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Empirical approach for estimating the pavement transition probabilities used in non-homogenous Markov chains

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An empirical approach is proposed to estimate the transition probabilities associated with non-homogenous Markov chains typically used in developing stochastic-based pavement performance prediction models. A reliable pavement performance prediction model is a key component of any advanced pavement management system. The proposed empirical approach is designed to account for two major factors that cause the transition probabilities (i.e. deterioration rates) to increase over time. The first major factor is the progressive increase in traffic loading as represented by the equivalent single axle load applications. The second major factor is the gradual decline in the pavement structural capacity which can be represented by an appropriate pavement strength indicator such as the structural number. The proposed empirical model can recursively estimate the non-homogenous transition probabilities for an analysis period of (n) transitions by simply multiplying the first-year (i.e. present) transition probabilities by two adjustment factors, namely the load and strength factors. Once the empirical model is calibrated, these two factors can capture the impact of traffic load increases and gradual pavement structural losses on the transition probabilities over time. The calibration process requires the estimation of the model two exponents to be obtained from the minimisation of sum of squared errors wherein the error is defined as the difference between the observed and predicted pavement distress ratings (DRs). The predicted DRs are mainly estimated based on the state probabilities, which are recursively derived from the non-homogenous Markov model. A sample empirical model is presented with results indicating its effectiveness in estimating the pavement non-homogenous transition probabilities.

Keywords: transition probabilities; Markovian processes; pavement performance; pavement management

Introduction

Pavement performance is mainly concerned with the prediction of future pavement conditions as a function of service time or axle load applications. Pavement typically deteriorates over time because of the action of traffic loading, weakening of the pavement structure and weather conditions. A reliable pavement performance prediction model is a key component of any advanced pavement management system designed for developing long-term pavement maintenance and rehabilitation (M&R) schedules. In the past, several pavement management models had incorporated a pavement performance prediction model for the purpose of developing optimum long-term M&R plans at both project and network levels (Abaza *et al.* 2004, Jorge and Ferreira 2012, Khan *et al.* 2014).

Pavement performance prediction models are typically classified as either deterministic or probabilistic models (Wang *et al.* 1994, Li *et al.* 1997, Shohel Reza Amin 2015). The deterministic models generally attempt to predict future pavement conditions with certainty, whereas the probabilistic models recognise the probabilistic performance of pavements, thus assigning different levels of uncertainty to various predicted pavement conditions. Researchers have used both types of prediction model in developing long-term pavement management models. Examples of the deterministic models are those deployed by Abaza (2004) and Jorge and Ferreira (2012), mainly relying on the AASHTO performance prediction model. However, the vast majority of researchers have used the probabilistic approach to model pavement performance, mainly utilising different versions of the Markov model (Butt *et al.* 1987, Li *et al.* 1996, Hong and Wang 2003, Abaza and Murad 2010, Mandiartha *et al.* 2012). Therefore, it seems that there is an overwhelming recognition of the effectiveness of the probabilistic approach in modelling pavement performance.

The discrete-time Markov model has been extensively used by researchers to model pavement performance using both homogenous and non-homogenous chains. A major element of the discrete-time Markov model is the transition probability matrix, which is assumed to remain unchanged for every transition (i.e. time interval) in the case of homogenous chains (Butt *et al.* 1987, Abaza *et al.* 2004, Mandiartha *et al.* 2012). However, a different transition probability matrix can be assigned to each transition when considering non-homogenous chains (Li *et al.* 1996, Hong and Wang 2003, Abaza and Murad 2010). It is unrealistic to assume a steady-state analysis wherein the transition probabilities remain constant over time mainly because of the progressive increase in traffic loading and gradual degradation of the pavement structural capacity. Estimation of a transition probability matrix requires conducting pavement distress assessment for two

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consecutive transitions; therefore, historical records of pavement distress must be available for the entire analysis period if the non-homogenous transition probabilities are to be estimated. A recent publication by Abaza (2014) has proposed a staged-homogenous Markov chain in which a new transition probability matrix can be introduced every five-year period, thus providing an economical alternative over the non-homogenous approach.

The effective application of any Markovian-based model greatly depends on the reliability of the deployed transition probabilities. There are generally two approaches to estimate the pavement transition probabilities. The first approach requires the acquisition of pavement distress records for every transition (i.e. time interval) within the analysis period while the second one is mainly derived from the experience and judgment of a panel of experts. Several researchers had made use of the first approach but deploying different techniques to arrive at the best estimates of the transition probabilities. For example, Butt *et al.* (1987) presented a procedure to estimate the homogenous transition probabilities based on the minimisation of sum of squared errors (SSE) obtained from pavement distress records acquired over (n) transitions. However, application of the same approach to estimating the non-homogenous transition probabilities is a very complex task since the minimisation process has to involve a very large number of variables (i.e. transition probabilities). Mishalani and Madanat (2002) developed a probabilistic model to estimate the time-spent in a given condition state using relevant parameters. They further derived a method to estimate the state transition probabilities from the developed time-spent models as applied to the deterioration of bridge structure. Abaza (2004) proposed a deterministic approach to estimate the homogenous transition probabilities based on the time durations spent in the various deployed condition states as derived from the performance curve generated using the AASHTO performance prediction model. Ortiz-García *et al.* (2006) proposed three different methods to estimate the homogenous transition probabilities essentially based on the minimisation of SSE. The first method requires the availability of original pavement condition data, the second method uses a regression curve generated from the original data and the third one assumes the yearly distributions of pavement condition are available. Finally, Kobayashi *et al.* (2010) deployed the exponential hazard models to describe the Markov transition probabilities between the defined deterioration states using non-uniform intervals between the inspection points in time.

