Stochastic Approach for Design of Flexible Pavement

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1 author:

Khaled Abaza
Birzeit University

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A Case Study for Low Volume Roads

Khaled A. Abaza

Department of Civil Engineering, Birzeit University
P.O. Box 14, Birzeit
West Bank, Palestine
kabaza@birzeit.edu

ABSTRACT. *A stochastic approach has been developed to estimate the required design thickness for flexible pavement using typical design factors and new additional stochastic-based factors. The stochastic design approach incorporates new design factors that can better control the long-term performance of flexible pavement. The new stochastic design factors are mainly the initial and terminal transition probabilities. These two transition probabilities, representing initial and terminal deterioration rates, have been shown to be essential in controlling the long-term performance of flexible pavement. The long-term performance of pavement has been traditionally defined using a pavement performance curve. Pavement performance curves can be stochastically developed using mainly the initial and terminal transition probabilities. The discrete-time Markov model typically applies the transition probabilities (transition matrix) along with the initial state probabilities to predict the future pavement distress ratings over an analysis period. The predicted pavement distress ratings are used to construct the corresponding performance curve. The transition probabilities can be estimated from a "back-calculation" of the Markov model using state probabilities obtained from two consecutive surveys of pavement distress. Sample empirical models have been developed for the case of low volume roads to predict the initial and terminal transition probabilities from relevant pavement design factors and to predict the required pavement design thickness based on the initial and terminal transition probabilities. In addition, two long-term performance indicators, namely the area under the performance curve and the corresponding average distress rating, have been used in developing sample empirical design models considering the case of low volume roads.*

KEYWORDS: *Flexible Pavement, Pavement Design, Stochastic-Based Performance Prediction, Performance-Based Pavement Design, Empirical Pavement Design.*

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1. Introduction

There are essentially three design approaches for flexible pavement: empirical, mechanistic, and mechanistic-empirical (Huang, 2004). The vast majority of design methods currently used by designers are either empirical or mechanistic-empirical. Recent research efforts have been focused towards the mechanistic-empirical approach which came about in response to the new proposed Mechanistic-Empirical Pavement Design Guide (MEPDG, 2004). The new MEPDG represents a major advancement in pavement design; however, it is substantially more complex than the 1993 AASHTO empirical design guide. The new MEPDG has been under intensive testing and evaluation since it became available in 2004 (Bari and Witczak, 2007; Wang *et al.*, 2008; Banerjee *et al.*, 2009). It requires significantly more input data that may not be readily available to the designer. It also requires the calibration of several MEPDG predictive models for performance, traffic loading, and material characteristics. Therefore, the author believes that the empirical approach will continue to play a significant role in the design of flexible pavement especially in developing countries which generally lack the resources and technical expertise to use a mechanistic-empirical approach such as the new MEPDG.

There are typically three major design factors used by most design procedures incorporating the empirical approach (AASHTO, 1993; Huang, 2004; Wright and Dixon, 2004). They include traffic loading as represented by the design ESAL (80kN Equivalent Single Axle Load repetitions), material characteristics, and climate or environment. A fourth design factor defining the pavement performance over the expected service life has been introduced by the 1993 AASHTO design guide through the introduction of the serviceability concept (AASHTO, 1993). The initial and terminal present serviceability indices are used to provide a partial control-mechanism for the trend of the pavement performance curve. It provides a partial control because it only controls the beginning and ending points of the performance curve but it does not control the long-term shape of the performance curve. The long-term performance of pavement has been directly related to the area falling under the performance curve (Abaza and Abu-Eisheh, 2003; Huang, 2004; Abaza, 2005). The larger the area, the better is the pavement performance over the same service life.

There are two types of models that have been widely used in predicting the long-term performance of pavements: probabilistic and deterministic models. The probabilistic model predicts the future pavement conditions with some degree of uncertainty whereas the deterministic model predicts them with certainty (Robinson *et al.*, 1998). The predicted pavement conditions can then be used to construct the corresponding performance curve. The probabilistic model that was extensively used by several researchers to predict pavement performance is the discrete-time Markov model (Way *et al.*, 1982; Butt *et al.*, 1987; Li *et al.*, 1996; Abaza and Murad, 2007; Abaza and Murad, 2009). The Markov model can be used with homogenous or non-homogenous transition chains. The homogenous chains require the same transition matrix whereas the non-homogenous chains can deploy a different transition matrix for each chain.

