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Investigating the geometry curriculum in Palestinian textbooks: towards multimodal analysis of Arabic mathematics discourse

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ABSTRACT
The analytical scheme used in the project The Evolution of the Discourse of School Mathematics (EDSM) was developed to analyse the change over time in examination texts. An adapted version of the EDSM scheme has been deployed to analyse the nature of mathematics construed in Palestinian schools' textbooks and the mathematical activity expected of students in the geometry textbooks for students aged 10 to 16 years. The adaptation includes adding further tools for analysing visual components of texts, as well as accounting for some differences between English and Arabic. This article outlines these adaptations and illustrates the use of the adapted scheme with a different genre of texts from those studied in the EDSM project. Some of the challenges in the adaptation process in relation to Arabic mathematics discourse are discussed.

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Arabic mathematics discourse; visual discourse analysis; multimodality

Introduction

The main aim of this article is to describe, and then reflect on, the adaptation of the analytic scheme developed in the project The Evolution of the Discourse of School Mathematics (EDSM) when applied to a new context (Palestine), language (Arabic) and a new purpose (analysis of geometry textbooks). There are a number of interrelated motivations behind this aim. First, there is an interest in analysing the way that *Palestinian textbooks present mathematics*. A second interest is to analyse *visual means*, which are of central importance to our ability to understand and evaluate mathematical texts, an element under-emphasised in the EDSM scheme. A third and related interest is the methods for analysing the *relationship between visual and verbal components of* school mathematics textbooks.

I begin with presenting the development of the tools for analysing visual aspects, showing their compatibility with the EDSM scheme. This is followed by an illustration of the elaborated analysis using an example from the Palestinian school mathematics textbook. The issues encountered in applying the analysis to Arabic mathematics texts will be the focus of the third section. In the final section I evaluate the usefulness of the adapted

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scheme to the study of geometry in Palestinian textbooks and then more generally consider its relevance within Palestinian mathematics education. Within Palestinian society, there is discussion and concern about student performance at both the national (Masad, 1998) and the international level as reflected in the Trends in International Mathematics and Science Study (TIMSS) (Mullis, Martin, Foy, & Arora, 2012), and related concerns about the mathematical knowledge of teachers (Al-Ramahi, 2006). The study reported here creates an opportunity to interpret and ultimately improve mathematics education in Palestine.

The opportunity for improvement emerges directly from the possibilities for reform that my textbook analysis suggests and crucially, if less directly, from the possibilities for changing pedagogy that emerge from the underlying view of mathematics as a social practice (Morgan, 2001; Sfard, 2008). This view requires us to look at the process of constructing meaning in teaching and learning mathematics and to understand the discourse of mathematics as deeply shaped by language (Sfard, 2008). In this framing, mathematics texts, and so school textbooks in particular, impact teaching and learning.

The analytic scheme: developing the tools for analysing visual aspects of texts within the EDSM scheme

Communication in mathematics has been the focus of research in mathematics education for some time now. In particular, two researchers have devoted their scholarship to this issue: Anna Sfard and Candia Morgan. Sfard focuses her work on mathematics discourse and its special features, and she developed her own (commognitive) approach (Sfard, 2008). According to Sfard, mathematics discourse has its own distinguishing characteristics which enable us to call it mathematics. Morgan, on the other hand, adopts Halliday's (1985) systemic functional grammar (SFG) (see below), thus focusing her work on the linguistic features of doing mathematics through which we can analyse the nature of mathematical activities constructed in verbal communication, whether written, as in textbooks, or in oral communication (Morgan, 1996). Sfard and Morgan have brought their theoretical frameworks together in the EDSM project to develop an analytical scheme for the analysis of school mathematics examination texts – the focus of this special issue.

The EDSM scheme was developed specifically to address the change that mathematics texts underwent over time. It was also used to analyse the picture that emerges of mathematics and the mathematical activities with which students engage (see Morgan, this issue; Morgan & Sfard, this issue; Tang, Morgan, & Sfard, 2012). Alshwaikh and Morgan (2013) used an adapted version of the EDSM scheme to analyse Palestinian school mathematics textbooks. While the EDSM analytical scheme does include attention to visual mediators, its main focus is on language. We therefore elaborated on that scheme by offering more refined classifications of the characteristics of visual forms (in particular, of geometric diagrams). The elaborated scheme is shown in Table 1 and is discussed in more detail below.

The analysis of visual communication has been under development especially in the social and cultural fields. Social semioticians consider communication to be inevitably multimodal, involving various means of communication including language, images and gestures (Kress & Van Leeuwen, 2006). In mathematics, too, communication is multimodal. The different modes employed include language, diagrams, graphs and other visual

Table 1. An elaborated version of the analytic framework (based on the EDSM scheme and Alshwaikh & Morgan, 2013).

