

[14] Rummier, B., Breitschuh, U. Berechnung der turbulenten Couette-Strömung mittels Galerkinäherung der Navier-Stokes-Gleichung - Ergebnisse und Probleme, ZAMM 64 (1984) 10, M 489-492

[15] Temam, R. Navier-Stokes equations, theory and numerical analysis. Amsterdam: North Holland (1979)

[16] Višik, M.J., Fursikov, A.V. Mathematische Probleme der statistischen Hydromechanik. Leipzig: Geest & Portig (1986)

ADDRESS: PD Dr. Bernd Rummier, Dept. of Mathematics, Otto-von-Guericke-Universität Magdeburg, PF 4120, D-39016 Magdeburg, Germany, email: bernd.rummier@mathematik.uni-magdeburg.de

# On Super and $\delta$ -Continuities

Mohammad Saleh\*

Mathematics Department, Birzeit University, P.O. Box 14,  
Birzeit, West Bank, Palestine  
Email: msaleh@birzeit.edu

## Abstract

In this paper, we further the study of super and  $\delta$ -continuities using  $\delta$ -open and  $\theta$ -open sets. Among others, it is shown that the graph mapping  $g$  of  $f : X \rightarrow Y$  is Super continuous iff  $f$  is super continuous and  $X$  is semi-regular.

**1. INTRODUCTION.** The concepts of  $\delta$ -closure,  $\theta$ -closure,  $\delta$ -interior and  $\theta$ -interior operators were first introduced by Veličko [14]. These operators have since been studied intensively by many authors. The collection of all  $\delta$ -open sets in a topological space  $(X, \Gamma)$  forms a topology  $\Gamma_s$  on  $X$ , called the semiregularization topology of  $\Gamma$ , weaker than  $\Gamma$  and the class of all regular open sets in  $\Gamma$  forms an open basis for  $\Gamma_s$ . Similarly, the collection of all  $\theta$ -open sets in a topological space  $(X, \Gamma)$  forms a topology  $\Gamma_\theta$  on  $X$ , weaker than  $\Gamma_s$ . So far, numerous applications of such operators have been found in studying different types of continuous like maps, separation of axioms, and above all, to many important types of compact like properties.

In the present paper, we further the study of  $\delta$ -continuity [8] and  $s.\delta.c$  or super continuity in the sense of Munshi and Bassan [6]. We give several characterizations to  $\delta.c$  and  $s.\delta.c$  maps, and we study the relations between these functions and their graphs. Theorem 2.13 proves that the graph mapping  $g = (x, f(x)) : X \rightarrow X \times Y$  is  $s.\delta.c$  iff  $f : X \rightarrow Y$  is  $\delta.c$  and  $X$  is

\*The author was supported by Birzeit University under grant 235-17-98-9.  
AMS Subject Classification: 54C08, 54D05, 54D30

semi-regular. Also, it is shown that the preimage of a Hausdorff injective  $\delta.c$  is Hausdorff, and the image of compact is n. compactness under  $\delta.c$  functions. Theorem 3.1 proves that an  $\delta.c$  retract of a Hausdorff space is  $\delta$ -closed. We get similar results to some of those in [3, ..., 13].