Predicted pavement performance curves were used in yielding optimum pavement design and estimating overlay design thickness for rehabilitation of flexible pavement (Abaza and Abu-Eisheh, 2003; Abaza, 2005; Abaza and Murad, 2009). The area falling under the performance curve has long been recognized as a direct measure of the pavement relative structural capacity (Abaza and Abu-Eisheh, 2003; Huang, 2004; Abaza, 2005; Abaza and Murad, 2009). The pavement performance curves can be generated using the discrete-time Markov model. Recent research has shown that the trend of the performance curve is directly related the values of the initial and terminal transition probabilities (Abaza and Murad, 2009). Therefore, the initial and terminal transition probabilities will be used as new additional design factors to better control the long-term performance of pavement, thus, forming the basis for the proposed stochastic design approach for flexible pavement. The initial and terminal transition probabilities can be estimated for individual projects from historical records of pavement distress using a "back-calculation" of the Markov model. These two controlling transition probabilities along with other relevant design factors will be deployed to develop sample empirical design models using regression techniques with nonlinear transformations.

2. Stochastic model for predicting pavement performance

The stochastic model that was extensively used in predicting future pavement conditions is the discrete-time Markov model (Way *et al.*, 1982; Butt *et al.*, 1987; Li *et al.*, 1996; Abaza and Murad, 2007; Abaza and Murad, 2009). The basic Markov model for discrete-time homogenous chains is presented in Equation [1]. The model predicts the state probabilities ($S^{(k)}$) after a period comprised of k discrete-time intervals (transitions) from multiplying the initial state probabilities ($S^{(0)}$) by the transition matrix ($P^{(k)}$) multiplied k times. The transition matrix used in Equation [1] contains the transition probabilities which remain unchanged if homogenous chains are assumed. The initial state probabilities ($S^{(0)}$) for new pavements can be assumed to have values as indicated by Equation [1]. This condition can be met if a reasonably large number of pavement states is used. A transition matrix with 10 states is generally adequate to ensure that this condition is satisfied. Otherwise, an initial pavement distress assessment is required to estimate the initial state probabilities for new pavements.

$$S^{(k)} = S^{(0)} P^{(k)} \qquad (k = 1, 2, \ldots, n) \qquad [1]$$

where: $S^{(k)} = (S_1^{(k)}, S_2^{(k)}, S_3^{(k)}, \ldots, S_m^{(k)})$

$$\sum_{i=1}^{m} S_i^{(k)} = 1.0$$

$$S^{(0)} = (S_1^{(0)}, S_2^{(0)}, S_3^{(0)}, ..., S_m^{(0)})$$

$$S^{(0)} = (1, 0, 0, ..., 0) \text{ for new pavements}$$

$S^{(k)}$ = column vector representing state probabilities after k transitions,

$S^{(0)}$ = row vector representing initial state probabilities,

$P^{(k)}$ = transition matrix raised to the k^th power,

m = number of deployed pavement condition states, and

n = number of deployed discrete-time intervals (transitions).

The homogenous transition matrix used in estimating the future state probabilities in the absence of any maintenance and rehabilitation (M&R) works is defined by Equation [2]. The transition matrix is a square matrix with size (m) representing the number of deployed pavement condition states. Each row of the transition matrix is typically assumed to only include the two transition probabilities $(P_{i,i})$ and $(P_{i,i+1})$ (Way *et al.*, 1982; Butt *et al.*, 1987; Abaza and Murad, 2007; Abaza and Murad, 2009). The transition probabilities along the matrix main diagonal $(P_{i,i})$ represent the probabilities that pavements presently in condition state (**i**) will remain in the same condition state after the elapse of one transition. The transition probabilities $(P_{i,i+1})$ represent pavement deterioration rates from a present condition state (**i**) to a worse state (**i+1**) after one transition. All matrix entries below the main diagonal represent pavement improvement rates which are assigned zero values in the absence of M&R works. The main objective in defining the transition matrix as presented in Equation [2] is to predict the future pavement conditions of new pavements.