How is the nature of mathematics and mathematical activity construed?			
Property of the discourse	*Specific questions guiding analysis*	*Indicators in verbal text*	*Indicators in visual text*
Specialisation	To what extent is specialised mathematical language used?	Vocabulary used in accordance with mathematical definitions "conventional" expressions mathematical symbols	"conventional" mathematical diagrams, charts, tables, graphs and labelling systems
Objectification	Does the text speak of properties of objects or of processes?	Nominalisation specialised nouns reifying processes relational processes (identifying or assigning properties) or material processes	Conceptual diagrams • classificational • identifying • spatial relationships • labels and colours Narrative diagrams • temporality indicators (arrows, dotted lines, shading, sequence, construction)
Alienation	To what extent is mathematics presented as a human activity?	presence of human agents in mathematical processes obscuring of agency through passive voice or nominalisation	• Presence of human figures, physical objects or context • High abstract diagrams (no practical activity or perspective, only geometric objects)
Logical structure	What kinds of logical relationships are present and how explicit are they?	the types and frequencies of conjunctions, disjunctions, implications, negations and quantifiers	Diagrammatic modality markers: • Abstract • Naturalistic and contextual
Status of mathematical knowledge	To what extent does the text indicate that decisions or choices are possible during mathematical activity?	modifiers indicating degree of certainty (e.g. *may*, *can*, *will* …) conditional clauses (e.g. *if* … or *when* …) explicit decisions have been or need to be made	• Labelling • Additional features (colour, arrows, words) • Neat / rough
	Is mathematics discovered or invented?	types of mental/verbal processes: observation or 'creation': e.g. "we notice … " or "we say … "	
How are the learners and their relationships to mathematics construed?			
Property of the discourse	*Specific questions guiding analysis*	*Indicators in verbal text*	*Indicators in visual text*
Activity	What kind of activity is the learner expected to engage in?	"thinker" or "scribbler" processes (imperatives, "you … ")	Contact: • demand
Authority	Where does authority lie (in relation to what aspects of mathematical activity)? Are choices or decisions available and who makes them?	personal pronouns (for which processes?) imperatives or questions modality markers of authority and certainty	• offer Modality: • abstraction/naturalism

(*Continued*)

Table 1. Continued.

How is the nature of mathematics and mathematical activity construed?

Property of the discourse	*Specific questions guiding analysis*	*Indicators in verbal text*	*Indicators in visual text*
Formality	What is the relationship between author and reader? Is there a pedagogic relationship?	inclusive "we" passive voice extent of specialised forms	Social distance: • neatness • labels • colour • redundancy

What role does the text play?

Property of the discourse	*Specific questions guiding analysis*	*Indicators in verbal text*	*Indicators in visual text*	*Verbal/visual relation*
Structure of knowledge	Does the text present facts or develop arguments? What is assumed?	Thematic progression Given-New	information value Given-New (horizontal) Ideal-Real (vertical) Centre-Margin	elaboration: • specification • explanation extension:
Topic	What are the significant ideas?	cohesion devices paralinguistic markers (e.g. colour, types face)	Salience: colour, size, position, perspective	• similarity • contrast • complement
Structure	What are the component parts of the text?	Paragraphing Headings	Framing: separation (frame lines, white space, colour) or connection (visual links, lack of framing)	

forms, algebraic notations, and gesture (Morgan & Alshwaikh, 2009; O'Halloran, 2005; Radford, Edwards, & Arzarello, 2009). Multimodal analysis requires consideration of the roles that different modes play in a specific text. In studying Palestinian mathematics textbooks, it was decided to focus in particular on the verbal and diagrammatic modes and the relationship between them. The choice to focus on verbal and diagrammatic aspects arose at least in part from the particular focus on geometry, the area that was studied most and also the area for which looking at visual modes seems most relevant. This consideration is shown in the final column of Table 1.

The elaborated analytical scheme shown in Table 1 adopts many features of the EDSM scheme. It makes use of properties of mathematics discourse and indicators suggested by the EDSM scheme in looking at the way in which mathematical activities are represented in the textbooks and the role of students in engaging with these activities. For instance, the elaborated scheme uses the properties and indicators for analysing how mathematics is specialised and objectified that appear in EDSM scheme (see Morgan & Sfard, this issue). However, it has a different structure and also includes some additional features. The focus of this article will be on the changes introduced to produce the elaborated scheme.

One main difference between the EDSM scheme and the elaborated scheme is the way in which each of them is structured. While the EDSM scheme is mainly structured according to Sfard's (2008) characteristics of mathematics discourse, the elaborated scheme was structured according to Halliday's three metafunctions (ideational, interpersonal and textual). In other words, the elaborated scheme was structured around three main questions corresponding to these metafunctions, respectively: How is the nature of mathematics and mathematical activity construed? How are learners and their relationships to mathematics construed? What role does the text play? The restructuring of the scheme in this way enabled integration of new tools for the analysis of visual elements drawn from multimodal social semiotics (Alshwaikh, 2011; Kress & Van Leeuwen, 2006). This component, including analytic tools for investigating the diagrammatic part of mathematical texts and the relationship between this and the verbal part, is shown in the final column of Table 1. In the next section, I briefly present the classifications of visual forms included in Table 1, explaining how they relate to the three metafunctions and the three main questions indicated above, and showing how they complement the EDSM scheme.

The nature of mathematical activities: narrative and conceptual diagrams

My scheme for analysing geometric diagrams in mathematical texts was inspired by SFG (Halliday, 1985) and Kress and Van Leeuwen's Reading Images (2006). Addressing the ideational metafunction, two categories of diagrams construe different visions of the nature of mathematical activity: narrative and conceptual. Kress and Van Leeuwen (2006) distinguish between narrative and conceptual representations based on the presence of vector or arrow that suggests action. Narrative representations, according to Kress and Van Leeuwen, always have one vector and conceptual representations "never do". In mathematics, since the presence of arrows is common (in geometry in particular), the distinction between narrative and conceptual is based on temporality – how time is represented visually. As the names may show, while narrative diagrams tell (by showing) a story by including some indictor of a timeline, conceptual diagrams contain no indication of temporal development. Narrative diagrams thus present mathematics

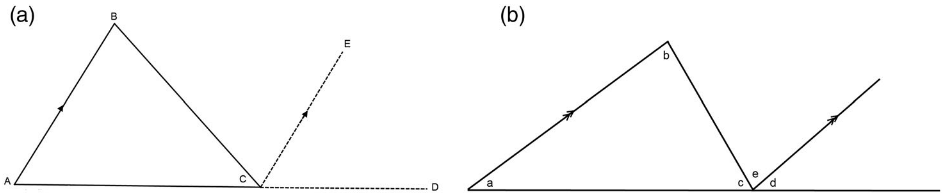


Figure 1. Narrative and conceptual diagrams represent the 'same' Proof of the Exterior Angle Theorem.

as an activity that may be seen as involving humans. Conceptual diagrams, in contrast, eliminate human actions and show atemporal objects or relationships, thus contributing to the *objectification* of mathematical discourse.

