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Centre of mass motion and the Mott transition in light nuclei

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Abstract

We calculate the effect of the inclusion of the centre of mass (CM) motion on the survival of light nuclear clusters in a vapour of nucleons at very low densities and moderate temperatures. The Mott transition refers to the dissolution of these clusters above a certain vapour density (the Mott density). The cluster CM motion has so far been included approximately or at fixed CM momentum in calculations of the Mott density. Often it is neglected altogether due to technical difficulties. In the present work we include the CM motion exactly for clusters up to $A=4$ and calculate its effect by averaging over all values of the CM momentum at a given temperature rather than at a fixed value of the CM momentum. We demonstrate that the inclusion of the CM momentum shifts the Mott density to higher values. The shift, however, decreases as the cluster mass increases.

Keywords: low density nuclear vapour, cluster formation, Mott transition, centre of mass effects

1. Introduction

Nuclear matter is customarily viewed as a collection of interacting nucleons. At very low densities (less than one tenth of nuclear saturation density) nuclear matter has been often described as a vapour consisting of widely separated nucleons while at higher densities nuclear matter is described as a liquid of closely interacting nucleons. Whereas the description in terms of interacting nucleons has been successful at elucidating the properties of nuclear matter at high densities, it has been found that the model breaks down at very low densities. The breakdown of the model is due to the formation of light nuclear clusters at these very low densities [1, 2]. These light nuclear clusters are essentially the same as ordinary light nuclei (which exist in vacuum), except that their binding energy and other properties are modified by the surrounding vapour of nucleons. The properties of these light nuclear clusters should

approach those of ordinary light nuclei with the same number of protons and neutrons as the density of the surrounding vapour of nucleons goes to zero. We use the term 'clusters' rather than 'nuclei' in order to emphasize the modification of their properties by the surrounding medium.

The abundance of light clusters in nuclear matter at very low densities and moderate temperatures has been confirmed by several theoretical and experimental studies [1–7]. However, as the density of nuclear matter increases the properties of the clusters are modified and their binding energy decreases until they finally dissolve into their constituent nucleons above a certain critical density (the Mott density) [8]. The decrease in the binding energy of the clusters is mainly due to the Pauli blocking effect which is caused by the indistinguishability between the nucleons inside the clusters and the free nucleons in the surrounding vapour. This indistinguishability means that the total wavefunction of the system should be antisymmetric not only under the exchange of any pair of identical 'internal' nucleons that constitute the cluster but also under the exchange of an 'internal' nucleon with an identical 'external' nucleon from the surrounding vapour. This leads to a reduction of the binding energy of the nuclear cluster as compared to a free nucleus existing in vacuum.

More recently, the effect of the surrounding medium on the binding energy of light nuclear clusters has been calculated in a quantum statistical approach [9]. Although the effect of the cluster's centre of mass (CM) momentum has been discussed in the first publications dealing with the Mott transition, e.g. [2], many later calculations have been carried out at zero total cluster CM momentum because of the difficulties inherent in the inclusion of this momentum in a many-body theory even in the mean-field approximation [10]. A detailed investigation of the effect of the CM momentum on the Pauli blocking term has recently been given by Röpke [11].

In order to estimate the effect of the cluster CM momentum, we propose to use a medium- independent wavefunction for the cluster and then carry out an exact separation of the wavefunction of the CM motion from the wavefunction of the internal degrees of freedom of the 'internal' nucleons that constitute the cluster. We then consider a system consisting of the cluster and a free 'external' nucleon in the surrounding medium. The choice of a single external nucleon is justified by the low density of the medium in which the cluster is immersed so that the effect of the other external nucleons is simply additive and thus linear in the density [8, 9]. This of course is a simplification since the change in the cluster's binding energy implies a modification of the cluster's internal wavefunction and leads to a nonlinear dependence on the density [11]. However, the present work is intended to investigate the shift in the Mott transition due to the correct inclusion of the CM motion and it is expected that this shift will not be affected by the nonlinear terms since these depend on the internal wavefunction of the cluster and not the CM momentum. It is hoped that in future work the nonlinear effects can be included.

The total wavefunction of the system consisting of the cluster and a free 'external' nucleon is constructed such that it is antisymmetric under the exchange of this external nucleon with each of the identical 'internal' nucleons. The Pauli blocking energy shift for the cluster is then evaluated by calculating the change in the expectation value of the Hamiltonian of the system resulting from the antisymmetrization of the total wavefunction. The calculation is then repeated with the same wavefunction but with the total CM momentum of the cluster set to zero. The difference between the two calculations gives the effect of the CM motion of the cluster on the binding energy of the cluster and hence on the Mott transition. The much smaller self-energy and Coulomb shifts are usually neglected, see for example [9], however they are not momentum dependent and so they do not affect the main result of the present work which is the dependence of the Mott transition on the CM momentum.