$$P = \begin{pmatrix} P_{1,1} & P_{1,2} & 0 & 0 & 0 & \cdots & 0 \\ 0 & P_{2,2} & P_{2,3} & 0 & 0 & \cdots & 0 \\ 0 & 0 & P_{3,3} & P_{3,4} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \ldots & P_{m-1,m-1} & P_{m-1,m} \\ 0 & 0 & 0 & 0 & 0 & \ldots & P_{m,m} \end{pmatrix} \qquad [2]$$

where: $P_{i,i} + P_{i,i+1} = 1.0$ and $P_{m,m} = 1.0$

The future pavement distress rating $(DR^{(k)})$ for a particular pavement project can be estimated as defined in Equation [3]. The future state probabilities $(S_i^{(k)})$ as determined from the Markov model are used as the main parameters for predicting the future pavement distress ratings. The pavement distress rating after k transitions $(DR^{(k)})$ is estimated as the mean of a compound uniform probability density function defined using the future state probabilities $(S_i^{(k)})$. The i[th] future state probability represents a uniform probability density function with its ordinate being represented by $(S_i^{(k)})$ and its random variable range defined using the lower and upper distress ratings (LDR_i) and (UDR_i), respectively. The state mean distress rating (B_i) is defined as the average of the lower and upper distress ratings used to define the pavement condition state (**i**) according to a deployed pavement distress indicator.

$$DR^{(k)} = \sum_{i=1}^{m} B_i S_i^{(k)} \qquad (k = 0, 1, 2, \ldots, n) \qquad [3]$$

where:

$$B_i = \frac{LDR_i + UDR_i}{2}$$

$$B_m \le DR^{(k)} \le B_1$$

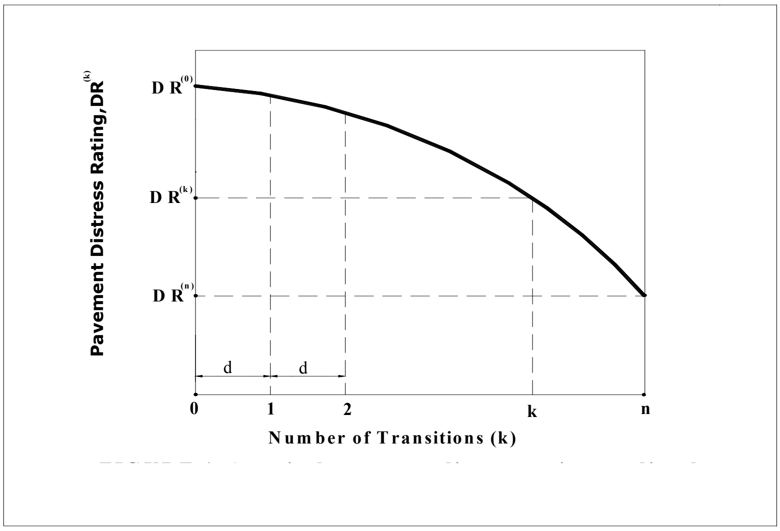

Figure 1. *A typical pavement distress rating predicted using stochastic modeling*

The predicted distress ratings $(DR^{(k)})$ can be used to construct a distinct performance curve for a particular pavement project as shown in Figure 1. The predicted distress ratings are plotted against the corresponding number of transitions (k) as shown or using the equivalent service time (t) in years obtained from multiplying the number of transitions (k) by the time interval length (d) in years. The length of time interval (transition) is typically taken to be equal to one or two years.