The difference between narrative and conceptual diagrams is illustrated in Figure 1 in which a proof of the Exterior Angle Theorem is presented in two different ways. In Figure 1(a), dotted lines are distinguished from solid lines, signifying the actions of constructing a line parallel to side AB of the triangle and of extending the side AC. The proof of the theorem is thus represented as an activity carried out, possibly by a human agent, over a period of time. In contrast, in Figure 1(b), only the product of the construction is shown, representing the proof as an object rather than a process, with no indicator of the temporal activity of drawing the triangle. See Alshwaikh (2011) for a full account of the properties and types of narrative and conceptual diagrams.

This classification of narrative and conceptual diagrams is in harmony with the EDSM scheme. The distinction arises from social semiotics, one of the theoretical approaches that underpin the EDSM scheme. My thinking about the classification has also been influenced by Sfard's commognition and, in particular, Sfard's identification of the role of *objectification* in mathematical discourse. For example, the analysis above highlighted the process of objectification by pointing out the difference between Figure 1(a), showing the process of proof, and Figure 1(b), showing the final product or the proof as object. The question of the extent to which mathematical activity is presented as a human activity is also a focus of the EDSM scheme.

The role of the learner

While the distinction between narrative and conceptual diagrams corresponds to Halliday's ideational metafunction, other features in the scheme were developed to match the other two metafunctions: the interpersonal and the textual. The second question of the elaborated scheme addresses the interpersonal metafunction, focusing on the role of learners and their relationships to mathematics. Drawing on the Hallidayan SFG, Kress and Van Leeuwen (2006) developed an approach to investigate such relationships between the reader (viewer) and the (visual) text. They suggested looking for a *contact* between the viewer and the visual text, and they distinguished two types of contact: *demand* and *offer*. In written mathematics text a *demand* contact usually asks for information, generally using a question "*what is the value of x?*" or an instruction "*find the value of x*". *Offer* contact, in contrast, provides information, e.g. "*rABC is a right triangle at B*".

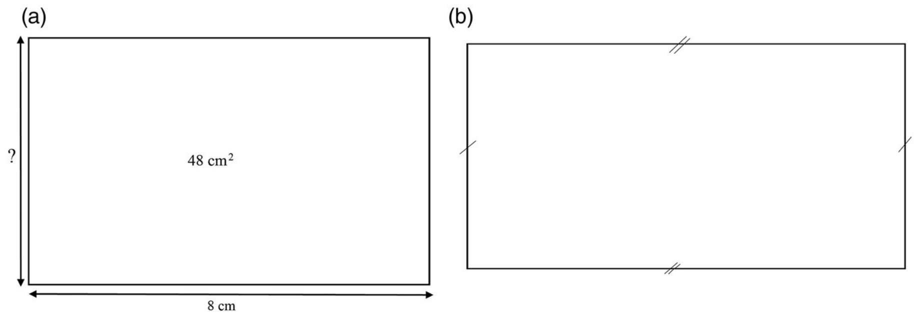


Figure 2. Demand and offer diagrams.

Demand and offer contact may be represented in (geometrical) diagrams by visual cues such as question marks or labels (Figure 2[1]). For example, Figure 2(a) asks for the length of the side next to the question mark. Figure 2(b), in contrast, offers information about the shape presented; it has four sides, the opposite sides are of equal length and there are four right angles.

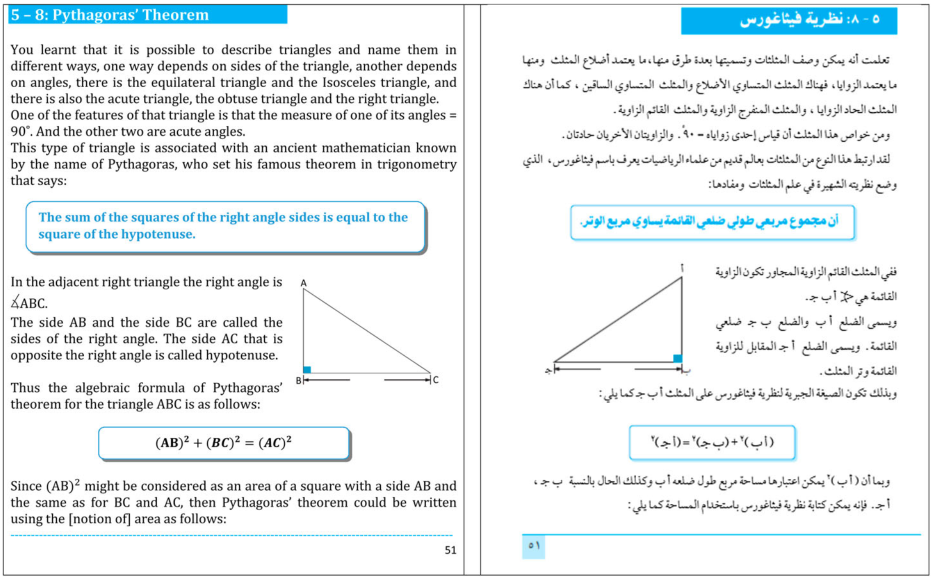
The text as a whole: the relationship between the verbal and the visual