The application of the non-homogenous Markov chains over (n) transitions requires the estimation of (n) distinct sets of transition probabilities. This paper presents a simplified empirical approach that mainly deploys the minimisation of SSE as done by former researchers but has the potential to yield reliable estimates of the non-homogenous transition probabilities with minimal time and effort. The deployed empirical approach recognises that the transition probabilities representing pavement deterioration are expected to increase over time mainly due to the progressive increase in traffic loading and gradual decrease of the pavement structural capacity. The developed empirical model attempts to recursively estimate the non-homogenous transition probabilities by capturing the impact of these two significant factors affecting pavement deterioration. It can yield estimates of the non-homogenous transition probabilities at the project level mainly utilising the annual pavement condition ratings obtained over the analysis period (i.e. pavement performance curve).

Methodology

The discrete-time Markov model has been extensively used in modelling pavement performance. The main elements of the Markov models are the number of pavement condition states, state probabilities denoting pavement proportions in various deployed condition states, transition probabilities indicating pavement deterioration rates, transition length defining the time interval between two successive transitions typically selected to be one or two years, and a number of transitions corresponding to the length of the analysis period. Generally, there are two types of discrete-time Markov models, namely the homogenous and non-homogenous models. An overview of these two models is provided in the following sections.

Homogenous discrete-time Markovian chain

The basic homogenous discrete-time Markov chain is indicated by Equation (1). This Markov model is used to determine the state probabilities after (k) transitions from the product of the initial state probabilities and transition matrix raised to the kth power. The transition matrix remains unchanged over the entire analysis period when considering a homogenous discrete-time Markov chain (Butt *et al.* 1987, Abaza *et al.* 2004, Mandiartha *et al.* 2012). This means the transition probabilities associated with the transition matrix remain unchanged over time which is an unrealistic assumption when modelling pavement performance. The transition probabilities representing pavement deterioration rates are generally expected to increase over time due to the increase in traffic loading and weakening of the pavement structure (Abaza 2014).

$$S^{(k)}_{1xm} = S^{(0)}_{mx1} P^{(k)}_{mxm} \quad (k = 1, 2, \ldots, n), \tag{1}$$

where

$$S^{(k)}_{1xm} = \begin{pmatrix} S^{(k)}_1 \\ S^{(k)}_2 \\ S^{(k)}_3 \\ \vdots \\ S^{k1)}_m \end{pmatrix} \quad S^{(0)}_{mx1} = \left(S^{(0)}_1, S^{(0)}_2, S^{(0)}_3, \ldots, S^{(0)}_m\right)$$

$$\sum_{i=1}^{m} S_i^{(k)} = 1.0$$

$S_{mx1}^{(0)} = (1, 0, 0, \ldots, 0)$ for new pavements, where $S_{1xm}^{(k)} = (1xm)$ column vector representing state probabilities after k transitions, $S_{mx1}^{(0)} = (mx1)$ row vector representing initial state probabilities, $P_{mxm}^{(k)} = (mxm)$ transition matrix raised to the kth power, $m =$ number of deployed pavement condition states and $n =$ number of deployed discrete-time intervals (transitions).

The homogenous transition matrix as presented in Equation (2) was used before to only include the two transition probabilities ($P_{i,i}$) and ($P_{i,i+1}$) in the absence of any M&R works (Butt *et al.* 1987, Abaza *et al.* 2004). The transition probability ($P_{i,i}$) represents the probability of pavements remaining in the same condition state (i) after one time interval, while ($P_{i,i+1}$) denotes the probability of pavements transiting into the next worse state ($i+1$) after one transition. The assumption of using only these two transition probabilities becomes realistic if the number of condition states is adequately small and the time interval between successive transitions is sufficiently large. This means it is very unlikely under these conditions that pavements will transit into a state worse than state ($i+1$) in one transition. A Markov model with 10 condition states and a one-year time interval was reported to be satisfactory in meeting the aforementioned assumption (Butt *et al.* 1987, Abaza and Murad 2010).

$$P_{mxm} = \begin{pmatrix} P_{1,1} & P_{1,2} & 0 & 0 & 0 & \cdots & 0 \\ 0 & P_{2,2} & P_{2,3} & 0 & 0 & \cdots & 0 \\ 0 & 0 & P_{3,3} & P_{3,4} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & P_{m-1,m-1} & P_{m-1,m} \\ 0 & 0 & 0 & 0 & 0 & \cdots & P_{m,m} \end{pmatrix}, \tag{2}$$

where

$$P_{i,i} + P_{i,i+1} = 1.0, \quad P_{m,m} = 1.0$$
$$0 \le P_{i,i} \le 1.0, \quad 0 \le P_{i,i+1} \le 1.0.$$