For a set  $A$  in a space  $X$ , let us denote by  $Int(A)$  and  $\bar{A}$  for the interior and the closure of  $A$  in  $X$ , respectively. Following Veličko [14], a point  $x$  of a space  $X$  is called a  $\theta$ -adherent point of a subset  $A$  of  $X$  iff  $\bar{U} \cap A \neq \emptyset$ , for every open set  $U$  containing  $x$ . The set of all  $\theta$ -adherent points of  $A$  is called the  $\theta$ -closure of  $A$ , denoted by  $cl_{\theta}A$ . A subset  $A$  of a space  $X$  is called  $\theta$ -closed iff  $A = cl_{\theta}A$ . The complement of a  $\theta$ -closed set is called  $\theta$ -open. Similarly, the  $\theta$ -interior of a set  $A$  in  $X$ , written  $Int_{\theta}A$ , consists of those points  $x$  of  $A$  such that for some open set  $U$  containing  $x$ ,  $\bar{U} \subseteq A$ . A set  $A$  is  $\theta$ -open iff  $A = Int_{\theta}A$ , or equivalently,  $X - A$  is  $\theta$ -closed. A point  $x$  of a space  $X$  is called a  $\delta$ -adherent point of a subset  $A$  of  $X$  iff  $Int(\bar{U}) \cap A \neq \emptyset$ , for every open set  $U$  containing  $x$ . The set of all  $\delta$ -adherent points of  $A$  is called the  $\delta$ -closure of  $A$ , denoted by  $cl_{\delta}A$ . A subset  $A$  of a space  $X$  is called  $\delta$ -closed iff  $A = cl_{\delta}A$ . The complement of a  $\delta$ -closed set is called  $\delta$ -open. Similarly, the  $\delta$ -interior of a set  $A$  in  $X$ , written  $Int_{\delta}A$ , consists of those points  $x$  of  $A$  such that for some regularly open set  $U$  containing  $x$ ,  $U \subseteq A$ . A set  $A$  is  $\delta$ -open iff  $A = Int_{\delta}A$ , or equivalently,  $X - A$  is  $\delta$ -closed. It is well-known that one of the most weaker forms of compactness is closure compactness ( $QHC$ ). A subset  $A$  of a space  $X$  is called a closure compact subset of  $X$  if every open cover of  $A$  has a finite subcollection whose closures cover  $A$ . A closure compact Hausdorff space is called  $H$ -closed, first defined by Alexandroff and Urysohn.

A function  $f : X \rightarrow Y$  is almost continuous (briefly,  $a.c$ ) ( resp., almost strongly  $\theta$ -continuous (briefly,  $a.s.c$ ), closure or  $\theta$ -continuous in the sense of Fomin [2] (briefly,  $c.c.$ ), weakly continuous (briefly,  $w.c$ ),  $\delta$ -continuous (briefly,  $\delta.c$ ) strongly continuous or strongly  $\theta$ -continuous (briefly,  $s.c$ ) if for any open set  $V$  in  $Y$ , there exists an open set  $U$  in  $X$  such that  $f(U) \subseteq Int(\bar{V})$  ( resp.,  $f(\bar{U}) \subseteq Int(\bar{V})$ ,  $f(\bar{U}) \subseteq \bar{V}$ ,  $f(Int(\bar{U})) \subseteq Int(\bar{V})$ ,  $f(\bar{U}) \subseteq V$ ). A function  $f : X \rightarrow Y$  is strongly  $\delta$ -continuous (briefly,  $s.\delta.c$ ) ( or super continuous in the sense of B.M. Munshi, and D. Basan ) if for any open set  $V$  in  $Y$ , there exists an open set  $U$  in  $X$  such that  $f(Int(\bar{U})) \subseteq V$ . A space  $X$  is called Urysohn if for every  $x \neq y \in X$ , there exist an open set  $U$  containing  $x$  and an open set  $V$  containing  $y$  such that  $\bar{U} \cap V = \emptyset$ . A space  $X$  is called semi-regular if for every  $x \in X$  and for every

open set  $U$  of  $x$  there exists an open set  $W$  such that  $x \in W \subseteq Int(\bar{W}) \subset U$ , or equivalently,  $\Gamma_s = \Gamma$ . A space  $X$  is called almost regular if for every regularly closed set  $F$  and for every  $x \notin F$  there exist disjoint open sets  $U, V$  such that  $x \in U, F \subset V$ . Equivalently, a space  $X$  is almost regular if for every  $x \in X$  and for every regularly open set  $U$  containing  $x$  there exists a regularly open set  $W$  containing  $x$  such that  $x \in W \subseteq \bar{W} \subset U$ , or equivalently,  $\Gamma_s$  is regular.