In analysing the textual meaning, which the third question in the elaborated EDSM scheme addresses, the focus is on the mathematical text itself. Here I analyse the placement of the visual in relation to the whole text, or what Kress and Van Leeuwen (2006) call the "*information value*", which corresponds to the way that mathematical knowledge is structured in the text. For instance, if the visual part is in the centre of the page, then it is the focus of that text. In addition, the relationship between the visual and the verbal is important in order to understand the argument as a whole. This is shown in the final column under the third question in Table 1.

An illustrative example

This section presents an example to demonstrate how the elaborated scheme has been applied to a Palestinian school mathematical text. The focus will be on one property of mathematics discourse for each of the three main features discussed above: The nature of mathematical activities, the role of the learner and the relationship between the verbal and the visual. These properties are, respectively: *objectification*, *activity* and *structure of knowledge*.

The example is a section taken from the textbook for grade 7 (13-year-olds) entitled "Pythagoras Theorem". Figure 3 shows only the first page of that section; the rest of the section includes three Examples, In-class activities and Exercises and problems. This is a typical example of the structure of Palestinian school mathematics textbooks: each section starts with a theoretical part (theorem, definition, etc.) followed by illustrative examples accompanied by diagrams, then activities (mainly using artefacts such as scissors or paper, etc.), with exercises and problems presented at the end of the text.



5 – 8: Pythagoras' Theorem

You learnt that it is possible to describe triangles and name them in different ways, one way depends on sides of the triangle, another depends on angles, there is the equilateral triangle and the Isosceles triangle, and there is also the acute triangle, the obtuse triangle and the right triangle.
One of the features of that triangle is that the measure of one of its angles = 90°. And the other two are acute angles.
This type of triangle is associated with an ancient mathematician known by the name of Pythagoras, who set his famous theorem in trigonometry that says:

The sum of the squares of the right angle sides is equal to the square of the hypotenuse.

In the adjacent right triangle the right angle is ∡ABC.
The side AB and the side BC are called the sides of the right angle. The side AC that is opposite the right angle is called hypotenuse.

Thus the algebraic formula of Pythagoras' theorem for the triangle ABC is as follows:

$$(\mathbf{AB})^2 + (\boldsymbol{BC})^2 = (\boldsymbol{AC})^2$$

Since $(AB)^2$ might be considered as an area of a square with a side AB and the same as for BC and AC, then Pythagoras' theorem could be written using the [notion of] area as follows:

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٥ - ٨: نظرية فيثاغورس

تعلمت أنه يمكن وصف المثلثات وتسميتها بعدة طرق منها، ما يعتمد أضلاع المثلث ومنها ما يعتمد الزوايا، فهناك المثلث المتساوي الأضلاع والمثلث المتساوي الساقين، كما أن هناك المثلث الحاد الزوايا، والمثلث المنفرج الزاوية والمثلث القائم الزاوية.
ومن خواص هذا المثلث أن قياس إحدى زواياه = ٩٠°. والزاويتان الأخريان حادتان.
لقد ارتبط هذا النوع من المثلثات بعالم قديم من علماء الرياضيات يعرف باسم فيثاغورس، الذي وضع نظريته الشهيرة في علم المثلثات ومفادها:

أن مجموع مربعي طولي ضلعي القائمة يساوي مربع الوتر.

ففي المثلث القائم الزاوية المجاور تكون الزاوية القائمة هي ∡ أ ب جـ.
ويسمى الضلع أ ب والضلع ب جـ ضلعي القائمة. ويسمى الضلع أ جـ المقابل للزاوية القائمة وتر المثلث.
وبذلك تكون الصيغة الجبرية لنظرية فيثاغورس على المثلث أ ب جـ كما يلي:

(أ ب)^٢ + (ب جـ)^٢ = (أ جـ)^٢

وبما أن (أ ب)^٢ يمكن اعتبارها مساحة مربع طول ضلعه أ ب وكذلك الحال بالنسبة ب جـ، أ جـ. فإنه يمكن كتابة نظرية فيثاغورس باستخدام المساحة كما يلي:

٥١

Figure 3. Extract of a Palestinian school mathematics textbook (Grade 7, Part 2, p. 51), translated by the author. © Palestinian Ministry of Education

The analysis applied the elaborated scheme to the original Arabic text, shown on the right of Figure 3 with a translated version on the left. See Alshwaikh (2015) for a fuller account of the analysis of Arabic school mathematics texts using these analytic tools.

The nature of mathematical activity: objectification

Objectification, according to Sfard (2008), is a key feature in mathematical discourse. In her linguistic approach, it is the creation of mathematical objects, implemented in two discursive moves. The first move is *reification*, turning processes into objects by using nouns instead of verbs. One of the ways in which this is done is through the grammatical metaphor of nominalisation, converting a verb into a noun. Halliday and Martin (1993) have shown that nominalisation is a dominant feature in scientific writing. The second discursive move of objectification is *alienation*, meaning concealing the role of human being in actions. Verbally, this is done through the use of impersonal voice. Again, this is a common characteristic of scientific writing. Visually, objectification may be done by the use of abstract (conceptual) diagrams that present the properties and relationships of mathematical objects rather than actions on them.