Non-homogenous discrete-time Markovian chain

The non-homogenous discrete-time Markov chain for modelling pavement performance in the absence of M&R works is indicated by Equation (3) wherein a different transition matrix can be introduced for every transition (time interval). This means the associated transition probabilities can be different for each transition, thus providing a more realistic approach for estimating the long-term pavement performance (Li *et al.* 1996, Hong and Wang 2003, Abaza and Murad 2010). However, the data requirements for using the non-homogenous Markov model are more extensive compared to the homogenous model because estimates of the transition probabilities are required over the entire analysis period comprised of (n) transitions. This means (n) sets of transition probabilities are needed to represent an analysis period of (n) transitions. The estimation of one set of transition probabilities requires two consecutive cycles of pavement distress assessment separated by one time interval. Therefore, it requires extensive historical records of pavement distress collected over (n) transitions to estimate the corresponding non-homogenous transition probabilities. Alternatively, the proposed empirical approach mainly relies on the annual pavement condition rating obtained over the analysis period (i.e. pavement performance curve) to yield estimates of the non-homogenous transition probabilities at the project level. Pavement distress assessment is typically conducted on pavement sections small in length and then an average DR can be obtained for the entire pavement project.

$$S^{(n)} = S^{(0)} \left(\prod_{k=1}^{n} P(k) \right), \tag{3}$$

where

$$P(k) = \begin{pmatrix} P(k)_{1,1} & P(k)_{1,2} & 0 & 0 & 0 & \cdots & 0 \\ 0 & P(k)_{2,2} & P(k)_{2,3} & 0 & 0 & \cdots & 0 \\ 0 & 0 & P(k)_{3,3} & P(k)_{3,4} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & P(k)_{m-1,m-1} & P(k)_{m-1,m} \\ 0 & 0 & 0 & 0 & 0 & \cdots & P(k)_{m,m} \end{pmatrix}$$

$$P(k)_{i,i} + P(k)_{i,i+1} = 1.0, \quad P(k)_{m,m} = 1.0$$
$$0 \le P(k)_{i,i} \le 1.0, \quad 0 \le P(k)_{i,i+1} \le 1.0.$$

Analytical solution of non-homogenous transition probabilities

The non-homogenous transition probabilities, $P(k)_{i,i+1}$, representing pavement deterioration rates, can analytically be derived from the outlined non-homogenous Markov model as a function of the state probabilities associated with two consecutive transitions, (k) and $(k-1)$ (Abaza 2014). The result of this matrix multiplication product is presented in Equation (4). The initial and terminal transition probabilities, $P(k)_{1,2}$ and $P(k)_{m-1,m}$, can directly be estimated as defined in Equations (4a) and (4c), respectively, using only relevant state probabilities. However, the remaining non-homogenous transition probabilities are to be recursively solved as indicated by Equation (4b). Application of Equation (4) mainly requires the estimation of state probabilities (i.e. pavement proportions) associated with the various deployed condition states considering an analysis period of (n) transitions. Equation (4b) can also be used to estimate the terminal transition probability.

$$P(k)_{1,2} = \frac{S_1^{(k-1)} - S_1^{(k)}}{S_1^{(k-1)}}, \quad S_1^{(k-1)} \geq S_1^{(k)} \quad (k = 1, 2, \ldots, n), \tag{4a}$$

$$P(k)_{i,i+1} = \frac{S_i^{(k-1)} - S_i^{(k)} + S_{i-1}^{(k-1)} P(k)_{i-1,i}}{S_i^{(k-1)}}, \quad (i = 2, 3, \ldots, m-1; \quad k = 1, 2, \ldots, n), \tag{4b}$$

$$P(k)_{m-1,m} = \frac{S_m^{(k)} - S_m^{(k-1)}}{S_{m-1}^{(k-1)}}, \quad S_m^{(k)} \geq S_m^{(k-1)} \quad (k = 1, 2, \ldots, n). \tag{4c}$$

The state probabilities associated two consecutive transitions (k and $k-1$) can be estimated from the corresponding pavement distress assessments. The state probabilities at a given transition can easily be estimated from the distress assessment of an adequately large number of pavement sections annually surveyed. However, according to Equation (4), two consecutive cycles of pavement distress assessment are required to estimate the corresponding transition probabilities. Therefore, estimation of the non-homogenous transition probabilities requires conducting distress assessment on an annual basis as the transition length is typically assumed equal to one year.

Pavement condition assessment

Pavement condition has been traditionally assessed based on prevailing pavement distresses which can be classified as load and non-load related (Huang 2004). Pavement distresses (defects) such as those related to cracking, deformation and surface texture can be assessed manually using visual inspection and simple linear measurements. The outcome of the distress assessment is the derivation of a numerical index that can describe the overall pavement condition. An example of that is the pavement condition index developed by ASTM (2007), which ranges from zero for a totally damaged pavement to one hundred for a perfect pavement. Pavement distress data can also be obtained from the analysis of digital images captured using automated systems. For example, the Virginia Department of Transportation annually collects its distress data using a vehicle equipped with continuous digital imaging and sensors for crack detection, and measurement of roughness and rutting (VDOT 2010). Automatic collection of pavement distress data provides a viable alternative; however, questions related to the accuracy and consistency of obtained results still need to be resolved. Therefore, it is recommended that automated distress results be compared against the manual ones, which may lead to applying certain adjustment factors (Underwood *et al.* 2011).