There is an additional difference between the results of the present work and those of previous calculations that dealt with the effect of the CM momentum on the Mott transition. The previous calculations usually estimated the shift of the Pauli blocking term at a given CM momentum. In the present work we obtain the average shift of the Pauli blocking term at a given temperature T by averaging over all possible values of the CM momentum at that temperature. The average Pauli blocking shift is calculated by multiplying the shift at a given CM momentum by the appropriate Fermi–Dirac or Bose–Einstein distribution function (depending on whether the cluster is a fermion or boson) and then averaging over all CM momenta assuming the cluster belongs to an ideal gas at temperature T in thermal and chemical equilibrium with the nucleons in the surrounding vapour. This averaging at a given temperature is suggested by experimental results which usually refer to a situation where the momenta of the clusters are not fixed but are dependent on the temperature of the surrounding vapour (see for example [12]).

2. The Mott transition for the deuteron cluster

The deuteron is the simplest case to consider as the separation of the internal and CM wavefuctions is straightforward in this two body system. Assuming that the deuteron has a CM momentum $\hbar\vec{K}$ and that it is confined inside a cubic box of volume L^3, where L is much larger than the size of the cluster, then the spatial wavefunction of the deuteron is given by

$$\psi_d\left(\vec{r}_1,\ \vec{r}_2\right) = \frac{1}{L^{3/2}} \mathrm{e}^{\mathrm{i}\vec{K}.\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)} g(1,\ 2), \tag{1}$$

where the internal wavefunction $g(1,\ 2) = g\left(\left|\vec{r}_1 - \vec{r}_2\right|\right)$ is assumed to be a pure s state. The small contribution of the d state, typically less than or about 4%, can be safely neglected. The exact form of $g(1,\ 2)$ depends on the type of potential used to describe the interaction between the proton located at $\vec{r}_1$ and the neutron located at $\vec{r}_2$. In the present work we take a simple square well potential.

Assuming, without loss of generality, that the external nucleon is a neutron with position $\vec{r}_3$ and momentum $\hbar\vec{k}$ then the spatial wavefunction of this 3-nucleon system is given by

$$\Phi(123) = \frac{1}{L^3}\left[g(1,\ 2)\mathrm{e}^{\mathrm{i}\vec{K}.\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}_3} \right]. \tag{2}$$

This wavefunction is however not properly symmetrized and we will need to construct antisymmetric and symmetric spatial combinations:

$$\Psi_{\mathrm{a}}(123) = N\left[\Phi(123) - \Phi(132)\right], \tag{3}$$

$$\Psi_{\mathrm{s}}(123) = N'\left[\Phi(123) + \Phi(132)\right]. \tag{4}$$

The normalization constants N and N' are given by

$$N = \frac{1}{\sqrt{2}\sqrt{1-\left[|\mathrm{J}|^2/L^3\right]}} \quad \text{and} \quad N' = \frac{1}{\sqrt{2}\sqrt{1+\left[|\mathrm{J}|^2/L^3\right]}}, \tag{5}$$

where

$$\mathrm{J} = \int g(r)\mathrm{e}^{\mathrm{i}\left(\frac{\vec{K}}{2}-\vec{k}\right)\cdot\vec{r}}\mathrm{d}^3r. \tag{6}$$

It is found that typically $|\mathrm{J}|^2 \ll L^3$ and we can safely set $N = N' = \frac{1}{\sqrt{2}}$.

The completely antisymmetric total wave function of the system is then given by

$$\Psi_{\rm tot}(123) = \sqrt{\frac{1}{4}}\ \left\{ \Psi_{\rm a}(123) \sum_{m=-1}^{1} \chi_{1\ {\rm m}}(2,\ 3) + \Psi_{\rm s}(123)\chi_{00}(2,\ 3) \right\}, \tag{7}$$

where $\chi_{1{\rm m}}(2,\ 3)$ and $\chi_{00}(2,\ 3)$ are the 3 triplet and 1 singlet spin states for the two neutrons occurring, on average, with equal amplitudes. The various amplitudes can have arbitrary phase factors. However these have no effect on the calculations. The spin state of the proton does not affect the calculation and is not included in (7).

Note that $\Psi_{\rm a}$ and $\Psi_{\rm s}$ are, by their very nature, orthogonal to each other and so are the various spin states. Therefore

$$\left\langle \Psi_{\rm tot}\middle|\mathscr{H}\middle|\Psi_{\rm tot}\right\rangle = \frac{3}{4}\left\langle \Psi_{\rm a}\middle|\mathscr{H}\middle|\Psi_{\rm a}\right\rangle + \frac{1}{4}\left\langle \Psi_{\rm s}\middle|\mathscr{H}\middle|\Psi_{\rm s}\right\rangle, \tag{8}$$

where $\mathscr{H} = \mathscr{H}(123) = \mathscr{H}(132)$ is the system's Hamiltonian which must be symmetric in the two neutrons 2 and 3. As such

$$\begin{aligned}\left\langle \Psi_{\rm a}\middle|\mathscr{H}\middle|\Psi_{\rm a}\right\rangle = N^2\{&\langle\Phi(123)|\mathscr{H}(123)|\Phi(123)\rangle - \langle\Phi(123)|\mathscr{H}(123)|\Phi(132)\rangle\\ &-\langle\Phi(132)|\mathscr{H}(132)|\Phi(123)\rangle + \langle\Phi(132)|\mathscr{H}(132)|\Phi(132)\rangle\}.\end{aligned} \tag{9}$$

Since we are integrating over $\vec{r}_2$ and $\vec{r}_3$ and $\mathscr{H}(123) = \mathscr{H}(132)$, we can exchange the labels of 2 and 3 in the last two terms. It is then easy to see that the second and third terms are equal and that the first and fourth terms are equal.