2. BASIC RESULTS.

$s.c \Rightarrow s.\delta.c \Rightarrow continuity \Rightarrow a.c \Rightarrow c.c \Rightarrow w.c, s.c \Rightarrow s.\delta.c \Rightarrow \delta.c \Rightarrow a.c$ , and  $s.c \Rightarrow a.s.c \Rightarrow a.c$ , but neither continuity implies  $\delta.c$  nor  $\delta.c$  implies continuity, neither  $a.s.c$  implies continuity nor  $a.s.c$  implies continuity, and neither  $a.s.c$  implies  $\delta.c$  nor  $s.\delta.c$  implies  $a.s.c$ .

Example 2.1. Let  $X = \{1, 2, 3\}$ , with  $S_X = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$ ,  $Y = \{1, 2, 3\}$  with  $S_Y = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}$ . Let  $f : X \rightarrow Y$  be the identity map then  $f$  is continuous but  $f$  is not  $\delta.c$  nor  $a.s.c$  since  $Int(\bar{\{1\}}) = \{1\}$  in  $Y$ , but  $Int(\overline{\{1\}}) = \{1, 3\}$  in  $X$  and  $\overline{\{1\}} = \{1, 3\}$  in  $X$ , but  $Int(\overline{\{1\}}) = \{1\}$  in  $Y$ .

Example 2.2. Let  $f : (R_U) \rightarrow (R_C)$  be the identity map, where  $R_U, R_C$  be the usual and the countable topologies, respectively. Then  $f$  is  $a.s.c$  and  $\delta.c$  but neither  $s.\delta.c$  nor continuous.

Example 2.3. Let  $X = R$  with the topology  $S$  generated by a basis with members of the form  $(a, b)$  and  $(a, b) - K$ , where  $K = \{\frac{1}{n} : n \in Z^+\}$ . Let  $f : (X, S) \rightarrow (X, S)$ , be the identity map. Then  $f$  is continuous but not strongly continuous.

The proofs of the next results follow directly from the definitions and thus will be omitted.

Theorem 2.1. [8, Theorem 4.6(2)]. Let  $f : X \rightarrow Y$  be an  $a.c$  function and let  $X$  be a semi-regular space then  $f$  is  $\delta.c$ .

Theorem 2.2. Let  $f : X \rightarrow Y$  be  $\delta.c$  and let  $Y$  be a semi-regular space then  $f$  is  $s.\delta.c$ .

Theorem 2.3. Let  $f : X \rightarrow Y$  be an  $a.s.c$  function and let  $Y$  be a semi-regular space then  $f$  is  $s.c$ .

Theorem 2.4. Let  $f : X \rightarrow Y$  be an  $c.c$  function and let  $Y$  be an almost-

regular space then  $f$  is *s.a.c.*

**Theorem 2.5.** An open map  $f$  is *c.c.* iff  $f$  is  $\delta.c.$

**Theorem 2.6.** Let  $f : X \rightarrow Y$  be  $\delta.c.$  and let  $X$  be an almost regular space then  $f$  is *a.s.c.*

**Theorem 2.7.** Let  $f : X \rightarrow Y$  be an  $\delta.c.$  function and let  $Y$  be a semi-regular space then  $f$  is *s.d.c.*

**Theorem 2.8.** Let  $f : X \rightarrow Y$  be a continuous function and let  $X$  be an almost regular space then  $f$  is *s.d.c.*

**Theorem 2.9.** Let  $f : X \rightarrow Y$  be an *c.c.* function and let  $Y$  be a regular space then  $f$  is *s.d.c.*

**Theorem 2.10.** Let  $X, Y$  be regular spaces. Then the following concepts on a function  $f : X \rightarrow Y : w.c., c.c., a.c., a.s.c., s.c., \delta.c., s.d.c., continuity$  are equivalent.

Most of the following characterization of  $\delta.c., s.d.c.$  are given in [8], [6].

