Resort Pricing and Bankruptcy

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Abstract We introduce a resort pricing model, where different types of tourists choose between different resorts. We study the influence of the resort prices on the choices of the different types of tourists. We characterize the coherent strategies of the tourists that are Nash equilibria. We find the prices that lead to the bankruptcy of the resorts and, in particular, their dependence on the characteristics of the tourists.

1 Introduction

The distribution of different types of tourists reaching a destination affects both the demand and supply side of the tourism industry. From the demand perspective, the choice of a particular destination will depend greatly on the beliefs of the agent about which kind of tourists will share the resort with him/her (see [4, 5]). On the supply side, resorts try to sell their destination based on reputation, and a large factor that determines the character and reputation of a resort is the type of tourists who frequent that resort (see [6]). Brida et als. [1, 2] presented a tourism model where the

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choice of a resort by a tourist depends not only on the product offered by the resort, but also depends on the characteristics of the other tourists present in the resort. In order to explore the effect the type of resident tourist has on other potential tourists selecting the same resort, they introduced a game theoretical model and described some relevant Nash equilibria. We add to the previous models the influence of resort prices on the tourist's choice of a resort (see [7]). We characterize the prices that lead to the bankruptcy of the resorts and, in particular, their dependence on the characteristics of the tourists.

2 Resort Pricing Model

The *resort pricing model* has two types $\mathbf{T} = \{t_1, t_2\}$ of tourists $i \in \mathbf{I}$ that have to choose between two goods or services. For instance, the tourists have to choose between spending their holidays in a beach resort B or in a mountain resort M, i.e. $r \in \mathbf{R} = \{B, M\}$. Let $n_q \geq 1$ be the number of tourists with type t_q. Let $\mathscr{P}$ be the *price vector* whose *coordinates* p^r indicates the *standard price* of the resort r for each tourist, independently of its type,

$$\mathscr{P} = (p^B, p^M).$$

Let $\mathscr{L}$ be the *preference location matrix* whose *coordinates* ω_q^r indicate how much the tourist, with type t_q, likes, or dislikes, to choose resort r

$$\mathscr{L} = \begin{pmatrix} \omega_1^B & \omega_1^M \\ \omega_2^B & \omega_2^M \end{pmatrix}.$$

The preference location matrix indicates, for each type, the resort that the tourists prefer, i.e. the tourists taste type (see [1, 2, 3]).

Let $\mathscr{N}_r$ be the *preference neighbors matrix* whose *coordinates* $\alpha_{qq'}^r$ indicate how much the tourist, with type t_q, likes, or dislikes, that tourist, with type $t_{q'}$, chooses resort r

$$\mathscr{N}_r = \begin{pmatrix} \alpha_{11}^r & \alpha_{12}^r \\ \alpha_{21}^r & \alpha_{22}^r \end{pmatrix}.$$

The preference neighbors matrix indicates, for each type of tourists, whom they prefer to be with or to not be with at each resort, i.e. the tourists crowding type (see [1, 2, 3]).

Let

$$\mathscr{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

be the coordinates of the *partial threshold order matrix*, where $A_{ij} = \alpha_{ij}^B + \alpha_{ij}^M$, for $i, j \in \{1, 2\}$. The partial thresholds encode all relevant information for the existence of Nash equilibria strategies.

We describe the tourists' location by a *strategy map* $S : \mathbf{I} \to \mathbf{R}$ that associates to each tourist $i \in \mathbf{I}$ its location $S(i) \in \mathbf{R}$. Let $\mathbf{S}$ be the space of all strategies S. Given a strategy S, let $\mathscr{O}_S$ be the *strategic occupation matrix*, whose coordinates $l_q^r = l_q^r(S)$ indicate the number of tourists, with type t_q, that choose resort r

$$\mathscr{O}_S = \begin{pmatrix} l_1^B & l_1^M \\ l_2^B & l_2^M \end{pmatrix}.$$

The *strategic occupation vector* $\mathscr{V}_S$, associated to a strategy S, is the vector $(l_1, l_2) = (l_1^B(S), l_2^B(S))$. Hence, l_1 (resp. $n_1 - l_1$) is the number of tourists, with type t_1, who choose the resort B (resp. M). Similarly, l_2 (resp. $n_2 - l_2$) is the number of tourists, with type t_2, that choose the resort B (resp. M). The set $\mathbf{O}$ of all possible *occupation vectors* is

$$\mathbf{O} = \{(l_1, l_2) : 0 \le l_1 \le n_1 \quad \text{and} \quad 0 \le l_2 \le n_2\}.$$