Therefore

$$\left\langle \Psi_{\rm a}\middle|\mathscr{H}\middle|\Psi_{\rm a}\right\rangle = 2N^2\{\langle\Phi(123)|\mathscr{H}(123)|\Phi(123)\rangle - \langle\Phi(123)|\mathscr{H}(123)|\Phi(132)\rangle\}. \tag{10}$$

The expression for $\left\langle \Psi_{\rm s}|\mathscr{H}\ |\Psi_{\rm s}\right\rangle$ is similar but with the − replaced by a +.

Note that the binding energy B of the deuteron in the presence of the third (vapour) nucleon is defined by

$$-B = \left\langle \Psi_{\rm tot}\middle|\mathscr{H}\middle|\Psi_{\rm tot}\right\rangle - \left\langle \Psi_{\rm tot,un}\middle|\mathscr{H}\middle|\Psi_{\rm tot,un}\right\rangle, \tag{11}$$

where the second term on the right corresponds to the case that the deuteron is unbound inside the vapour.

The Hamiltonian $\mathscr{H}(123)$ can be written as:

$$\mathscr{H}(123) = h_{12} - \frac{\hbar^2}{2M}\nabla^2_{R_{12}} - \frac{\hbar^2}{2m}\nabla^2_{r_3} + V(r_{13}) + V(r_{23}), \tag{12}$$

where h_{12} is the internal Hamiltonian of the deuteron and V is the nucleon–nucleon interaction. We use a simple spin-dependent square well potential with a depth of 35 MeV and range 2.05 fm for the triplet interaction $V_{\rm t}$, and a depth of 16 MeV and range 2.4 fm for the singlet interaction $V_{\rm s}$ [13]. Using equation (12) we get:

$$\begin{aligned}\left\langle \Psi_{\rm a}\middle|\mathscr{H}\middle|\Psi_{\rm a}\right\rangle = -B_0 + \frac{\hbar^2K^2}{2M} + \frac{\hbar^2k^2}{2m} + \frac{2N^2}{L^6}\iiint |g(r_{12})|^2\left[V(r_{13}) + V(r_{23})\right]{\rm d}\Omega\\ -\frac{2N^2}{L^6}\iiint g^*(r_{12}){\rm e}^{{\rm i}\left(\vec{k}-\frac{\vec{K}}{2}\right)\cdot(\vec{r}_2-\vec{r}_3)}\left[V(r_{13}) + V(r_{23})\right]g(r_{13}){\rm d}\Omega,\end{aligned} \tag{13}$$

where B_0 is the binding energy of an isolated deuteron, the second and third terms are the kinetic energies of the cluster and the free neutron, the fourth is the interaction of the deuteron with the free nucleon (the self-energy term) and the last integral is the Pauli blocking term.

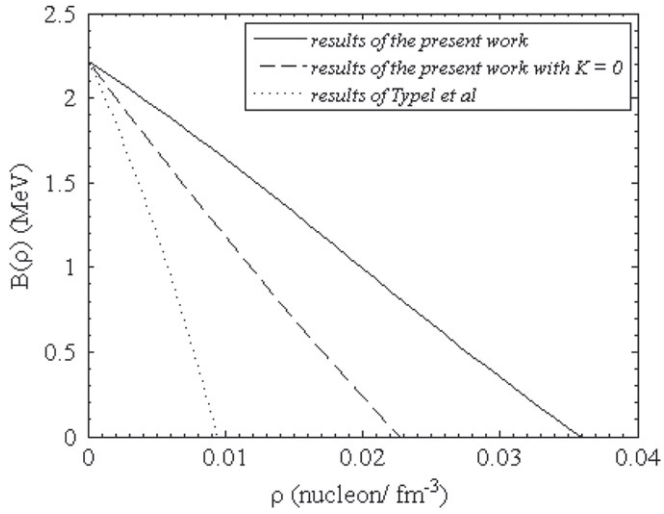

Figure 1. The deuteron binding energy as a function of vapour density at $T = 20$ MeV.