**Theorem 2.11.** Let  $f : X \rightarrow Y$ . Then the following are equivalent:

- a)  $f(\text{cls}_s A) \subset \text{cls}_s f(A)$  for every  $A \subset X$ ,
- b) The inverse image of *regularly* closed set is  $\delta$ -closed,
- c) The inverse image of *regularly* open set is  $\delta$ -open,
- d) The inverse image of  $\delta$ -closed set is  $\delta$ -closed,
- e) The inverse image of  $\delta$ -open set is  $\delta$ -open,
- f) For each  $x \in X$  and for each filter base  $F$   $\delta$ -converges to  $x$ ,  $f(F)$   $\delta$ -converges to  $f(x)$ ,
- g) For each  $x \in X$  and for each net  $\{x_\alpha\}_{\alpha \in D}$   $\delta$ -converges to  $x$ ,  $\{f(x_\alpha)\}_{\alpha \in D}$   $\delta$ -converges to  $f(x)$ ,
- h)  $f$  is  $\delta.c.$

**Theorem 2.12.** Let  $f : X \rightarrow Y$ . Then the following are equivalent:

- a)  $f(\text{cls}_s A) \subset f(A)$  for every  $A \subset X$ ,
- b) The inverse image of a closed set is  $\delta$ -closed,
- c) The inverse image of an open set is  $\delta$ -open,
- d) For every  $x \in X$ , and for every  $V$  open subset of  $Y$  containing  $f(x)$  there exists a regularly open set  $U_x$  such that  $f(U_x) \subset V$ ,
- e) For each  $x \in X$  and for each net  $\{x_\alpha\}_{\alpha \in D}$   $\delta$ -converges to  $x$ ,  $\{f(x_\alpha)\}_{\alpha \in D}$  converges to  $f(x)$ ,
- f) For each  $x \in X$  and for each filter base  $F$   $\delta$ -converges to  $x$ ,  $f(F)$  converges to  $f(x)$ ,

$f$ )  $f$  is *s.d.c.*

In [3] it is shown that a function  $f$  is almost continuous iff its graph mapping  $g$ , where  $g(x) = (x, f(x))$  is almost continuous. In [7], [8] this result was extended to weak continuity and  $\delta.c.$  In the present paper we extend this result to *s.d.c.*

**Theorem 2.13.** Let  $f : X \rightarrow Y$  be a mapping and let  $g : X \rightarrow X \times Y$  be the graph mapping of  $f$  given by  $g(x) = (x, f(x))$  for every point  $x \in X$ . Then  $g : X \rightarrow X \times Y$  is *s.d.c.* iff  $f : X \rightarrow Y$  is *s.d.c.* and  $X$  is semi-regular.

**Proof.** Suppose  $g$  is *s.d.c.* Let  $x \in X$  and let  $V$  be an open set in  $Y$  containing  $f(x)$ . Then  $X \times V$  is an open set in  $X \times Y$  containing  $g(x)$ . Since  $g$  is *s.d.c.*,  $g^{-1}(X \times V)$  is regularly open. But  $g^{-1}(X \times V) = f^{-1}(V)$ , since  $g$  is the graph mapping of  $f$ . Hence,  $f^{-1}(V)$  is regularly open, proving that  $f$  is *s.d.c.* To prove that  $X$  is semi-regular, let  $x \in X$  and let  $U$  be an open set containing  $x$ . Then  $U \times Y$  is an open set containing  $g(x)$ . By *s.d.c.* of  $g$ , there exists a regularly open set  $W$  containing  $x$  such that  $g(W) \subseteq U \times Y$ . Thus  $x \in W \subseteq U$ , proving that  $X$  is semi-regular. Conversely, assume  $f$  is *s.d.c.* and let  $A \subset X$ . Then  $g(\text{cls}_s A) \subseteq \text{cls}_s A \times f(\text{cls}_s A)$ , since  $g(x)$  is the graph mapping of  $f$ . By *s.d.c.* of  $f$ ,  $f(\text{cls}_s A) \subset f(A)$ . Since  $X$  is semi-regular,  $\overline{A} = \text{cls}_s A$ . Therefore,  $g(\text{cls}_s A) \subseteq \text{cls}_s A \times f(\text{cls}_s A) = \overline{A} \times f(\text{cls}_s A) \subset \overline{A} \times f(A) = A \times f(A) = g(A)$ , proving that  $g$  is *s.d.c.*