3. Simplified approach for predicting pavement performance

Abaza and Murad (2009) developed a simplified approach for estimating the transition matrix using only three transition probabilities. They include initial, middle, and terminal transition probabilities $(P_{1,2})$, $(P_{x,x+1})$, and $(P_{m-1,m})$, respectively. Therefore, three transition probabilities are to be estimated from pavement distress assessment conducted on pavements in condition states (**1**), (**x**), and (**m-1**). The condition state (**x**) is selected to be the middle state between states **(1)** and (**m-1)**. Estimates of these three transition probabilities (deterioration rates) are required to approximate the remaining transition probabilities $(P_{i,i+1})$ under the assumption of either uniformly increasing or uniformly decreasing deterioration rates. Typically, there are three distinct performance prediction models that can be derived from the transition matrix depending on the relationship among the deployed transition probabilities as described below (Abaza and Murad, 2009).

3.1. *Increasing rate of deterioration model*

The first performance prediction model is represented by a polynomial with second degree as indicated by Equation [4]. The model for pavement distress rating $(DR(t))$ at a service time (t) is obtained from fitting a second degree polynomial to the predicted performance curve shown in Figure 1. The second derivative for this model is negative indicating that the corresponding performance curve is concave upward. The model coefficients a, b, and c are constants obtained from curve fitting the predicted distress ratings $(DR^{(k)})$ using either best fitting or regression techniques.

$$DR(t) = -at^2 - bt + c \qquad [4]$$

The performance prediction model indicated by Equation [4] is associated with progressively increasing deterioration rates. This deterioration trend takes place when the initial transition probability $(P_{1,2})$ is smaller than the middle transition probability $(P_{x,x+1})$ which is in turn smaller than the terminal transition probability $(P_{m-1,m})$. It is not too unrealistic to assume that the deterioration rates associated with this model increase uniformly especially that the corresponding performance curve is parabolic in nature with consistently increasing slopes. Therefore, the transition probabilities $(P_{i,i+1})$ are determined using Equation [5a] for condition states falling between (**2**) and (**x-1**) based on the assumption that deterioration rates uniformly increase from an initial value of $(P_{1,2})$ to a terminal value of $(P_{x,x+1})$. Similarly, the transition probabilities associated with condition states falling between (**x+1**) and (**m-2**) are determined using Equation [5b] assuming that deterioration rates uniformly increase from an initial value of $(P_{x,x+1})$ to a terminal value of $(P_{m-1,m})$.

$$P_{i,i+1} = P_{1,2} + (i-1)\left(\frac{P_{x,x+1} - P_{1,2}}{x-1}\right) \qquad (i = 2, 3, \ldots.., x - 1) \qquad [5a]$$

$$P_{i,i+1} = P_{x,x+1} + (i-x)\left(\frac{P_{m-1,m} - P_{x,x+1}}{m-x-1}\right) \qquad (i = x+1, x+2, \ldots.., m - 2) \qquad [5b]$$

where: $P_{1,2} < P_{2,3} < \ldots.. < P_{x,x+1} < P_{x+1,x+2} < \ldots.. < P_{m-1,m}$

3.2. *Uniform rate of deterioration model*

The second type of deterioration models is associated with a uniform rate of deterioration as presented in Equation [6]. The corresponding prediction model takes on a linear form with coefficients b and c are determined from the predicted distress ratings $(DR^{(k)})$. The transition probabilities associated with this model are essentially the same. Therefore, the initial and terminal transition probabilities are sufficient to construct the corresponding project transition matrix.

$$DR(t) = -bt + c \qquad [6]$$

where: $P_{1,2} = P_{2,3} = P_{3,4} = \ldots.. = P_{m-1,m}$

3.3. *Decreasing rate of deterioration model*

The third type prediction model is associated with progressively decreasing deterioration rates. The corresponding prediction model is a second degree polynomial as defined by Equation [7]. The second derivative for this model is positive indicating that the corresponding parabolic performance curve is concave downward with a consistently decreasing slope. Similarly, the model coefficients a, b, and c are to be estimated from the predicted distress ratings $(DR^{(k)})$ using best fitting or regression techniques.