The kinetic energy terms do not contribute to the binding energy. The self-energy term is almost the same whether the constituents of the deuteron are bound or unbound and so is negligibly small when we subtract the two terms in equation (11) from each other. The major contribution to the binding energy of the deuteron then comes from the Pauli blocking term. The second part of the Pauli blocking term that contains the term $V\left(r_{23}\right)$ can be shown to be negligibly small so that we are left with the $V\left(r_{13}\right)$ term. Noting that the bound nucleons interact with the free neutron via the triplet interaction V_{t} (with 75% probability) or the singlet interaction V_{s} (with 25% probability) we can evaluate the $V\left(r_{13}\right)$ term

$$-\frac{2N^2}{L^6}\iiint g^*(r_{12})\mathrm{e}^{\mathrm{i}\left(\vec{k}-\frac{\vec{K}}{2}\right)\cdot(\vec{r}_2-\vec{r}_3)}\left[V(r_{13})\right]g(r_{13})\mathrm{d}\Omega = -\frac{2N^2}{L^3}J^*\left[\frac{3}{4}J_{2\mathrm{t}}+\frac{1}{4}J_{2\mathrm{s}}\right], \tag{14}$$

where J is defined in equation (6) and

$$J_{2\mathrm{t}} = \int g(r_{13})V_{\mathrm{t}}(r_{13})\mathrm{e}^{\mathrm{i}\left(\vec{k}-\frac{\vec{K}}{2}\right)\cdot\vec{r}_{13}}\mathrm{d}^3r_{13}, \tag{15}$$

with a similar expression for $J_{2\mathrm{s}}$ involving the singlet interaction V_{s}. Note that equation (14) gives the contribution to the Pauli blocking term coming from the antisymmetric spatial wavefunction Ψ_{a} and so must be multiplied by a weight factor of ¾ as in equation (8). There is a similar contribution coming from the symmetric spatial wavefunction Ψ_{s} which has the opposite sign and with a weight factor of ¼ as in equation (8).

Adding the contribution of all the nucleons in the vapour, assuming that there are n free nucleons in the volume L^3 with number density $\rho = \frac{n}{L^3}$ we get:

$$\begin{aligned} B(\rho) &= B_0 + \frac{3}{4}\rho J^*\left[\frac{3}{4}J_{2\mathrm{t}}+\frac{1}{4}J_{2\mathrm{s}}\right] - \frac{1}{4}\rho J^*\left[\frac{3}{4}J_{2\mathrm{t}}+\frac{1}{4}J_{2\mathrm{s}}\right] \\ &= B_0 + \frac{3}{8}\rho J^*J_{2\mathrm{t}} + \frac{1}{8}\rho J^*J_{2\mathrm{s}}. \end{aligned} \tag{16}$$

The integrals J, $J_{2\mathrm{t}}$ and $J_{2\mathrm{s}}$ in equation (16) involve the momenta $\vec{K}$ and $\vec{k}$ of the deuteron cluster and the free nucleon as can be seen from equations (6) and (15). After evaluating these

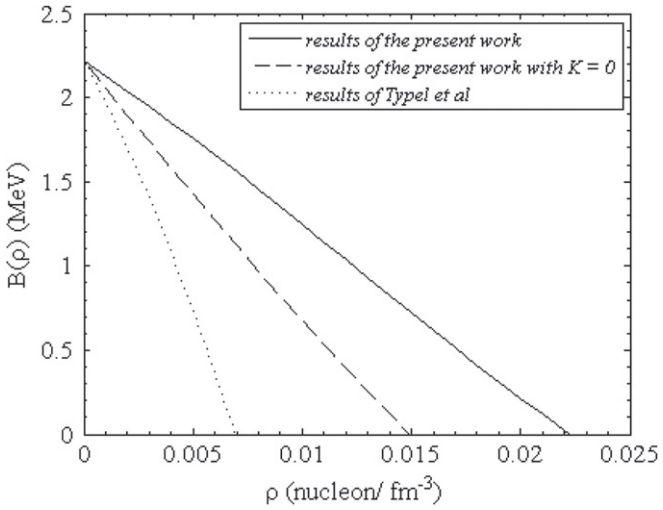

Figure 2. The deuteron binding energy as a function of vapour density at $T = 15$ MeV.

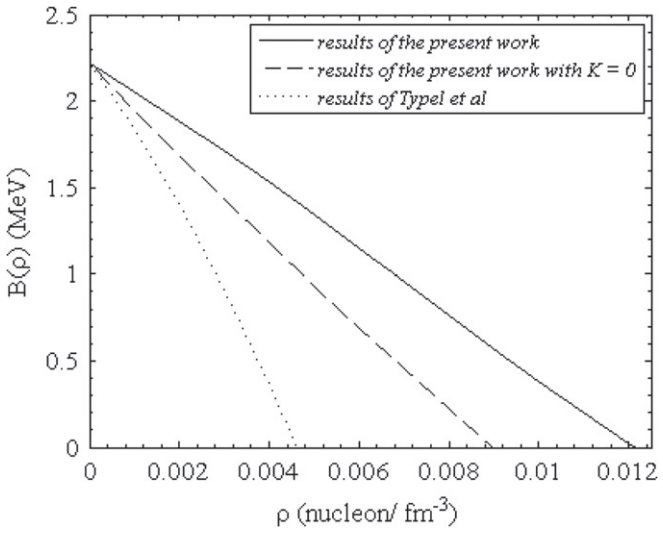

Figure 3. The deuteron binding energy as a function of vapour density at $T = 10$ MeV.

integrals, their values must therefore be averaged over all momentum space assuming the deuteron is part of an ideal Bose gas in thermal and chemical equilibrium with the surrounding ideal Fermi gas of vapour nucleons at a temperature T. Using the nuclear statistical equilibrium (NSE) model [14, 15] and the chemical potentials for ideal Bose and Fermi gases [16] these averages can be carried out in a straight forward manner.