Let $U_1 : \mathbf{R} \times \mathbf{O} \to \mathbb{R}$ the *utility function*, of the tourist with type t_1, be given by

$$\begin{aligned} U_1(B; l_1, l_2) &= -p^B + \omega_1^B + \alpha_{11}^B (l_1 - 1) + \alpha_{12}^B l_2 \\ U_1(M; l_1, l_2) &= -p^M + \omega_1^M + \alpha_{11}^M (n_1 - l_1 - 1) + \alpha_{12}^M (n_2 - l_2). \end{aligned}$$

Let $U_2 : \mathbf{R} \times \mathbf{O} \to \mathbb{R}$ the *utility function*, of the tourists with type t_2, be given by

$$\begin{aligned} U_2(B; l_1, l_2) &= -p^B + \omega_2^B + \alpha_{22}^B (l_2 - 1) + \alpha_{21}^B l_1 \\ U_2(M; l_1, l_2) &= -p^M + \omega_2^M + \alpha_{22}^M (n_2 - l_2 - 1) + \alpha_{21}^M (n_1 - l_1). \end{aligned}$$

Given a strategy $S \in \mathbf{S}$, the *utility* $U_i(S)$, of the tourist i with type $t_{p(i)}$, is given by $U_{p(i)}(S(i); l_1^B(S), l_2^B(S))$.
We note that, if the price can depend on the tourist type, then the prices can be encoded in the preference decision matrix and, therefore, the model can be studied as the yes-no decision model presented in [8].

Definition 1. A strategy $S^* : \mathbf{I} \to \mathbf{R}$ is a *Nash equilibrium* if, for every tourist $i \in \mathbf{I}$ and for every strategy S, with the property that $S^*(j) = S(j)$ for every tourist $j \in I \setminus \{i\}$, we have

$$U_i(S^*) \ge U_i(S).$$

A *coherent strategy*[1] is a strategy in which all tourists, with the same type, prefer to choose the same resort. A *coherent strategy* is described by a map $C : \mathbf{T} \to \mathbf{R}$ where, for every tourist i with type $t_{q(i)}$, $C(q(i))$ indicates its location. Hence, a coherent strategy $C : \mathbf{T} \to \mathbf{R}$ determines an unique strategy $S : \mathbf{I} \to \mathbf{R}$ given by $S(i) = C(q(i))$.

Let $x = \omega_1^B - \omega_1^M$ be the *horizontal relative location preference* of the tourists with type t_1 and let $y = \omega_2^B - \omega_2^M$ be the *vertical relative location preference* of the tourists with type t_2. Let $p = p^B - p^M$ be the *relative price*. Given a pair (x, y) of relative location preferences, the *Nash equilibrium prices interval* $P(R_1, R_2) = P(x, y; R_1, R_2)$ of a coherent strategy (R_1, R_2) is the set of all relative prices p for which the strategy

[1] or equivalently, *no-split strategy* or *heard strategy*

(R_1,R_2) is a Nash equilibrium. Our aim is to determine and characterize all Nash equilibrium prices interval.

3 Nash Equilibrium Prices

We observe that there are four distinct *coherent* strategies:

- (B,B) *strategy* - all tourists choose the resort B;
- (B,M) *strategy* - all tourists, with type t_1, choose the resort B, and all tourists, with type t_2, choose the resort M;
- (M,B) *strategy* - all tourists, with type t_1, choose the resort M and all tourists, with type t_2, choose the resort B;
- (M,M) *strategy* - all tourists choose the resort M.

The *horizontal* $H(B,B)$ and *vertical* $V(B,B)$ *strategic thresholds* of the (B,B) strategy are given by

$$H(B,B) = -\alpha_{11}^{B}(n_1-1) - \alpha_{12}^{B} n_2 \quad \text{and} \quad V(B,B) = -\alpha_{22}^{B}(n_2-1) - \alpha_{21}^{B} n_1 .$$

The *(B,B) Nash equilibrium prices interval* $P(B,B)$ is the semi-line

$$P(B,B) = \{p \in \mathbb{R} : p \leq x - H(B,B) \quad \text{and} \quad p \leq y - V(B,B)\}.$$

In the red half-plane of the upper left section of Figure 1, for a given relative preferences pair (x,y), the first coordinate of the blue vector, i.e. the yellow horizontal projection, represents the maximum price in $P(B,B)$; in the green half-plane, for a given relative preferences pair (x,y), the second coordinate of the blue vector, i.e. the orange vertical projection, represents the maximum price in $P(B,B)$.