In the example text (Figure 3), the 'right triangle' is the focus of the text. This concept is objectified both verbally and visually. It is objectified verbally by the use of the noun *measure* (قياس) and the use of passive voice. The verb *measure* (يقيس) represents a process carried out to determine the dimensions of observable entities.[2] This process involves actions usually done by human agents using tools such as rulers and other calibrated devices. Assigning a number or a quantity (with a unit) to this process indicates that the process is completed, and we can talk about the product as a 'thing'. For instance, the final product of the process of measuring the dimensions (length and width) of a table is found to be that the length is 120 cm and the width is 70 cm. We can say that the measure of the dimensions of that table is *120 cm × 70 cm*, or, as in Figure 3, *the measure of one of its angles = 90°*. Thus, the use of the noun *measure* reifies the process of measuring as a 'thing'. Use of the passive voice also conceals the role of the human agent, as in *The side AB and the side BC are called the sides of the right angle*, where the process of 'calling' has been concealed.

Visually, the diagram shown in the text is conceptual, meaning it does not present the process of measuring. The double arrow next to BC might be seen a sign of measuring process; however, there is no number assigned to it in the diagram (indicating a completed act of measurement) nor is there any reference in the written text to such a process. Instead, the diagram presents the right triangle in an abstract, atemporal way with no humans involved. Furthermore, the whole text uses a specialised language with specific vocabulary (e.g. *triangle*, *angle*, *hypotenuse*), conventional expressions (e.g. *equilateral triangle*, *acute triangle*, *algebraic formula*) and symbols (e.g. *90°*, ∡). This phenomenon (specialised and objectified mathematical text) is common in Palestinian school mathematics texts and in many other countries. Given these specialised and objectified properties of mathematical texts, one would ask about the role these texts anticipate for students interacting with them. This role is analysed in the next section.

The role of the learner: learner activity

The elaborated scheme (Table 1) addresses learner activity by asking the question: What kind of activity is the learner expected to engage in? The suggested indicator follows the distinction Rotman (1988) made between "scribbler", where the learner of mathematics engages in material processes, and "thinker", in which the engagement is in mental processes. In the verbal part of the selected text (Figure 3), the majority of processes construe the learner as a "scribbler" (e.g. *use*, *cut*, *write*, *measure*, *calculate*), but there is also an expectation that the learner will be a "thinker" (e.g. *show*, *notice*), engaged in observation, reflection and reasoning. In the Examples and In-class activities sections of the text (not shown in Figure 3), the visual component of the text includes diagrams labelled by marks indicating equality, symbols showing areas or with specific measures. The learner's role is thus construed as observing and reasoning about the properties of the shapes, triangles or Pythagorean numbers. Moreover, the learner's activity is shown not only through imperatives but also through the use of tasks which allow choice in the approach to the task, e.g. *Write a group of Pythagorean numbers that were not mentioned in the question.*

Visually, the way in which diagrams are presented in the text also indicates that the role of the learner is of both scribbler and thinker. The diagrams are abstract and mostly *offer* information to the learner (e.g. showing specific measures on the sides of triangles). There is, however, one diagram (not shown in Figure 3) that involves *demand*, indicating by the use of x on one side of a right triangle that the learner is required to calculate when the hypotenuse and the other side are known. Such demands encourage learners to engage in problem solving in nontrivial ways.

Palestinian school mathematics textbooks not only objectify mathematical activity and emphasise its specialised properties, as shown above; they also tend to present the learner as *scribbler*, with some exceptions in which they encourage learners to think.

The relation between the visual and the verbal: the structure of knowledge

As mentioned earlier, Palestinian school mathematics textbooks present the theoretical part at the beginning of the text followed by examples and then exercises for students to solve. The property of the elaborated scheme dealt with in this section addresses the role of the text as a whole in structuring mathematical knowledge. Consideration of the visual part of mathematical texts is extended to include the layout of the text. In addition, the relationship between the visual and the verbal parts is considered. Therefore, the placement of these two parts in the text in Figure 3 will be analysed. If we take a close look at the theoretical part of the Palestinian text, which is the upper part, it starts by writing the theorem highlighted in colour and large bold font, framed and centred on the page. Then we see written text on the right in Arabic (left in English) with a diagram to the left (right in English), then, again, the Pythagorean Theorem is written in algebraic form, framed and centred. To analyse how knowledge is structured in this part, we need to look at the verbal part of the texts, the visual part including the diagram and the relationship between the verbal and the visual.

First, most of the text here is verbal, occupying most of the page with just one diagram and some visual frames around the writing. The question guiding the analysis here is whether the text presents facts or develops an argument. Mathematical facts are statements that participants in mathematics discourse believe or agreed to be valid or "endorsed narrative" in Sfard's terms (2008). Arguments, on the other hand, are statements yet to be approved or endorsed. Thus the question becomes whether the text presents something that is taken for granted to be true or something new yet to be proven (see below the discussion about *Given-New* structure). The first few lines of the text offer information about types of triangles learned in previous grades, and in particular about the right triangle, its features and association with Pythagoras. Then Pythagoras' theorem itself is presented and distinguished by visual marks (colour and frame, see below). The verbal text next presents an example to demonstrate the theorem and to present its algebraic formula at the end of that example. Finally, the theorem is presented as a sum of areas on the next page (not shown in Figure 3), stating that "the sum of the constructed squares on the two sides of the right angle is equal to the square constructed on the hypotenuse", accompanied by a diagram and followed by an example and an activity.