The results for various temperatures T are shown in figures 1–3 where we plot the present results and compare them with the case where the deuteron is assumed to be at rest. The latter case corresponds to assuming that $\vec{K} = 0$ throughout the present calculations so that there is no need to average over the CM momentum of the deuteron. It is seen that neglecting the CM motion of the deuteron cluster can lead to a significant change in its estimated Mott density.

For example, as can be seen from figure 1, the Mott density at $T = 20$ MeV is reduced from about 0.036 nucleons fm^{-3} when the CM motion of the cluster is included properly to 0.023 nucleons fm^{-3} when the cluster is assumed to be at rest. Also included in these figures are the results of the calculations of Typel *et al* [9] which are carried out assuming that the deuteron is at rest. However a direct comparison with these results is not very meaningful because of the different methods used in the calculations including the addition of empirical quadratic terms [9].

3. The Mott transition for the helion cluster

The method used above to calculate the Mott transition in the deuteron can be extended to the case of the helion (^{3}He) cluster. We use the harmonic oscillator shell model to obtain the internal spatial wavefunction of an isolated ^{3}He nucleus moving with momentum $\hbar\vec{K}$ inside a cubic box of volume L^3

$$\psi_{\rm space}(123) = \frac{1}{27^{1/4}L^{3/2}}\left(\frac{\beta}{\sqrt{\pi}}\right)^3 {\rm e}^{-\frac{\beta^2}{2}\left[\left(\vec{r}_1-\vec{R}\right)^2+\left(\vec{r}_2-\vec{R}\right)^2+\left(\vec{r}_3-\vec{R}\right)^2\right]}{\rm e}^{{\rm i}\vec{K}\cdot\vec{R}}\,, \tag{17}$$

where $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ are the position vectors of the two protons and the neutron respectively and $\vec{R} = \sum_{i=1}^{3}\vec{r}_i/3$ is the position of the CM of the system. The choice of the origin for the harmonic oscillator potential at the location of the CM ensures that this wave function does not include any unphysical oscillatory CM motion. The value of $\beta = 0.696$ fm^{-1} is determined by fitting the experimental value of the rms radius of the ^{3}He nucleus which is 1.76 fm [8].

Assuming that the external free nucleon is a neutron with position $\vec{r}_4$ and momentum $\hbar\vec{k}$ and introducing the variables $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\vec{s} = \vec{r}_3 - \vec{r}_1$ then the spatial wavefunction of the 4-nucleon system can be written as

$$\psi_{\rm space}(1234) = \frac{1}{27^{1/4}L^{3/2}}\left(\frac{\beta}{\sqrt{\pi}}\right)^3 {\rm e}^{-\frac{\beta^2}{2}\left[\frac{r^2}{2}+\frac{2}{3}\left(\vec{s}+\frac{\vec{r}}{2}\right)^2\right]}{\rm e}^{{\rm i}\vec{K}\cdot\vec{R}}\;\;\frac{1}{L^{3/2}}{\rm e}^{{\rm i}\vec{k}\cdot\vec{r}_4}, \tag{18}$$

which clearly shows the separation of the CM motion from the internal motion. We can now construct a spatial wavefunction $\Psi_{\rm a}(1234)$ for the 4-nucleon system that is antisymmetric in the two neutrons (nucleons 3 and 4):

$$\Psi_{\rm a}(1234) = \frac{N}{27^{1/4}L^3}\left(\frac{\beta}{\sqrt{\pi}}\right)^3 {\rm e}^{-\frac{\beta^2 r^2}{4}}{\rm e}^{\frac{2}{3}\,{\rm i}\vec{K}\cdot\vec{\rho}}\,[\Phi(34) - \Phi(43)], \tag{19}$$

where $\vec{\rho} = \frac{\vec{r}_1+\vec{r}_2}{2}$, $\Phi(34) = {\rm e}^{-\frac{\beta^2}{3}(\vec{r}_3-\vec{\rho})^2}{\rm e}^{{\rm i}\frac{\vec{K}}{3}\cdot\vec{r}_3}{\rm e}^{{\rm i}\vec{k}\cdot\vec{r}_4}$ and N is a normalization constant. Similarly we can construct a spatial wavefunction $\Psi_{\rm s}(1234)$ that is symmetric in the two neutrons by using $[\Phi(34) + \Phi(43)]$ instead of $[\Phi(34) - \Phi(43)]$ in equation (19) and replacing the normalization constant N by N'. As in the case of the deuteron we can safely set $N \cong N' \cong \frac{1}{\sqrt{2}}$.