The horizontal $H(B,M)$ and vertical $V(B,M)$ *strategic thresholds* of the (B,M) strategy are given by

$$H(B,M) = -\alpha_{11}^{B}(n_1-1) + \alpha_{12}^{M} n_2 \quad \text{and} \quad V(B,M) = \alpha_{22}^{M}(n_2-1) - \alpha_{21}^{B} n_1 .$$

The *(B,M) Nash equilibrium prices interval* $P(B,M)$ is the segment line (that can be empty)

$$P(B,M) = \{p \in \mathbb{R} : p \leq x - H(B,M) \quad \text{and} \quad p \geq y - V(B,M)\}.$$

In the blue half-plane of the upper right section of Figure 1, for a given relative preferences pair (x,y), the second coordinate of the blue vector, i.e. the orange vertical projection, represents the minimum price in $P(B,M)$ and the first coordinate of the blue vector, i.e. the yellow horizontal projection, represents the maximum price in $P(B,M)$; in the purple half-plane, there are no Nash equilibrium prices.

The horizontal $H(M,B)$ and vertical $V(M,B)$ *strategic thresholds* of the (M,B) strategy are given by

$$H(M,B) = \alpha_{11}^{M}(n_1-1) - \alpha_{12}^{B} n_2 \quad \text{and} \quad V(M,B) = -\alpha_{22}^{B}(n_2-1) + \alpha_{21}^{M} n_1.$$

The *(M,B) Nash equilibrium prices interval* $P(M,B)$ is the segment line (that can be empty)

$$P(M,B) = \{p \in \mathbb{R} : p \geq x - H(M,B) \quad \text{and} \quad p \leq y - V(M,B)\}.$$

In the blue half-plane of the lower left section of Figure 1, for a given relative preferences pair (x,y), the first coordinate of the blue vector, i.e. the yellow horizontal projection, represents the minimum price in $P(M,B)$ and the second coordinate of the blue vector, i.e. the orange vertical projection, represents the maximum price in $P(M,B)$; in the purple half-plane, there are no Nash equilibrium prices.

The horizontal $H(M,M)$ and vertical $V(M,M)$ strategic thresholds of the (M,M) strategy are given by

$$H(M,M) = \alpha_{11}^{M}(n_1-1) + \alpha_{12}^{M} n_2 \quad \text{and} \quad V(M,M) = \alpha_{22}^{M}(n_2-1) + \alpha_{21}^{M} n_1.$$

The *(M,M) Nash equilibrium prices interval* $P(M,M)$ is the semi-line

$$P(M,M) = \{p \in \mathbb{R} : p \geq x - H(M,M) \quad \text{and} \quad p \geq y - V(M,M)\}.$$

In the red half-plane of the lower right section of Figure 1, for a given relative preferences pair (x,y), the first coordinate of the blue vector, i.e. the yellow horizontal projection, represents the minimum price in $P(M,M)$; in the green half-plane of the lower right section of Figure 1, for a given relative preferences pair (x,y) the second coordinate of the blue vector, i.e. the orange vertical projection, represents the minimum price in $P(M,M)$.

4 Bankruptcy Nash Equilibrium Prices

Let the *coherent uniqueness Nash equilibria prices* be the regions $U(B,B) \subset P(B,B)$, $U(B,M) \subset P(B,M)$, $U(M,B) \subset P(M,B)$ and $U(M,M) \subset P(M,M)$, where for every point in these regions, there is a unique coherent Nash equilibrium. We call the prices in $U(B,B)$ the *bankruptcy* Nash equilibrium prices of the mountain resort M, because, for every price in $U(B,B)$, there are no tourists choosing the mountain resort M. Similarly, we call the prices in $U(M,M)$ the *bankruptcy* Nash equilibrium prices of the beach resort B, because, for every price in $U(M,M)$ there are no tourists choosing the beach resort B. We call the prices in $U(B,M)$ and $U(M,B)$ the *competitive business* Nash equilibrium prices, because, for every price in $U(B,M)$ and in $U(M,B)$, one type of tourists choose the beach resort B and the other type of tourists choose the mountain resort M. We note that, the bankruptcy Nash equilib-