Equation [7] may theoretically result in an increasing distress rating when (t) is greater than (b/2a) but that is not possible as applied in this paper. Equation [7] will be derived based on distress ratings predicted using the stochastic model (*i.e.*, Equation [3]) wherein the predicted distress rating for a particular transition will be less than the value associated with the preceding transition as will be demonstrated by the three sample models provided in Figure 2. In this regard, Equation [7] will only be valid for a service life (t) that is less than or equal to the number of deployed transitions (n).

$$DR(t) = at^2 - bt + c \quad [7]$$

The prediction model indicated by Equation [7] requires progressively decreasing deterioration rates. This performance trend typically occurs when the initial transition probability $(P_{1,2})$ is larger than the middle transition probability $(P_{x,x+1})$ which is in turn larger than the terminal transition probability $(P_{m-1,m})$. Equation [8a] generates the transition probabilities $(P_{i,i+1})$ for condition states falling between (**2**) and (**x-1**) based on the assumption that deterioration rates uniformly decrease from an initial value of $(P_{1,2})$ to a terminal value of $(P_{x,x+1})$. Similarly, Equation [8b] yields the transition probabilities $(P_{i,i+1})$ for condition states falling between (**x+1**) and (**m-2**) assuming that the deterioration rates uniformly decrease from an initial value of $(P_{x,x+1})$ to a terminal value of $(P_{m-1,m})$.

$$P_{i,i+1} = P_{1,2} - (i-1)\left(\frac{P_{1,2} - P_{x,x+1}}{x-1}\right) \qquad (i = 2, 3, ..., x-1) \quad [8a]$$

$$P_{i,i+1} = P_{x,x+1} - (i-x)\left(\frac{P_{x,x+1} - P_{m-1,m}}{m-x-1}\right) \qquad (i = x+1, x+2, ..., m-2) \quad [8b]$$

where: $P_{1,2} > P_{2,3} > ..... > P_{x,x+1} > P_{x+1,x+2} > ..... > P_{m-1,m}$

The transition probabilities $(P_{i,i+1})$ can also be approximated using only the initial and terminal transition probabilities. In this case, the middle transition probability $(P_{x,x+1})$ is estimated as the average of the initial and terminal transition probabilities. Then, Equations [5] and [8] are used as outlined to yield the remaining transition probabilities.

Figure 2 shows sample pavement performance curves (models) derived from only the initial and terminal transition probabilities using 10 condition states. The state mean distress ratings (B_i) are assumed to be 95, 85,…., 5 for condition states **1**, **2**, …., **10**, respectively. The distress ratings, DR(t) = DR(k), for each model are predicted using Equation [3] for 15 transitions (t = k = 0, 1, 2, …., 15) which results in 15 data points used in the construction of each performance curve shown in

Figure 2. Equations [4], [6] and [7] can easily be fitted to the corresponding curves shown in Figure 2 using best fitting or regression techniques. The performance curve for the concave-up model is derived using 0.2 and 0.9 initial and terminal transition probabilities, respectively. The straight-line performance curve is associated with 0.4 uniform transition probabilities. The performance curve for the concave-down model is generated using 0.8 and 0.1 initial and terminal transition probabilities, respectively. The deployed length of time interval (d) is one year which represents the transition length.

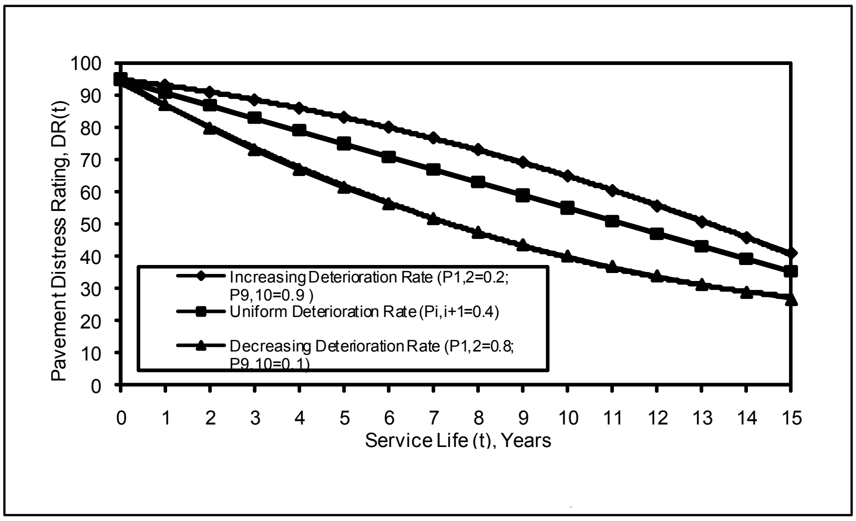

Figure 2. *Sample pavement deterioration models predicted using stochastic modeling*