A total wavefunction that is antisymmetric in the two neutrons (nucleons 3 and 4) is then given by an expression similar to that in equation (7) for the deuteron case:

$$\Psi_{\rm tot}(1234) = \sqrt{\frac{1}{4}}\left\{\Psi_{\rm a}(1234)\sum_{m=-1}^{1}\chi_{1\,{\rm m}}(3,\;4) + \Psi_{\rm s}(1234)\chi_{00}(3,\;4)\right\}, \tag{20}$$

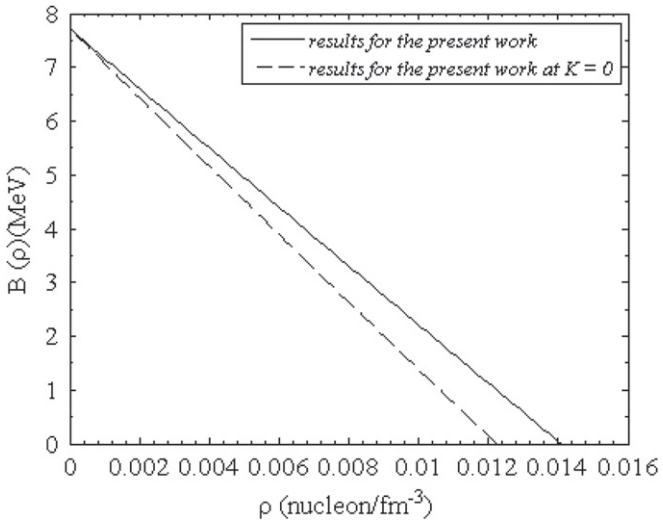

Figure 4. Helion binding energy as a function of vapour density at $T = 20$ MeV.

which takes into consideration the equal average probabilities of the two neutrons being in the singlet or one of the triplet states. The two protons (nucleons 1 and 2) are always in a singlet state and their spin wavefunction is not written explicitly.

For the interaction between the nucleons we will use a simplified form of the Skyrme interaction [17, 18] consisting of an attractive two-body interaction

$$v_{ij} = -t_0\left(1 + x_0 P_\sigma\right)\delta\left(\vec{r}_i - \vec{r}_j\right), \tag{21}$$

and a repulsive three-body interaction

$$v_{ijk} = t_3\delta\left(\vec{r}_i - \vec{r}_j\right)\delta\left(\vec{r}_j - \vec{r}_k\right). \tag{22}$$

The parameters $t_0 = 865$ MeV fm^3 and $t_3 = 25152$ MeV fm^6 of the interaction are obtained by fitting the binding energy (7.718 MeV) of an isolated ^{3}He nucleus. Note that because we are not dealing with an even–even nucleus we cannot replace the three-body interaction by a density-dependent one [18]. The parameter x_0 can be related to the symmetry energy [18]. In the present work we use the value $x_0 = 0.47$ used in previous calculations [16, 19].

The Pauli-blocking term and its effects on the binding energy of a ^{3}He cluster can then be easily evaluated using the same steps mentioned earlier for the deuteron from equation (7) to equation (16). The delta functions associated with the zero range of the Skyrme interaction simplify the calculation. The density-dependence of the binding energy of the cluster due to the surrounding vapour is then given by:

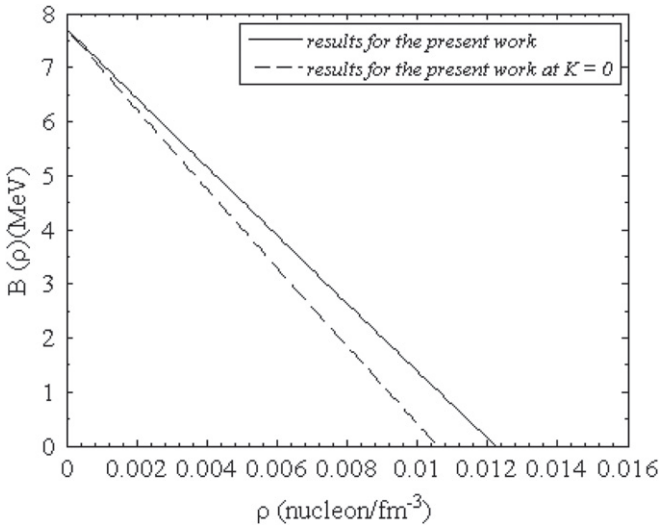

Figure 5. Helion binding energy as a function of vapour density at $T = 15$ MeV.