One of the indicators that may be used to investigate how knowledge is structured is to look at the progression of *themes* in the text (Morgan, 1996). The *theme* is the focus of the clause,indicated by being positioned at the beginning of a clause (Halliday, 1985). More specifically, the first part of a sentence plays a different role in structuring knowledge from the part at the end of that sentence. Halliday refers to the structure of information as the *Given-New* structure. The *Given* part comes at the beginning of the clause and is what the reader is expected to be familiar with or take for granted, while the *New* part comes at the end of the clause, and is what the author wants to show or prove. Consider the example "*The side AB and the side BC are called the sides of the right angle*", the *Given* part is "*The side AB and the side BC*" where the two sides are known to the reader, and the *New* part is "*are called the sides of the right angle*"where a new name for these two sides is presented.

In a similar way, to analyse how knowledge is structured visually, the *Given-New* structure is used. This is done by considering the placement of the diagram in relation to the whole text and the verbal text from different directions –horizontally (from right to left, in Arabic) and vertically (up to down) – and by asking questions such as: what comes first, what comes next, etc. For instance, if we look at the horizontal structure of the page in Figure 3, the verbal text comes first (on the right side in Arabic) and the diagram comes second, on the left side. Here the *Given* part is the verbal, and the *New* part is the diagram. This segment of the text is followed by the algebraic formula of the theorem. If we want to use words to describe this structure, we may say: in the diagram shown here, one can see that the information presented in the verbal part about this triangle (e.g. *the right angle is at B*) is true. Thus the visual and the verbal parts have "*similar*" content (see below), providing information about the right triangle. Furthermore, the use of the word "*Thus*" brings these two parts together to provide the algebraic form for Pythagoras' theorem, as if the verbal and the visual are brought together in order to construe the algebraic form.

A further approach to structuring mathematical knowledge is done visually through *salience* (e.g. the use of colour) and *framing* (e.g. frame lines). Making some elements more salient contributes to their importance in the text. On the other hand, separating or connecting elements of a text affects its unity as a coherent message (Kress & Van Leeuwen, 2006). In the text analysed here (Figure 3), both types of indicator are present. The use of colour in the wording of the Pythagorean Theorem, its bold font and central alignment, signify its importance as a focus of this text. The frame lines around both the wording of the theorem and its symbolic form separate them from the rest of the text. The two indicators (*salience* and *framing*) draw attention to Pythagoras' theorem as a fact rather than developing argument about it. They both declare its content, using different forms of representation.

The relationship between the verbal and the visual parts of the text also contributes to structuring knowledge. Van Leeuwen (2005) categorised different relationships between the verbal and the visual as *elaboration* and *extension* types (the final column of Table 1), each of which has different subtypes. In the mathematics lesson analysed here, the diagrams are presented in a *similar* relationship with words, a subtype of the extension relationship, meaning they present similar information, introducing a new representation but no new information. In presenting diagrams in a *similar* relationship to words, this text is consistent with the conventional view among mathematicians who have traditionally viewed diagrams as "pedagogical illustrations, not part of the real subject matter" (Hardy, 2004, p. 125). As in the case of *salience* and *framing*, discussed above, the *similarity* relationship between the verbal and the visual parts presents and *reinforces* facts about the Pythagoras' theorem rather than introducing and forming *relationships between* facts in order to develop argument about it.

To conclude this section, a partial analysis of ideational, interpersonal and textual aspects has been presented. Within each aspect I looked at several of the textual indicators and interpreted them in terms of the nature of mathematics, the role of the learner and the way in which mathematical knowledge is structured. Mathematics is abstract and specialised, the activities that students are expected to engage with are of a scribbling nature more than thinking, and mathematical knowledge is structured as a collection of facts. See Morgan (this issue) for detailed discussion of how mathematical knowledge is construed in mathematics examination texts and the potential impact on the way in which students may perceive their role in learning mathematics.

The analysis presented above shows the potential of the elaborated scheme to analyse texts from a different genre and context to that used in the original EDSM study. This application beyond the original design of the scheme includes addressing questions about mathematics discourse in a different language and socio-cultural context. However, applying the scheme to a different language and context presents its own challenges, especially applying it to Arabic texts. These challenges are the focus of the next section.

Challenges: Arabic mathematics discourse

In attempting to use the EDSM scheme and the elaborated scheme, including the visual mode, as described above, I became interested in the issue of mathematical discourse in different languages (Arabic in particular). I became sensitive to the challenges of the task already earlier while translating a number of mathematics school textbooks from Arabic to English as part of the "Analysing Palestinian school mathematics textbooks" project. We had some successes, but we also encountered difficulties. When I presented a paper about mathematics, language and communication in Arabic language at a local conference about Arabic language in 2012, the feedback I got encouraged further investigation. And yet, I was unsuccessful in trying to find Arabic language scholars who could help me refine my work. Currently, in the Palestinian (and Arabic) context, the notion of mathematics discourse and communication seems exotic because of the dominant view of mathematics as a 'universal language' and context-free subject. There are many examples illustrating this view, whether expressed by academics (https://uqu.edu.sa/hnahmed/ar/21415), media (http://www.albayan.ae/science-today/studies-research/2012-12-16-1.1785416) or public opinion (http://thevoiceofreason.de/article/380).