Empirical solution of non-homogenous transition probabilities

An empirical approach is presented for estimating the non-homogenous transition probabilities which takes into consideration the impact of two major factors influencing the transition probabilities, $P(k)_{i,i+1,}$ representing pavement deterioration rates over time. These two factors include the progressive increase of traffic loading and gradual weakening of the pavement structure over the analysis period. The deterioration transition probabilities, $P(k)_{i,i+1,}$ are expected to increase over time due to these two major influencing factors. Therefore, Equation (5) is proposed to adjust the deterioration transition probabilities, $P(k)_{i,i+1}$, associated with the kth transition to yield the transition probabilities, $P(k+1)_{i,i+1}$, associated with the subsequent transition by introducing two multiplication factors, namely load factor (F_L) and pavement strength factor (F_S).

$$P(k+1)_{i,i+1} = P(k)_{i,i+1} \times F_L(k) \times F_S(k) \leq 1.0 \quad (k = 1, 2, 3, \ldots, n), \tag{5a}$$

$$P(k+1)_{i,i} = 1 - P(k+1)_{i,i+1} \quad (i = 1, 2, \ldots, m-1). \tag{5b}$$

The load factor (F_L) is expected to account for the effect of traffic load increases over the analysis period, which is defined in Equation (6) as a function of the expected 18 k equivalent single load axle (ESAL) applications. The load factor as presented in Equation (6) is calculated as a ratio of the incremental ESAL applications associated with two consecutive transitions, (k) & ($k+1$), raised to power (A). It is expected that ESAL increases over time due to traffic growth will cause this ratio to be greater than 1, resulting in higher transition probabilities.

$$F_{\mathrm{L}}(k) = \left(\frac{\mathrm{ESAL}(k+1)}{\mathrm{ESAL}(k)}\right)^{A} \quad (k = 1, 2, 3, \ldots, n). \quad (6)$$

Similarly, the strength factor (F_{S}) is introduced to account for the weakening of pavement structure over time as indicated by Equation (7). The structural capacity of a pavement structure, $S(k)$, which naturally decreases over time in the absence of M&R works can be used to calculate the strength factor. Therefore, the pavement strength factor is defined in Equation (7) as a ratio of the pavement structural capacities associated with two consecutive transitions, (k) & ($k+1$), raised to power (B). Again, this ratio is greater than one which results in higher transition probabilities over time. For example, the structural number (SN) used by the former AASHTO design guides can be used to represent the structural capacity of pavements (AASHTO 1993).

$$F_{\mathrm{S}}(k) = \left(\frac{S(k)}{S(k+1)}\right)^{B} \quad (k = 1, 2, 3, \ldots, n). \quad (7)$$

The overall empirical model for estimating the non-homogenous transition probabilities, accounting for both the progressively increasing traffic loading and gradually degrading pavement structural capacity, is presented in Equation (8). The model requires the estimation of incremental ESAL and incremental structural capacity, ESAL(k) and $S(k)$, respectively, and the model exponents (A & B). The incremental ESAL can be estimated from traffic data as explained next, the incremental pavement structural capacity can either be estimated from periodical testing of pavement as outlined later or based on experience and engineering judgment, and the model exponents are to be estimated from the calibration process as described in a subsequent section. Equation (8) essentially applies the same adjustment factors to all transition probabilities, an assumption made to simplify model calibration. Alternatively, the model exponents (A & B) can be replaced by exponents (A_i & B_i) which could result in estimating two distinct exponents for each deployed pavement condition state.

$$P(k+1)_{i,i+1} = P(k)_{i,i+1} \times \left(\frac{\mathrm{ESAL}(k+1)}{\mathrm{ESAL}(k)}\right)^{A} \left(\frac{S(k)}{S(k+1)}\right)^{B} \quad (8)$$

$$(k = 1, 2, 3, \ldots, n) \quad (i = 1, 2, \ldots, m-1).$$

Estimation of incremental ESAL

The incremental 18 k, EASL(k), applications can be estimated from the traffic data including design ESAL (ESAL$_D$), analysis period comprised of (n) transitions with each transition being equivalent to one year and annual traffic growth rate (r). The design ESAL is generally estimated based on axle load distribution, load equivalency factors, design daily truck number and traffic growth factor GF(n). Equation (9) outlines the procedure for estimating the incremental ESAL using mainly the first-year ESAL, ESAL(1), and other outlined parameters. Alternatively, the first-year incremental ESAL can directly be computed from relevant traffic data.

$$\mathrm{ESAL}(k) = \mathrm{ESAL}(1) \times (1+r)^{k-1} \quad (k = 1, 2, 3, \ldots, n), \quad (9a)$$

$$\mathrm{ESAL}(k+1) = \mathrm{ESAL}(1) \times (1+r)^{k} \quad (k = 1, 2, 3, \ldots, n), \quad (9b)$$

where

$$\mathrm{ESAL}(1) = \frac{\mathrm{ESAL}_D}{\mathrm{GF}(n)}$$

$$\mathrm{GF}(n) = \frac{(1+r)^{(n)} - 1}{r}$$

$$\mathrm{ESAL}_D = \sum_{k=1}^{n} \mathrm{ESAL}(k).$$

It can be noted that the incremental load factor, $F\mathrm{L}(k)$, as obtained from dividing Equation (9b) by Equation (9a) is essentially constant with a value equal to (1 + r). This means that only the annual traffic growth rate (r), in decimal form, is required to compute the load factor in addition to the model exponent (A) to be estimated from the calibration process. This is true, provided that the annual traffic growth rate is uniform over the analysis period.