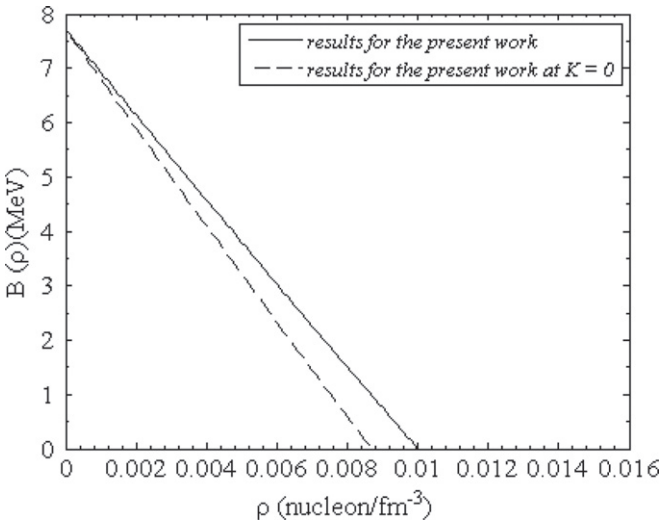

Figure 6. Helion binding energy as a function of vapour density at $T = 10$ MeV.

$$B = B_0 + \rho\left\{ -t_0\left[\frac{1}{2} + x_0 - \left(1 + \frac{x_0}{2}\right)\frac{1}{\sqrt{27}}\left(\frac{6}{\sqrt{7}}\right)^3 \mathrm{e}^{-\frac{}{7\beta^2}\left(\frac{\vec{K}}{3}-\vec{k}\right)^2}\right] + \frac{\beta^3}{\pi^{3/2}}t_3\left[\frac{1}{\sqrt{8}} + \frac{1}{2}\mathrm{e}^{-\frac{3}{4\beta^2}\left(\frac{\vec{K}}{3}-\vec{k}\right)^2}\right]\right\}. \tag{23}$$

As with equation (16) for the deuteron, equation (23) involves the momenta $\vec{K}$ and $\vec{k}$ of the cluster and the free nucleons. It must therefore be averaged over all possible values of

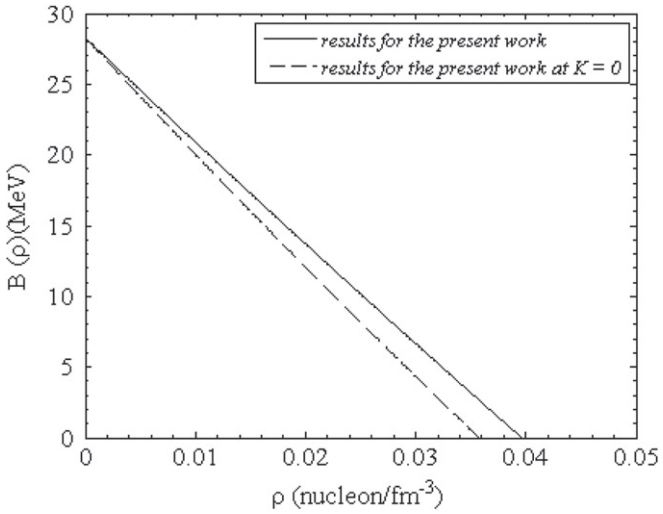

Figure 7. Alpha binding energy as a function of vapour density at $T = 20$ MeV.

these momenta assuming the ^{3}He cluster is part of an ideal Fermi gas in thermal and chemical equilibrium with the surrounding ideal Fermi gas of nucleons at a temperature T. Using the NSE model [14, 15] and the chemical potentials for ideal Fermi gases [16] these averages can be carried out in a straight forward manner. The results for various temperatures are shown in figures 4–6. Also shown in these figures are the results obtained by assuming $\vec{K} = 0$ which corresponds to the case when the cluster is assumed to be at rest.

4. The Mott transition for the alpha cluster

The method used above to calculate the Mott transition for the helion cluster can be extended in a straightforward manner to the case of the alpha (^{4}He) cluster. We extend the harmonic oscillator shell model wavefunction of equation (17) by adding a fourth nucleon (a neutron) and using $\beta = 0.714\ \mathrm{fm}^{-1}$ [20] corresponding to an rms radius of 1.45 fm for an isolated alpha particle. We also use the Skyrme interaction given by equations (21) and (22) but with the parameters [20] $t_0 = 855$ MeV fm^3 and $t_3 = 13250$ MeV fm^6 which reproduce the binding energy (28.30 MeV) of an isolated ^{4}He nucleus.