While mathematics had been developed in the Arab culture for centuries, especially in the period of the eighth to twelfth centuries (e.g. al-Khwarizmi in algebra from the Arabic word *al-jabr*, Lyons, 2009), its development, and that of Arabic science in general, has slowed down after that era. The downturn in contribution to mathematics is a reflection of a more general decline in contribution to scientific knowledge and is evident in contemporary Arab life and academia as manifested in many writings (e.g. Hanafi & Arvanitis, 2016; United Nations Development Programme [UNDP], 2003). For instance, in its report about the state of knowledge in the current Arab countries, the Arab Human Development Report (UNDP, 2003) shed some light, stating that while Arabs represent 5% of the world's population they produce only 1.1% of the world production of books. Because of the slow-down in the development of original mathematical thought, the mathematical register in Arabic began lagging behind its counterparts in other languages such as English. This means that some parts of the mathematical register have to be imported from other languages.

A second significant issue is the diglossia of Arabic: the separation between formal written Arabic (fus-ha فصحى) and spoken everyday language (ammiyya عامية). Unlike English, Arabic is essentially two languages, written and spoken, with only limited overlap between the two. The formal language is the language of reading and writing and the everyday language is the language of 'speaking'. This entails that children need to learn mathematics in a formal language that is not as familiar to them as the English of mathematics is for English-speaking learners. Of course, diglossia influences the teaching and learning process in general, not just in mathematics. However, there is a lack of research about its impact. The issue of Arabic language was touched upon in the UNDP report, which highlighted that Arabic faces "severe challenges and a real crisis in theorization, grammar, vocabulary, usage, documentation, creativity and criticism" (p. 7).

The connection between Arabic language and mathematics is a novel topic of investigation and requires additional research. In the Palestinian context, at least, I surmise that this area is undeveloped because the common view is that mathematics is a universal language. This view was manifested in creating mathematics textbooks mainly by borrowing from other Arabic (e.g. Jordanian) or English (e.g. American) textbooks in the mid-1990s. The lack of development of that connection between language and mathematics has continued even after Palestinians started writing textbooks themselves (around 2000) since most of the textbook authors (academic mathematicians, mathematics educators and teachers) hold this view of mathematics. I hope that this article may be a step towards introducing the topic of Arabic mathematics discourse into scholarly discussion. There are two primary challenges in doing so. First, there is a linguistic challenge: finding grammatical expressions in Arabic that are 'similar' to those included in the English language scheme. Second, there is a methodological challenge: applying an analytic scheme, which was developed in English for application to English language texts, to Arabic text.

The translation challenge

This challenge refers to the difficulties that emerge in the translation process in general, including the challenges of translating technical terms used in the scheme and of translating the words and grammatical expressions used in the school textbook.

Technical terms in the scheme

In order to engage more fully with the elaborated analytic scheme and to make it available for other researchers in Palestine, I translated a version of it into Arabic. While doing this translation and interacting more with it, I needed to translate the specialised vocabulary of the scheme, including specialised words such as *mathematical objects* and non-specialised words such as *scribbler*. *Mathematical objects* was translated into Arabic as *mathematical entities* (كائنات رياضية) – an expression which, to the best of my knowledge, is not used in Arabic. The lack of a relevant familiar expression in Arabic suggests that the construct itself is not likely to be familiar, leading to difficulties in communicating both the methods and the outcomes of analysis. The closest literal translation for *scribbler* was a word in Arabic (مخربش) which connotes something chaotic or messy. This word cannot be used to function in the way intended in the analytic scheme to indicate engagement in a material process rather than a mental process. Ultimately, the translation that seemed most apt is an expression, not a single word, indicating the person who follows instructions without thinking (الشخص الذي يتبع التعليمات دون تفكير); this expression provides the desired contrast between material and mental processes.

Arabic syntax versus English syntax

After translating the technical terms in the scheme, I faced the challenge of finding equivalent Arabic grammatical forms that could be used as textual indicators. In a personal communication (April, 21, 2015) Sfard commented that:

> … expressions [that] are translated from other languages, such as English or Hebrew, verbatim, often [lose] their metaphorical comprehensibility in the translation. This aggravates the difficulty stemming from the fact that no two different-language mathematical discourses are fully homeomorphic.

That homeomorphism was one of the challenges I encountered while trying to find Arabic grammatical forms that preserve the suggested meaning of the properties or the indicators in the scheme (cf. Barton, 2008; Morris, 2014). Consider the example of a property such as *objectification*. After deciding the translation as discussed above, I addressed the challenge of how to express it in terms of Arabic grammar. This was a hard task because: (1) I am not an expert in Arabic grammar; and (2) it was difficult to explain the problem to colleagues in the Arabic department of Birzeit University maybe because the dominant approach to Arabic grammar is normative rather than descriptive. Nominalisation, turning a verb into a noun, is the first indicator of the *objectification* property, and it took some time to reach a satisfactory parallel grammatical expression in Arabic. Another challenge was defining the textual indicators for the *activity* property in Arabic. While pronouns and imperatives are the key indicators in the scheme, Palestinian school mathematical texts use three main pronouns: we, you and I,[3] but their use is not consistent. The next section discusses this issue.

The challenge of applying the scheme to Arabic text

After the linguistic challenges outlined above were addressed and the scheme was translated, the next step was to apply the scheme to the mathematical text under analysis. In some cases, where the vocabulary and syntax of Arabic and English are sufficiently similar, application of the scheme was straightforward. For example, the analysis presented above of the mathematical text in Figure 3 the claim was made that it was through the use of language that the right triangle became the focus. Nominalisation was found in the text in words such as *measure*. In Arabic, as in English, the noun *measure* is a nominalisation of the verb *measure* (see the discussion above about *measure* as reification of the process of measuring). This is similar to the word *sum* in "*The sum of the squares…* ". The verb *sum* in Arabic (يجمع) is a process of adding two (or more) numbers or things together – a process which may involve human actors. The product of the process of summing (جمع) is a number that enables us to talk about the sum as a 'thing', obscuring any human activity. These cases of nominalisation, as well as the further alienation through the use of the passive voice 'is called' were easy to identify because of similarities between the two languages.