The next example shows that the graph mapping of *s.d.c.* need not be *s.d.c.*

**Example 2.4.** Let  $X = Y = \{1, 2, 3\}$  with topologies  $S_X = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, X\}$ ,  $f(x) = 3$ , for all  $x \in X$ . Then  $f$  is *s.d.c.* but the graph mapping  $g$  of  $f$ , where  $g(x) = (x, f(x))$  is not *s.d.c.* at  $x = 1$ .

Similar to  $\delta.c.$  [8, Theorems 3.3, 3.4] and following a similar argument as in [4, Theorems 6, 7], we get the following results.

**Theorem 2.14.** Let  $f : X \rightarrow \prod_{\alpha \in I} X_\alpha$  be given. Then  $f$  is *s.d.c.* iff the composition with each projection  $\pi_\alpha$  is *s.d.c.*

**Theorem 2.15.** Define  $\prod_{\alpha \in I} f_\alpha : \prod_{\alpha \in I} X_\alpha \rightarrow \prod_{\alpha \in I} Y_\alpha$  by  $\{x_\alpha\} \rightarrow$

$\{f_\alpha(x_\alpha)\}$ . Then  $\prod f_\alpha$  is  $s.\delta.c.$  iff each  $f_\alpha : X_\alpha \rightarrow Y_\alpha$  is  $s.\delta.c.$

### 3. HAUSDORFF SPACES AND $\delta$ -CONTINUITIES.

**Definition 3.1.** A space  $X$  is said to be  $\theta$ -Hausdorff if for every  $x \neq y \in X$ , there exist  $\theta$ -open sets  $U_x, V_y$  such that  $U_x \cap V_y = \emptyset$ .

**Lemma 3.1.** Let  $X$  be a Hausdorff. Then for every  $x \neq y \in X$ , there exist *regularly* open sets  $U_x, V_y$  such that  $U_x \cap V_y = \emptyset$ .

**Proof.** Let  $X$  be a Hausdorff space and let  $x \neq y \in X$ . Thus, there exist two open sets  $U_x, V_y$  of  $x$  and  $y$ , respectively, such that  $U_x \cap V_y = \emptyset$ . It follows easily that  $\text{Int}(\overline{U_x}) \cap \text{Int}(\overline{V_y}) = \emptyset$ .

By an  $\delta$ -retraction we mean a  $\delta$ -continuous function  $f : X \rightarrow A$  where  $A \subset X$  and  $f|_A$  is the identity function on  $A$ . In this case,  $A$  is said to be a  $\delta$ -retraction of  $X$ .

**Theorem 3.1.** Let  $A \subset X$  and let  $f : X \rightarrow A$  be an  $\delta$ -retraction of  $X$  onto  $A$ . If  $X$  is Hausdorff, then  $A$  is an  $\delta$ -closed subset of  $X$ .

**Proof.** Suppose not, then there exists a point  $x \in \text{cls}_\delta A \setminus A$ . Since  $f$  is an  $\delta$ -retraction we have  $f(x) \neq x$ . Since  $X$  is Hausdorff, there exist regularly open sets  $U$  and  $V$  containing  $x$  and  $f(x)$ , respectively, such that  $U \cap V = \emptyset$ . Now let  $W$  be any open set in  $X$  containing  $x$ . Then  $\text{Int}(\overline{U}) \cap \text{Int}(\overline{W})$  is a regularly open set containing  $x$  and hence  $\text{Int}(\overline{U}) \cap \text{Int}(\overline{W}) \cap A \neq \emptyset$ , since  $x \in \text{cls}_\delta A$ . Therefore, there exists a point  $y \in \text{Int}(\overline{U}) \cap \text{Int}(\overline{W}) \cap A$ . Since  $y \in A$ ,  $f(y) = y \in \text{Int}(\overline{U})$  and hence  $f(y) \notin V$ . This shows that  $f(\text{Int}(\overline{W}))$  is not contained in  $V$ . This contradicts the hypothesis that  $f$  is  $\delta$ -continuous. Thus  $A$  is  $\delta$ -closed as claimed.