The final result for the shift in the binding energy is given by:

$$B = B_0 + \rho\left\{-\left(\frac{1}{2} + x_0\right)t_0 - 12\left(1 + \frac{x_0}{2}\right)t_0\left(\sqrt{\frac{2}{5}}\right)^3 \mathrm{e}^{-\frac{4}{5\beta^2}\left(\frac{\vec{K}}{4} - \vec{k}\right)^2} + \frac{3\sqrt{2}}{8}t_3\frac{\beta^3}{\pi^{3/2}} + \frac{96}{\left(\sqrt{17}\right)^3}t_3\frac{\beta^3}{\pi^{3/2}}\mathrm{e}^{-\frac{12}{17\beta^2}\left(\frac{\vec{K}}{4} - \vec{k}\right)^2}\right\}. \tag{24}$$

This result must be averaged over all momenta $\vec{K}$ and $\vec{k}$ of the cluster and the free nucleons assuming the alpha cluster is part of an ideal Bose gas in thermal and chemical equilibrium with the surrounding ideal Fermi gas of vapour nucleons at a temperature T. Using the NSE model [14, 15] and the chemical potentials for ideal Bose and Fermi gases

Table 1. Mott densities ρ_M for the deuteron at different temperatures obtained in the present work and the corresponding values ρ_M* obtained if the cluster is assumed to be at rest.

T (MeV)	ρ_M (nucleons fm^{-3})	ρ_M* (nucleons fm^{-3})
10	0.0121	0.00881
15	0.0222	0.0148
20	0.0359	0.0226

Table 2. Mott densities ρ_M for the helion at different temperatures obtained in the present work and the corresponding values ρ_M* obtained if the cluster is assumed to be at rest.

T (MeV)	ρ_M (nucleons fm^{-3})	ρ_M* (nucleons fm^{-3})
10	0.0099	0.0087
15	0.0122	0.0106
20	0.0141	0.0121

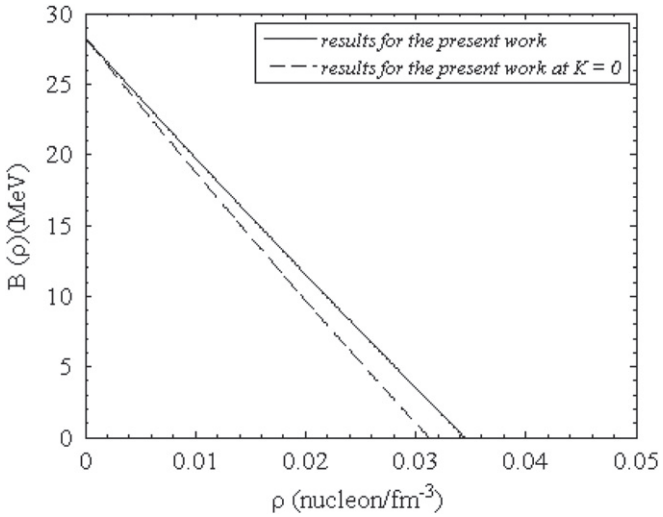

Figure 8. Alpha binding energy as a function of vapour density at $T = 15$ MeV.

[16] these averages can be carried out in a straight forward manner. The results at the two temperatures $T = 20$ MeV and $T = 15$ MeV are shown in figures 7 and 8.

5. Discussion and conclusions

The above treatments of the cases of the deuteron, the helion and the alpha show that the proper inclusion of the CM motion of the clusters has a significant effect on the values of their Mott densities, especially in the case of the deuteron. The treatment of the triton is very similar to that of the helion except for the difference in the binding energy of an isolated triton. The results for the four cases are summarized in tables 1–4.

Table 3. Mott densities ρ_M for the triton at different temperatures obtained in the present work and the corresponding values ρ_M* obtained if the cluster is assumed to be at rest.

T (MeV)	ρ_M (nucleons fm^{-3})	ρ_M* (nucleons fm^{-3})
10	0.0111	0.0097
15	0.0136	0.0118
20	0.0156	0.0135

Table 4. Mott densities ρ_M for the alpha at different temperatures obtained in the present work and the corresponding values ρ_M* obtained if the cluster is assumed to be at rest.

T (MeV)	ρ_M (nucleons fm^{-3})	ρ_M* (nucleons fm^{-3})
15	0.0345	0.0310
20	0.0395	0.036

In conclusion, the correct inclusion of the cluster CM momentum leads to an increase in the estimated value of the Mott density. Depending on the temperature, the increase in the case of the deuteron can be in the range of 40–60% while for the helion and triton clusters the increase is in the range 13–16%. For the alpha the increase is in the range 10–11%. This indicates that, as expected, the effect of the CM motion tends to get smaller as the mass of the cluster increases and therefore its neglect can be justified for heavy clusters. A direct comparison with other theoretical estimates of the Mott density is however not possible because of the neglect in the present work of nonlinear and other effects as mentioned previously. The focus in the present work is not on the value of the Mott density itself but on estimating the shift or inaccuracy in its value introduced by setting the CM momentum $K = 0$. We expect that our conclusions about the effect of the inclusion of the CM motion should hold in general and that the magnitude of the shift due to the CM momentum is not affected by the nonlinear terms which depend on the internal wavefunction only. There has been a first attempt at an experimental determination of the Mott densities in light clusters from the results of heavy ion collisions [12]. The reported values are close to those predicted by Typel *et al* [9]. However the large uncertainties in the extraction of these experimental values from the data make it impossible to determine if there is or there is not an agreement with our conclusion about the effect of CM motion. Hopefully future experiments will throw more light on the situation.

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