Estimation of incremental structural capacity

The strength factor (F_{S}) is to be estimated using the incremental pavement structural capacity, $S(k)$, as defined in Equation (7). The structural capacity of a pavement structure is expected to decrease over time mainly due to strength degradation affecting the asphalt concrete layer. The underlying pavement layers will typically experience negligible strength losses. For example, the structural capacity of the asphalt concrete layer can be represented using the incremental SN, $\mathrm{SN}_1(k)$, used in former AASHTO design guides as defined in Equation (10), wherein $a_1(k)$ and D_1 are the incremental strength coefficient and thickness in inches, respectively, associated with the asphalt concrete layer (AASHTO 1993).

$$\mathrm{SN}_1(k) = a_1(k)D_1, \tag{10a}$$

$$\mathrm{SN}_1(k+1) = a_1(k+1)D_1. \tag{10b}$$

The incremental strength factor, $F_S(k)$, determined according to Equation (7) is presented in Equation (11). The use of Equation (11) requires the estimation of the incremental strength coefficient, $a_1(k)$, typically correlated to compressive strength parameters such as elastic modulus or Marshall stability (AASHTO 1993). Therefore, it requires periodical testing of the asphalt concrete layer which can be performed using destructive or non-destructive testing procedures (Huang 2004). Alternatively, the incremental structural capacity of the asphalt concrete layer can be estimated based on experience and engineering judgment.

$$F_S(k) = \left(\frac{a_1(k)}{a_1(k+1)}\right)^B \quad (k = 1, 2, 3, \ldots, n). \tag{11}$$

Calibration of empirical model

The empirical model for estimating the non-homogenous transition probabilities as presented in Equation (8) has to be calibrated for the purpose of identifying the appropriate values of the model exponents (A & B). This can be accomplished by minimising the SSE defined in Equation (12) based on the difference between the observed distress ratings, $\mathrm{DR_O}(k)$, and the predicted ones, $\mathrm{DR_P}(k)$, summed over an analysis period comprised of (n) transitions. Therefore, historical records of pavement distress are required for this calibration process. In particular, it is required that an average distress rating, $\mathrm{DR_O}(k)$, be annually assigned for each project; hence, the model exponents can be developed at the project level. Alternatively, they can be generated for various pavement systems based on average DRs representing a given pavement system.

$$\text{Minimise} \quad \mathrm{SSE} = \sum_{k=1}^{n} [\mathrm{DR_O}(k) - \mathrm{DR_P}(k)]^2. \tag{12}$$

The predicted distress ratings, $\mathrm{DR_P}(k)$, can be estimated from the state probabilities, $S_i^{(k)}$, as indicated by Equation (13). The predicted DR for a particular transition is essentially estimated as the mean of a compound uniform probability density function (Abaza and Murad 2010). Each uniform probability function is defined by its own state probability, $S_i^{(k)}$, which is applicable over a range of DR defined with lower and upper limits (LDR_i & UDR_i) as presented in Equation (13). Therefore, the predicted DR is determined as the sum product of the state probabilities and the corresponding state mean DRs (B_i) defined as the average of state DR range. The state probabilities are to be computed using the non-homogenous Markov model outlined in Equation (3) as a function of the non-homogenous transition probabilities estimated using the empirical model indicated by Equation (8).

$$\mathrm{DR_P}(k) = \sum_{i=1}^{m} B_i S_i^{(k)}, \quad (k = 0, 1, 2, \ldots, n), \tag{13}$$

where

$$S^{(k)} = \begin{cases} S_1^{(k)}, & \mathrm{LDR}_1 < \mathrm{DR} \le \mathrm{UDR}_1 \\ S_2^{(k)}, & \mathrm{LDR}_2 < \mathrm{DR} \le \mathrm{UDR}_2 \\ \vdots & \vdots \\ S_i^{(k)}, & \mathrm{LDR}_i < \mathrm{DR} \le \mathrm{UDR}_i \\ \vdots & \vdots \\ S_m^{(k)}, & \mathrm{LDR}_m \le \mathrm{DR} \le \mathrm{UDR}_m \end{cases}$$

$$B_i = \frac{\mathrm{LDR}_i + \mathrm{UDR}_i}{2}.$$

The minimisation of SSE can be performed using a trial and error approach which is initiated by selecting initial values of the model exponents (A & B). They can be assigned initial values of 1, which allows for the calculation of the predicted DRs over the analysis period, hence resulting in an initial value of SSE. The trial and error approach can then proceed by incrementally changing the values of the model exponents (A & B) with tenth of a point increase/decrease is considered adequate. The new SSE is then compared against the old one with the approach terminated when no further reduction in the SSE value can be achieved. It is expected that the values of (A & B) are in the range of 1.0–1.5 for pavement performance with increasingly higher deterioration rates, whereas the range for decreasingly lower deterioration rates is 0.5–1.0 as demonstrated in the sample presentation. The software package 'Excel' has been used to formulate and solve the optimisation problem presented in Equation (12). Excel provides several optimisation algorithms that can effectively solve this type of problems.

Sample presentation

The empirical approach presented in this paper for estimating the non-homogenous transition probabilities is demonstrated using relevant data applicable to a four-lane major arterial located in the city of Nablus, West Bank, Palestine. This arterial was reconstructed in 1997 from international aid provided to assist the Palestinian Authority in rebuilding the road network under its jurisdiction. The corresponding pavement structure was constructed of 13 cm (5 inches) high-stability hot-mix asphalt surface ($a_1 = 0.44$) and 50 cm (20-inches) crushed limestone aggregate base ($a_2 = 0.14$) resulting in a design SN of 5. The design ESAL was computed to be approximately 5 million using 20,000 vpd average daily traffic, 6% trucks, 0.85 average truck factor (i.e. ESAL equivalent per truck), 20-year design period and 4% uniform annual traffic growth rate. Following reconstruction, distress data had been annually collected for the period 1998–2014 and used to estimate an average DR that represents the overall pavement condition for this arterial.