4. Pavement performance assessment

The procedures typically used to assess pavement performance are either based on visual inspection of pavement defects or measurement of roadway roughness. Inspection of pavement defects typically requires selecting pavement sections that are small in length ranging from 10 to 50 m lane length. Each section is individually inspected and assigned a distress rating determined according to a specified formula. For example, Equation [9] estimates the Distress Rating (DR) for each pavement section based on the defect rating (d_i) and the corresponding assigned weight (w_i) to yield a section rating on a scale of 100 points (Garber and Hoel, 1996).

$$DR = 100 - \sum_i w_i d_i \qquad [9]$$

Equation [10] was developed by Abaza and Murad (2009) to estimate the distress rating for a pavement section based merely on cracked and deformed areas. Equation [10] estimates the distress rating (DR) using the section surface area (A), cracked areas (A_c), deformed areas (A_d), cracking severity factor (F_c), and deformation severity factor (F_d). The severity factors are assumed 1, 2 and 3 for low, medium and high severity ratings, respectively. Equation [10] yields the section distress rating using a scale of 100 points. A defected pavement area can only be counted as cracked or deformed.

$$DR = \left(\frac{3A - \sum F_c A_c - \sum F_d A_d}{3A} \right) \times 100 \qquad [10]$$

where: $\sum A_c + \sum A_d \leq A$

There are other procedures to assess pavement performance which use partially or totally the International Roughness Index (IRI). Several mathematical models have been developed to estimate the present serviceability index (PSI) from the roadway longitudinal roughness measurements (Paterson, 1986; Al-Omari and Darter, 1994). The present serviceability Index (PSI) is a well-known pavement condition indicator. It was used by AASHTO in all its empirical pavement design guides. For example, Equations [11] and [12] were developed by Paterson (1986), and Al-Omari and Darter (1994), respectively, to estimate the PSI from the IRI for asphalt pavement.

$$PSI = 5e^{(-0.18 IRI)} \qquad [11]$$

$$PSI = 5e^{(-0.24 IRI)} \qquad [12]$$

Equations [11] and [12] both have the same exponential form but the value of one of the two coefficients differs slightly.

5. Estimation of transition probabilities

The transition probabilities associated with the transition matrix presented in Equation [2] can be estimated using the pavement distress assessment outcomes. Two consecutive cycles of pavement distress assessment are required as a minimum to estimate the transition probabilities. The two cycles must be separated by a time interval that is equal to the length of one transition. The typical transition length is one or two years. The pavement distress assessment is used to estimate the state probabilities associated with each cycle. The state probabilities represent the proportions of pavement that exist in each condition state at the end of a specified time interval (transition). The pavement sections are assigned to the various condition states according to the section condition rating estimated as outlined in the previous section. The state probabilities $(S_i^{(k)})$ at the k[th] transition are estimated using Equation [13] based on the ratio of the number of pavement sections $(N_i^{(k)})$ belonging to state (**i**) to the total number of sections existing in all deployed condition states for a given pavement project.

$$S_i^{(k)} = \frac{N_i^{(k)}}{\sum_{i=1}^{m} N_i^{(k)}} \qquad [13]$$

The transition probabilities can now be estimated from a "back-calculation" of the Markov model presented in Equation [1] using the state probabilities derived from two consecutive cycles of pavement distress assessment. The initial transition probability $(P_{1,2})$ is obtained from multiplying the first-cycle state probability row vector $(S_i^{(1)})$ by the first column in the transition matrix. The result of this multiplication product is equal to the state probability $(S_1^{(2)})$ associated with state (**1**) at the end of the 2[nd] transition (cycle). The initial transition probability is then derived as defined in Equation [14a]. The state probabilities required for using Equation [14a] can be replaced by their corresponding values as defined in Equation [13] to yield Equation [14b].

