set is closed.*

Recall that *a Hausdorff space is called C-compact* if each closed set in X is closure compact.

Theorem 4.18. *Let $f : X \to Y$ be continuous and let X be C-compact and Y is a Hausdorff space. If f is bijective, then X is homeomorphic to Y and Y is C-compact.*

Proof. Since f is one-to-one and Y is Hausdorff, X is Hausdorff. Let A be a closed subset of X. Since X is C-compact, A is closure compact. Since f is continuous, it follows that $f(A)$ is closure compact and thus a closed subset of Y. It follows that f is a bijective closed continuous map and thus a homeomorphism. Now it follows easily that Y is C-compact.

The next theorem follows from the fact that an inverse image of a θ−closed subset under a closure continuous is θ−closed and from Theorem 2.1 in [5]. A function $f : X \to Y$ is called quasi *θ−continuous* if the inverse image of every θ−open is θ−open. Notice that every closure continuous is quasi θ−continuous.

Theorem 4.19. *Let X be a Hausdorff space and $A \subseteq X$. Then the following are equivalent:*

(a) *A is θ−closed in X;*

(b) *X/A is Hausdorff;*

(c) *A is the inverse image of a θ-closed subset of a continuous function from X into a Hausdorff space;*

(d) *A is the point inverse of a continuous function from X into a Hausdorff space;*

(e) *A is the inverse image of a θ-closed subset of a closure continuous function from X into a Hausdorff space;*

(f) *A is the point inverse of a closure continuous function from X into a Hausdorff space;*

(g) *A is the inverse image of a θ-closed subset of a quasi θ−continuous function from X into a Hausdorff space;*

(h) *A is the point inverse of a quasi θ−continuous function from X into a Hausdorff space.*

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REFERENCES

1 P. Alexandroff and P. Urysohn, *Mémoire sur les espaces topologiques compacts*, Verh. Nederl. Akad. Wetensch. Afd. Natuurk. Sect. I, 14(1929), 1 − 96.

2 D. Cameron, *Maximal QHC-Spaces*, Rocky Mount. J.Math, 7(1977), 313 − 322.

3 D. Cameron, *Properties of S-closed Spaces*, Proc. Amer. Math. Soc., 72(1978), 581−586.

4 R.F. Dickman and J.R. Porter, *θ−perfect and θ−absolutely Closed Functions*, Illinois. J. Math., 21(1977), 42 − 60.

5 R. Dickman and J. Porter, *θ−closed Subsets of Hausdorff Spaces*, Pacific J. Math., 59(1975), 407-415.

6 M. Girou, *Properties of Locally H-closed Spaces*, Proc. Amer. Math. Soc., 113(1991), 287 − 295.

7 L. Herrington, *Properties of Nearly-compact Spaces*, Proc. Amer. Math. Soc., 45(1974), 431 − 436.

8 L. Herrington and P. Long, *Characterizations of H-compact Spaces*, Proc. Amer. Math. Soc. 48(1975), 469 − 475.

9 J. Joseph, *Multifunctions and Cluster Sets*, Proc. Amer. Math. Soc., 74(1979), 329−337.

10 Kocinac, and Ljubisa, *On the Cardinality of Urysohn Spaces*, Questions and Answers in General Topology, 13(2)(1995), 211 − 216.

11 M.N. Mukherjee, and S. Raychaudhuri, *Some Applications of θ-closure Operators*, Indian Journal of Pure and Applied Mathematics, 26(5)(1995), 433 − 439.