**Theorem 3.2.** Let  $f : X \rightarrow Y$  be an  $\delta.c.$  and injective function. If  $Y$  is Hausdorff, then  $X$  is Hausdorff.

**Proof.** For any distinct points  $x_1, x_2 \in X$ , we have  $f(x_1) \neq f(x_2)$ , since  $f$  is injective. By Lemma 3.1, there exist regularly open sets  $V_1, V_2$  of  $f(x_1)$  and  $f(x_2)$ , respectively, such that  $V_1 \cap V_2 = \emptyset$ . But since  $f$  is  $\delta.c.$ , there exist regularly open  $U_1, U_2$  of  $x_1, x_2$ , respectively, such that  $f(U_1) \subset V_1$ , and  $f(U_2) \subset V_2$ . Thus  $U_1 \cap U_2 = \emptyset$ , proving that  $X$  is Hausdorff.

**Theorem 3.3.** Let  $f, g$  be  $\delta.c.$  from a space  $X$  into a Hausdorff space  $Y$ .

Then the set  $A = \{x \in X : f(x) = g(x)\}$  is an  $\delta$ -closed set.

**Proof.** We will show that  $X \setminus A$  is  $\delta$ -open. Let  $x \in A^c$ . Then  $f(x) \neq g(x)$ . Since  $Y$  is Hausdorff, there exist regularly open sets  $W_{f(x)}$  and  $V_{g(x)}$  such that  $W \cap V = \emptyset$ . By  $\delta.c.$  of  $f$  and  $g$  there exist regularly open sets  $U_1, U_2$  of  $x$  such that  $f(U_1) \subset W$  and  $g(U_2) \subset V$ . Clearly  $U = U_1 \cap U_2 \subset A^c$ . Thus  $A^c$  is  $\delta$ -open and hence  $A$  is  $\delta$ -closed.

Theorem 3.3 leads to a generalization of a well-known principle of the extension of identities.

**Definition 3.2.** A subset  $A$  of a space  $X$  is called  $\delta$ -dense if  $\text{cls}_\delta A = A$ .

**Corollary 3.1.** Let  $f, g$  be  $\delta.c.$  from a space  $X$  into a Hausdorff space  $Y$ . If  $f$  and  $g$  agree on a regularly dense subset of  $X$  then  $f = g$  every where.

**Theorem 3.4.** Let  $f : X \rightarrow Y$  be  $\delta.c.$  (resp.,  $s.\delta.c.$ ) map and let  $A \subset X$ . Then  $f : A \rightarrow Y$  is  $\delta.c.$  (resp.,  $s.\delta.c.$ )

**Proof.** Straightforward.

**Remark.** If a function  $f : X \rightarrow Y$  is  $\delta.c.$  Then  $f : X \rightarrow f(X)$  need not be  $\delta.c.$  However, it is true for  $s.\delta.c.$

**Example 3.1.** Let  $X = \mathbb{R}$  with the usual topology,  $Y = \mathbb{R}$  with the countable topology and let  $f : X \rightarrow Y$  be defined as  $f(\text{rationals}) = 1, f(\text{irrationals}) = 0$  then  $f$  is  $\delta.c.$ , but  $f : X \rightarrow f(X)$  is not  $\delta.c.$

## 4. APPLICATIONS

It is well-known that the image of compact is closure compact under weakly continuous functions, the image of closure compact is closure compact under closure continuous functions and the image of closure compact is compact under strongly continuous functions. The following results are similar to that applied to  $\delta$ -continuities.

**Definition 4.1.** A subset  $A$  of a space  $X$  is said to be  $\theta$ -compact if every cover of  $\theta$ -open sets has a finite subcover.