However, there are some nouns that are the product of the nominalisation process in Arabic but not in English. For example, in Arabic there is a cognate verb form related to the property 'congruence' (تطابق), used to mean to check whether two shapes are congruent or not. This process may be performed by a human agent. When facing the challenge of communicating this usage in English, Alshwaikh and Morgan (2013) coined a verb *to congruent** as in "*I cut the triangle and I congruent* it with the triangle of my colleague.*" This was necessary in order to demonstrate to an English-speaking audience that, in this part of the text, the specialised mathematical property of congruence is construed as a human activity. Unlike in English, the property *congruence* is a nominalisation of the process *to congruent**, and its use contributes to objectification of the text.

As mentioned above, using pronouns to investigate the student role also posed problems. These arose from the fact that in Arabic the reference of personal pronouns can be ambiguous. For example, the translated text in Figure 3 uses the pronoun 'you', but in the Arabic original, it is actually not clear whether the pronoun is first or second person. In Arabic, the pronoun is attached to the word 'learn' (تعلمت) in the first word in the text after the title. This may be read as either 'I learned' or 'You learned'. The decision to translate it as 'you learned' is actually based on other examples that appear in this lesson (not shown in the figure) in which most of the pronouns are in the imperative, such as *verify*, *use*, *cut*, etc. which refer unambiguously to the second person. There are, however, two examples of pronouns which refer to the learner in the first person: 'I write' and 'I indicate'. The textbooks are inconsistent in the way they include the learner as reader (using second person) or in solidarity with the author (using first person). This inconsistency heightens the challenge of interpreting ambiguous pronouns. The Palestinian school mathematics texts sometimes clarify whether they refer to 'you' or 'I' by using a diacritical mark called *hamza* with the letter *Alif* (the first letter of the alphabet in Arabic) to refer to the pronoun *I* (أنا), or to write the *Alif* without the *hamza*, which would refer to the pronoun *you* (انت). Unfortunately, the use of this mark is also not consistent throughout the textbooks, so the ambiguity is not resolved.

Conclusion

This article aimed to reflect on the use of the analytical scheme developed in the EDSM project, the focus of this special issue, in investigating the geometry curriculum in Palestinian mathematics school textbooks. Because mathematics texts use multiple modes of communication, and because the EDSM scheme focuses mainly on the verbal mode, my investigation used an elaborated version of that scheme which focused on developing the analysis of the diagrammatic components of the texts. This use was illustrated by an example from a Palestinian mathematics school textbook, focusing on three properties of the mathematics discourse suggested by the EDSM scheme: objectification, activity of the learner and structure of mathematical knowledge. Focusing on these properties demonstrates that the analytic scheme is applicable to other language and contexts and to a different goal (analysing mathematical textbooks rather than examination texts).

The illustrative example showed that the section of a Palestinian textbook geometry chapter that was analysed used a specialised discourse of mathematics and presented mathematics in an objectified way, primarily highlighting mathematical objects rather than the activity that produced them. Consideration of the role of the learner in this example suggested that this text was likely to encourage the learner of mathematics to be more scribbler than thinker. Further studies of substantial samples from Palestinian textbooks, using the full set of analytical tools in the scheme, have shown similar results (e.g. Alshurafa, 2015; Alshwaikh, 2015). This construed image of mathematics in Palestinian school mathematics textbooks (namely that mathematics is specialised and objectified) appears consistent with that shared by mathematics teachers and educators in Palestine and illustrated in textbooks that contain 'condensed' information with many mathematical concepts. Similarly, the image of learners of mathematics found in the textbook conforms to the general approach to education in the Palestinian educational system, in which teaching is teacher-led. These consistent images of mathematics and of learners may contribute to possible explanations for the poor performance of Palestinian students in learning mathematics.

To sum up, the potential of the EDSM scheme to analyse the image of the mathematical activities and of the learner of mathematics in Palestinian textbooks has been demonstrated. However, the adaptation of that scheme to Arabic mathematics textbooks created challenges. These challenges mainly related to translation of the analytical scheme into Arabic and its application to Arabic text. Most of the challenges mentioned in this article arise from the investigation of the geometry curriculum. They still need more elaboration and investigation, as additional challenges may arise in applying the scheme to other topic areas in mathematics. I hope this discussion of those challenges will contribute to the debate about mathematics and its relationship to (Arabic) language among Arab scholars interested in mathematics teaching and learning.

Notes

1. Figure 2(a) is taken from https://www.theguardian.com/science/2015/aug/03/alex-belllos-monday-puzzle-question-area-maze-smarter-than-japanese-schoolchild; and Figure 2(b) is taken from http://www.mathsisfun.com/quadrilaterals.html
2. In English, the words for more specific types of measure (e.g. length) are more commonly used. I use the noun *measure* here as a translation for the original Arabic in order to reflect as closely as possible the cognate relationship between the noun and verb in Arabic.
3. In Arabic pronouns may be either separated or attached to a subject or an object. The three mentioned pronouns come in different forms but I am not going to burden the reader by introducing these forms in detail here.

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