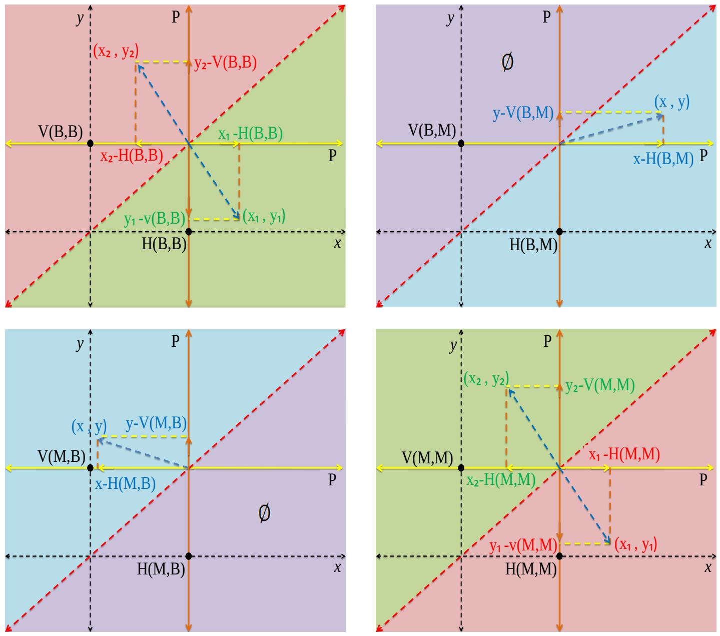

Fig. 1 Nash equilibrium prices

ria prices $U(B,B)$ and $U(M,M)$ are non-empty, but the competitive business Nash equilibrium price can be empty (see Figure 2).

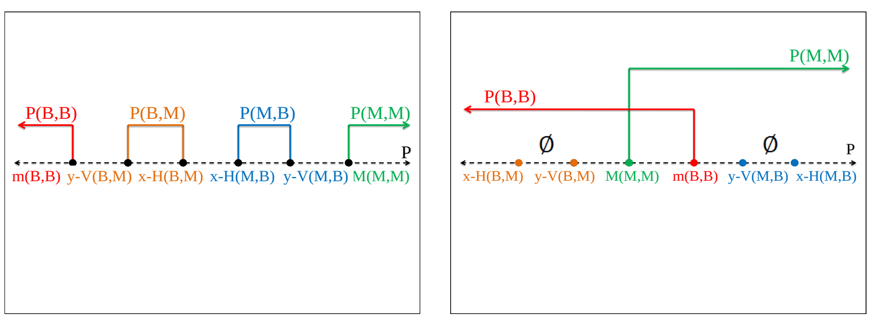

Fig. 2 Bankruptcy and competitive business Nash equilibria prices, where $M(M,M) = \max\{x - H(M,M),\ y - V(M,M)\}$ and $m(B,B) = \min\{x - H(B,B),\ y - V(B,B)\}$.

5 Conclusions

Small changes in the the coordinates of the preference location matrix, which indicates the resort that the tourists prefer, and of the preference neighbors matrix, which indicates who the tourists prefer to be with in each resort, can create and annihilate competitive business Nash equilibrium prices and change the bankruptcy Nash equilibria prices.

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References

1. Brida, J., Defesa, M., Faias, M. and Pinto, A., A Tourists Choice Model.Dynamics, Games and Science I (eds.: M. Peixoto, A. A. Pinto and D. Rand). Proceedings in Mathematics series, Springer-Verlag, Chapter 10, 159-167 (2011).
2. Brida, J., Defesa, M., Faias, M. and Pinto, A. A., Strategic Choice in Tourism with Differentiated Crowding Types. *Economics Bulletin*, 30 (2) 15091515 (2010).
3. Conley, John P. and Wooders, Myrna H., Tiebout Economies with Differential Genetic Types and Endogenously Chosen Crowding Characteristics. *Journal of Economic Theory* 98 261-294 (2001).
4. Ferreira, F. A., Ferreira, F. and Pinto, A. A., 'Own' price influences in a Stackelberg leadership with demand uncertainty. *Brazilian Journal of Business Economics*, 8 (1) 29-38 (2008).
5. Ferreira, F., Ferreira, F. A. and Pinto, A. A., Price-setting dynamical duopoly with incomplete information. In J. A. Tenreiro Machado et als. (eds.): *Nonlinear Science and Complexity*. Springer 1-7 (2009).
6. Liu, Z., Siguaw, J. A. and Enz, C. A., Using Tourist Travel Habits and Preferences to assess Strategic Destination Positioning: The case of Costa Rica. *Cornell Hospitality Quarterly*, 49 3 (2008).
7. Pinto, A.A.: *Game Theory and Duopoly Models*. Interdisciplinary Applied Mathematics Series. Springer, New York (2011)
8. Pinto A. A, Mousa A. S., Mousa M. S. and Samarah M. S., Tilings and Bussola for Making Decisions. *Dynamics, Games and Science I* (eds.: M. Peixoto, A. A. Pinto and D. Rand). Proceedings in Mathematics series, Springer-Verlag, Chapter 44, 689-710 (2011).