**Definition 4.2.** A subset  $A$  of a space  $X$  is called nearly compact (briefly,

n.compact) if every open cover has a finite subcollection whose interior of the closures cover  $A$ . Equivalently, a subset  $A$  of a space  $X$  is n.compact iff every cover of regularly open sets of  $A$  has a finite subcover.

**Lemma 4.1** [12, Corollary 2.1]. A subset  $A$  of a space  $X$  is n.compact iff every cover of  $\delta$ -open sets of  $A$  has a finite subcover.

**Remark 4.1.** A function  $f : (X, \Gamma) \rightarrow (Y, \mathfrak{S})$  is  $s.\delta.c$  iff  $f : (X, \Gamma_s) \rightarrow (Y, \mathfrak{S})$  is continuous and  $f : (X, \Gamma) \rightarrow (Y, \mathfrak{S})$  is  $\delta.c$  iff  $f : (X, \Gamma_s) \rightarrow (Y, \mathfrak{S}_s)$  is continuous.

**Remark 4.2.** A subset  $K \subset (X, \Gamma)$  is an n.compact subset iff  $K \subset (X, \Gamma_s)$  is compact and  $K \subset (X, \Gamma)$  is a  $\theta$ .compact subset iff  $K \subset (X, \Gamma_\theta)$  is compact.

The following results follow directly from Remarks 4.1 and 4.2.

**Theorem 4.1.** Let  $f : X \rightarrow Y$  be  $\delta.c$  and let  $K$  be an n.compact subset of  $X$ . Then  $f(K)$  is an n.compact subset of  $Y$ .

**Corollary 3.4.** Let  $f : X \rightarrow Y$  be an open  $a.c$  and let  $K$  be an n.compact subset of  $X$ . Then  $f(K)$  is an n.compact subset of  $Y$ .

**Theorem 4.2.** Let  $f : X \rightarrow Y$  be  $s.\delta.c$ . and let  $K$  be an n.compact subset of  $X$ . Then  $f(K)$  is a compact subset of  $Y$ .

**Theorem 4.3.** An n.compact subset of a Hausdorff space is  $\delta$ -closed.

**Theorem 4.4.** Every  $\delta$ -closed subset of an n.compact space is n.compact.

**Theorem 4.5.** An  $\theta$ .compact subset of an  $\theta$ .Hausdorff space is  $\theta$ -closed.

**Theorem 4.6.** Every  $\theta$ -closed subset of an  $\theta$ .compact space is  $\theta$ .compact.

**Theorem 4.7.** Let  $f : X \rightarrow Y$  be a surjective  $\delta.c$  and let  $X$  be connected. Then  $Y$  is connected.

**Proof.** Suppose  $Y$  is disconnected. Then there exists disjoint open sets  $V, W$  such that  $Y = Int(\bar{V}) \cup Int(\bar{W})$ . By  $\delta$ -continuity of  $f$ ,  $f^{-1}(Int(\bar{V})) = f^{-1}(V)$  and  $f^{-1}(Int(\bar{W})) = f^{-1}(W)$  are open in  $X$ . But  $X = f^{-1}(W) \cup f^{-1}(V)$  and  $f^{-1}(W) \cap f^{-1}(V) = \emptyset$ . Thus  $X$  is disconnected, a contradiction. Therefore,  $Y$  is connected.

**Theorem 4.8.** Let  $f : X \rightarrow Y$  be  $\delta.c$  1-1, onto. If  $X$  is n.compact, and  $Y$  Hausdorff, then the image of every  $\delta$ -open is  $\delta$ -open.

**Proof.** Let  $U$  be an  $\delta$ -open subset of  $X$ , and thus  $X \setminus U$  is an  $\delta$ -closed subset of  $X$ . Theorem 4.4 implies that  $X \setminus U$  is n.compact. Since  $f$  is  $\delta.c$ , Theorem 4.1 leads that  $f(X \setminus U)$  is n.compact. Therefore, Theorem 4.3 implies that  $f(X \setminus U) = Y \setminus f(U)$  is  $\delta$ -closed, and thus  $f(U)$  is  $\delta$ -open.