While collecting pavement distress data, it was observed that certain pavement segments of this major arterial had experienced rapid initial deterioration rates since they were constructed on relatively flat terrains with poor drainage. The remaining segments were associated with slow initial deterioration rates as they were built on relatively high grounds with good drainage. The observed annual pavement distress ratings, $DR_O(k)$, over the analysis period (17 transitions) are provided in Table 1 considering both slow and rapid initial deterioration rates. The pavement distress assessment was annually conducted using pavement sections of 50 m lane length while the corresponding DRs are averaged to yield the observed DRs at the project level as provided in Table 1. It can be noted that the pavement performance associated with slow initial deterioration rates (i.e. $P_{1,2} < P_{9,10}$) essentially represents increasingly higher deterioration rates while performance with rapid initial deterioration rates (i.e. $P_{1,2} > P_{9,10}$) denotes decreasingly lower deterioration rates as depicted in Figures 1 and 2, respectively.

Table 1. Sample observed and predicted distress ratings (DR_O & DR_P) for an analysis period of 17 transitions.

Slow initial deterioration rates[@]			Rapid initial deterioration rates[#]		
Transition no. (k)	$DR_O(k)$	$DR_P(k)$	Transition no. (k)	$DR_O(k)$	$DR_P(k)$
0	95.0	95.00	0	95.0	95.00
1	92.7	93.18	1	90.1	88.50
2	90.5	91.16	2	82.8	82.16
3	87.9	88.97	3	74.5	76.04
4	85.5	86.58	4	69.3	70.14
5	83.1	83.98	5	64.9	64.47
6	80.2	81.15	6	59.8	59.02
7	77.6	78.05	7	55.2	53.80
8	73.4	74.66	8	50.6	48.81
9	70.7	70.93	9	43.7	44.05
10	66.1	66.84	10	39.8	39.52
11	60.8	62.33	11	35.5	35.22
12	56.3	57.36	12	32.4	31.16
13	51.5	51.93	13	28.1	27.35
14	46.1	46.05	14	24.2	23.82
15	40.2	39.83	15	19.9	20.59
16	33.8	33.44	16	15.7	17.69
17	28.0	27.18	17	13.3	15.15

[@] The associated SSE = 11.62.
[#] The associated SSE = 22.37.

The pavement deterioration associated with the sample pavement structure is represented by a non-homogenous Markov chain similar to the one presented in Equation (3) using 10 pavement condition states ($m = 10$) with a transition length being equal to a one-year time interval. Estimation of state probabilities over (n) transitions requires recursively solving Equation (8) for the non-homogenous transition probabilities. Estimation of the non-homogenous transition probabilities, as defined in Equation (8), essentially requires four main input parameters, namely first-year (i.e. present) transition probabilities ($k = 1$), incremental ESAL, EASL(k), incremental structural capacity, $S(k)$, and specified values for the model exponents (A & B). The present transition probabilities ($P_{i,i+1}$) for the sample pavement structure considering both types of pavement deterioration are provided in Table 2. It must be pointed out that only the initial ($P_{1,2}$) and terminal ($P_{9,10}$) transition probabilities have been estimated using Equations (4a) and (4c), while the remaining transition probabilities are computed from the initial and terminal transition probabilities using linear interpolation (Abaza and Murad 2010). The incremental ESAL ratio, as defined in Equation (6), simply has a constant value of $(1 + r)$. The incremental structural capacity ratio, as indicated by Equation (7), is estimated based on the assumption that the design SN decreases from an initial value of 5 to 3.3 after 17 years with an annual uniform decrease of 0.1. This implies that the hot-mix asphalt surface would have lost most of its structural capacity at the end of 17 years of service life, while the underlying layers essentially retained their full structural capacity.

Therefore, the previously outlined procedure has been deployed for different values of the model exponents (A & B) in the search for the minimum SSE. The SSE as defined in Equation (12) requires, in addition to the observed annual DRs, the predicted annual DRs for an analysis period of (n) transitions. The predicted annual DRs for a given transition are calculated using Equation (13) based on the state probabilities, $S_i^{(k)}$, determined from Equation (3) and state mean DRs (B_i) defined equal to (95, 85, 75, ..., 5) for pavement condition states (1, 2, 3, ..., 10), respectively. The predicted annual DRs that resulted in the

Table 2. Sample first-year transition probabilities for both types of pavement deterioration.

First-year transition probabilities ($k = 1$) $P_{i,i+1}$ (slow initial deterioration rates[@])				First-year transition probabilities ($k = 1$) $P_{i,i+1}$ (rapid initial deterioration rates[#])			
$P_{1,2}$	0.183	$P_{6,7}$	0.308	$P_{1,2}$	0.650	$P_{6,7}$	0.356
$P_{2,3}$	0.207	$P_{7,8}$	0.334	$P_{2,3}$	0.591	$P_{7,8}$	0.298
$P_{3,4}$	0.232	$P_{8,9}$	0.359	$P_{3,4}$	0.532	$P_{8,9}$	0.239
$P_{4,5}$	0.258	$P_{9,10}$	0.384	$P_{4,5}$	0.474	$P_{9,10}$	0.179
$P_{5,6}$	0.283			$P_{5,6}$	0.415		

[@] Slow initial deterioration rates (i.e. $P_{1,2} < P_{2,3} < \ldots < P_{9,10}$).
[#] Rapid initial deterioration rates (i.e. $P_{1,2} > P_{2,3} > \ldots > P_{9,10}$).

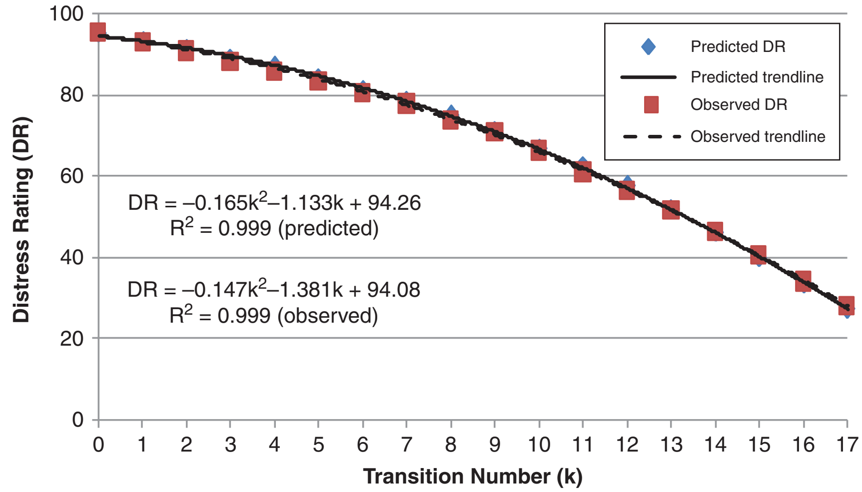

Figure 1. Sample predicted and observed pavement DRs for increasingly higher deterioration rates (i.e. $P_{1,2} < P_{2,3} < \ldots < P_{9,10}$).

minimum SSE have been obtained using the values of 1.4 and 1.2 for the model exponents (A & B) in the case of increasingly higher deterioration rates and 0.7 and 0.4 in the case of decreasingly lower deterioration rates, respectively. The two sets of predicted distress ratings, $\mathrm{DR}_P^{(k)}$, are also provided in Table 1 and plotted along with the corresponding observed DRs in Figures 1 and 2. It can be noted from Figures 1 and 2 that there is a good agreement between the predicted and observed DR values for both cases of pavement deterioration. The best-fit curves (models) for both predicted and observed DR values have been generated and provided in Figures 1 and 2 with both associated with near one R^2 values.

Figures 3 and 4 show the variation of the initial and terminal transition probabilities over the analysis period for both cases of pavement deterioration. The remaining transition probabilities for a given transition are estimated using linear interpolation as mentioned earlier. This represents a simplified approach for estimating the remaining transition probabilities from only two transition probabilities, namely the initial and terminal ones (Abaza and Murad 2010). However, if all transition probabilities are available for the first year, then Equation (8) can be applied to each transition probability yielding results similar to those depicted in Figures 3 and 4. Table 3 provides the values of the nine transition probabilities as obtained from Equation (8) for the last year ($k = 17$) considering both types of pavement deterioration. It can be noted that in relation to the first-year transition probabilities, the transition probabilities associated with the 17th transition have increased on average by a multiple of about 2.5 and 1.55 for slow and rapid initial deterioration rates, respectively.

Conclusions

The paper presented an effective empirical model for estimating the non-homogenous transition probabilities

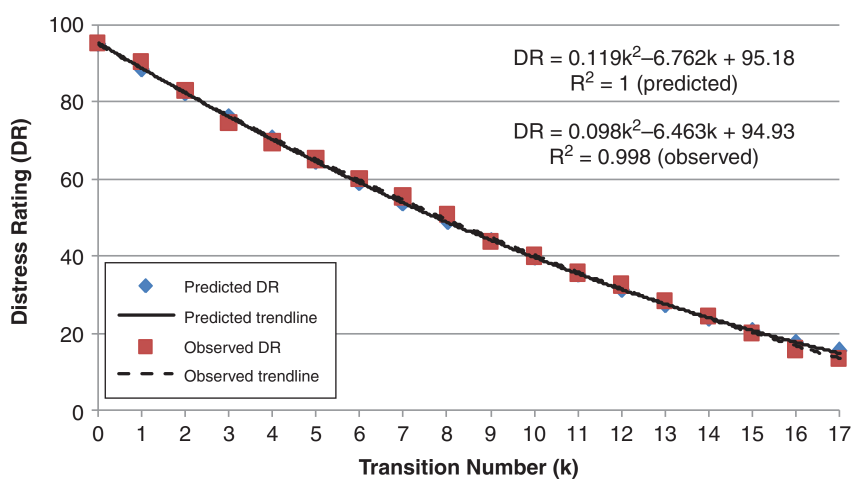

Figure 2. Sample predicted and observed pavement DRs for decreasingly lower deterioration rates (i.e. $P_{1,2} > P_{2,3} > \ldots > P_{9,10}$).