**Theorem 4.9.** Let  $f : X \rightarrow Y$  be  $\delta.c$ . If  $X$  is n.compact and  $Y$  Hausdorff, then the image of every  $\delta$ -closed is  $\delta$ -closed.

**Theorem 4.10.** Let  $f : X \rightarrow Y$  be  $s.\delta.c$  1-1, onto. If  $X$  is n.compact and  $Y$  Hausdorff, then the image of every  $\delta$ -open is  $\theta$ -open.

**Theorem 4.11.** Let  $f : X \rightarrow Y$  be  $s.\delta.c$ . If  $X$  is n.compact and  $Y$  Hausdorff, then the image of every  $\delta$ -closed is  $\theta$ -closed.

## REFERENCES

- 1 P. Alexandroff and P. Urysohn, Mémoire sur les espaces topologiques compacts, Verh. Nederl. Akad. Wetensch. Afd. Natuurk. Sect. I 14(1929), 1 - 96.
- 2 S. Fomin, Extensions of topological spaces, C. R. Dokl. Akad. Sci. URSS(M. S), 32(1941), 114 - 116.
- 3 P. E. Long and D. A. Carnahan, Comparing almost continuous functions, Proc. Amer. Math. Soc. 38(1973), 413 - 418.
- 4 P. E. Long and L. Herrington, Strongly  $\theta$  - continuous functions, J. Korean. Math. Soc. 18(1981), 21 - 28.
- 5 M.N. Mukherjee, and S. Raychaudhuri, Some applications of  $\theta$ -closure operators, Indian Journal of Pure and Applied Mathematics, 26(1995), 433 - 439.
- 6 B.M. Munshi, and D. Bassan, Super-continuous mappings, Indian Journal of Pure and Applied Mathematics, 13(1982), 229 - 236.
- 7 T. Noire, On weakly continuous mappings, Proc. Amer. Math. Soc., 46(1), 1974, 120 - 124.
- 8 T. Noire, On  $\delta$ -continuous functions, J. Korean. Math. Soc. 18(1980), 161 - 166.

- 9 M. Saleh, Some remarks on closure and strong continuity, *An Najah J.*, 1998, 7-20.
- 10 M. Saleh, Some applications of  $\delta$ -sets to H-closed spaces. *Q&A in Topology*, 1999, 203-211.
- 11 M. Saleh and M. Al Amleh, N-compactness and  $\theta$ -closed sets, *Proceedings of the Mathematics Conference*, World Scientific Press, 2000, 207-218.
- 12 M. Saleh and M. Al Amleh, On  $\theta$ -closed sets and faint continuity, *Proceedings of the Mathematics Conference*, World Scientific Press, 2000, 219-232.
- 13 M.K. Singal and A.R. Singal, Almost continuous mappings, *Yokohama Math. J.* 16(1968), 63 - 73.
- 14 N.V. Veličko, H-closed topological spaces, *Transl. AMS*, 78(1968), 103 - 118. 179(1973), 61 - 69.

# Design of user interface for computer-aided instruction of mathematics

K. Tintarev,  
Uppsala University  
P.O.Box 480  
751 06 Uppsala  
Sweden  
kyril@math.uu.se

## Abstract

Symbolic computations software designed for teaching college and high school mathematics should have a self-explanatory user interface that does not waste the learning effort on machine operation. The paper discusses available directions for such design within the constraints of available operating systems, GUI libraries and computer algebra algorithms.

## 1 Introduction

Use of technology in education is justified when it improves quality of education and when such improvement is not offset by negative side-effects. This paper deals with the question of improving technology and decreasing negative side-effects as a precondition for reaping the benefits of the technology for mathematical education.

Most of software used in instruction of mathematics falls in the following categories:

- a) Fixed content software or "hyperbook" (usually a sequence of screens that present material and calculate scores for student's answers);