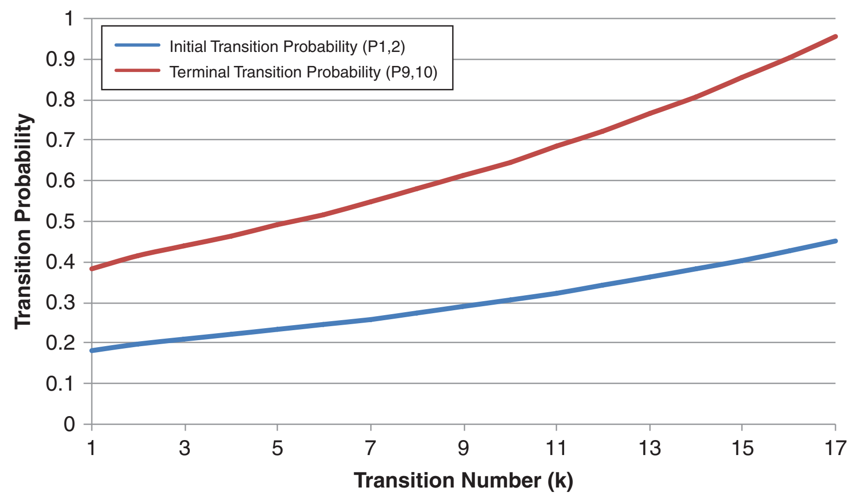

Figure 3. Sample initial and terminal transition probabilities for increasingly higher deterioration rates ($P_{1,2} < P_{9,10}$).

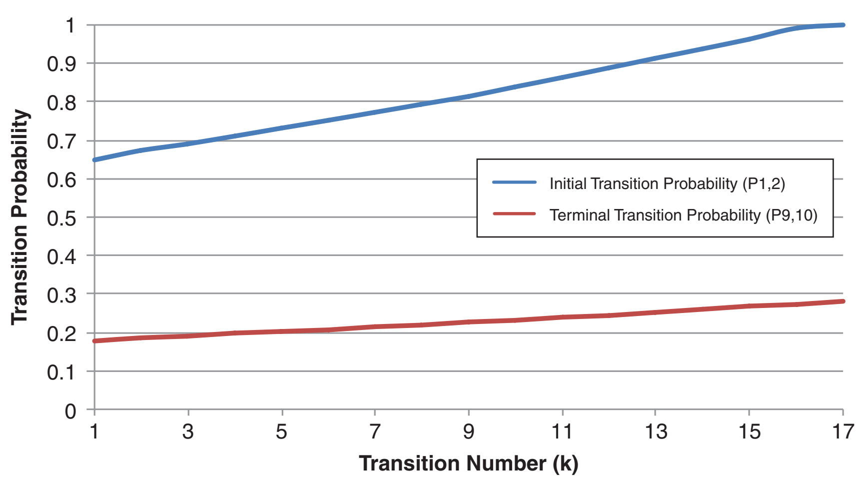

Figure 4. Sample initial and terminal transition probabilities for decreasingly lower deterioration rates ($P_{1,2} > P_{9,10}$).

used in the Markov model to predict future pavement conditions. The empirical approach accounts for the two main factors affecting pavement deterioration over time, namely the progressive increase in traffic loads and gradual degradation of the pavement structural capacity. The non-homogenous transition probabilities can be estimated at the project level provided that pavement distress assessment is annually collected over an analysis period of (n) transitions. An average DR at the project level is only required to use the empirical model presented in this paper. Distress assessment is typically conducted on pavement sections small in length wherein a DR value is assigned to each section while all DR values are then averaged out to yield an annual average DR value needed to use the outlined empirical model. The incremental ESAL and incremental structural capacity are also required to calibrate the model, which can be estimated from the initial pavement design parameters as indicated in the sample presentation. Therefore, the presented empirical model provides a simplified but yet efficient approach

Table 3. Sample last-year transition probabilities for both types of pavement deterioration.

Last-year transition probabilities ($k = 17$) $P_{i,i+1}$ (slow initial deterioration rates[@])				Last-year transition probabilities ($k = 17$) $P_{i,i+1}$ (rapid initial deterioration rates[#])			
$P_{1,2}$	0.454	$P_{6,7}$	0.768	$P_{1,2}$	1.000	$P_{6,7}$	0.552
$P_{2,3}$	0.517	$P_{7,8}$	0.831	$P_{2,3}$	0.910	$P_{7,8}$	0.462
$P_{3,4}$	0.570	$P_{8,9}$	0.894	$P_{3,4}$	0.821	$P_{8,9}$	0.372
$P_{4,5}$	0.642	$P_{9,10}$	0.957	$P_{4,5}$	0.731	$P_{9,10}$	0.283
$P_{5,6}$	0.705			$P_{5,6}$	0.641		

[@] Slow initial deterioration rates (i.e. $P_{1,2} < P_{2,3} < \ldots < P_{9,10}$).
[#] Rapid initial deterioration rates (i.e. $P_{1,2} > P_{2,3} > \ldots > P_{9,10}$).

to estimate the non-homogenous transition probabilities at the project level which can lead to significant improvements in all relevant pavement management applications.

The calibration of the empirical model requires estimating the model exponents (A & B), which can effectively be done by minimising the SSE. In the sample presentation, this has been accomplished using a trial and error approach which has converged to reliable estimates of the model exponents (A & B). Generally, there are two different value sets for the model exponents depending on the type of pavement performance. There are two types of pavement performance that are typically recognised when considering pavement deterioration, namely performance with slow initial deterioration rates which is superior to the second type of performance associated with rapid initial deterioration rates. The sample results have indicated that the values of the model exponents (A & B) are in the ranges of about 1.0–1.5 and 0.5–1.0 for the superior and inferior pavement performances, respectively. However, it is recommended that highway agencies, interested in using the outlined empirical model, develop their own estimates of the model exponents, thus yielding more reliable estimates of the non-homogenous transition probabilities that can better represent the pavement local conditions.

Disclosure statement

No potential conflict of interest was reported by the author.

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