$$P_{1,2} = \frac{S_1^{(1)} - S_1^{(2)}}{S_1^{(1)}} \qquad [14a]$$

$$P_{1,2} = \frac{N_1^{(1)} - N_1^{(2)}}{N_1^{(1)}} \qquad [14b]$$

Similarly, the terminal transition probability $(P_{m-1,m})$ is obtained from multiplying the first-cycle state probability row vector by the last column in the transition matrix. The result of this multiplication is equal to the second-cycle state probability $(S_m^{(2)})$ associated with state (**m**). The resulting terminal transition probability is as indicated by Equation [15a]. Similarly, the state probabilities used in Equation [15a] can be replaced by their corresponding values as obtained from Equation [13] to yield Equation [15b]. The other remaining transition probabilities can be derived in a similar way using a recursive approach. However, the initial and terminal transition probabilities are adequate to be used in estimating the other remaining transition probabilities as defined in Equations [5] and [8]. This implies less effort involved in conducting pavement distress assessment as Equations [14b] and [15b] only require the numbers of pavement sections $(N_i^{(k)})$ belonging to three condition states, namely, states **1**, **m-1,** and **m**. These numbers need to be obtained from conducting two consecutive cycles of pavement distress assessment.

$$P_{m-1,m} = \frac{S_m^{(2)} - S_m^{(1)}}{S_{m-1}^{(1)}} \qquad [15a]$$

$$P_{m-1,m} = \frac{N_m^{(2)} - N_m^{(1)}}{N_{m-1}^{(1)}} \qquad [15b]$$

Mishalani and Madanat (2002) have developed a probabilistic-based model to estimate the transition probabilities based on the time spent (duration) in a given condition state. Other researchers have developed methods which minimize the sum of residuals (errors) defined as the difference between the observed distress ratings and their corresponding predicted values obtained from the Markov model (Butt *et al.*, 1987; Shahin, 1994; Ortiz-Garcia *et al.*, 2006). These methods are expected to yield reliable estimates of the transition probabilities provided that sufficient historical records of pavement distress are available.

6. Stochastic approach for design of flexible pavement: a case study of low volume roads

The empirical stochastic-based approach for the design of flexible pavement is presented using four major design factors which include the design ESAL, material characteristics, roadway drainage condition, and the initial and terminal transition probabilities. The use of the initial and terminal transition probabilities (deterioration rates) to better control the long-term pavement performance represents the main contribution of this paper to the field of pavement design. It is well documented in the literature that pavement performance is probabilistic rather than deterministic (Way *et al.*, 1982; Butt *et al.*, 1987; Li *et al.*, 1996; Abaza and Murad, 2007; Abaza and Murad, 2009). This provides the justification for using the empirical stochastic-based approach in the design of flexible pavement. In addition, sample empirical design models will be presented using two long-term performance indicators derived from the initial and terminal transition probabilities. These two long-term performance indicators include the area falling under the performance curve and the corresponding average distress rating.

The empirical stochastic-based approach will be presented using a sample of 12 village access roads selected from the northern districts of West Bank, Palestine. These roads were constructed under the administration of the Palestinian Authority during the period of 1997-1998 with international donor funding provided to assist the Palestinian people rebuilding their infrastructure system. The 12 roads to be investigated have been randomly selected from a larger sample of over fifty roads that were built during the same period. These roads are mainly two-lane low-volume rural roads with length ranging from 3 to 7 km and are used by local residents to reach the nearest main highways.

6.1. *Sample pavement performance prediction models*

A 10x10 transition matrix similar to the one outlined in Equation [2] has been used to represent the deterioration mechanism of each investigated roadway. The corresponding 10 condition states (m=10) are defined using equal 10 points distress rating (DR) range on a scale of 100 points with higher ratings indicating better pavements. Therefore, the DR ranges for states 1, 2,…., 10 are 100-90, 90-80,…, 10-0, respectively. Pavement sections of 10 m lane length and 3 m width were used in the distress assessment resulting in 30 m^2 section surface area (A). The distress rating (DR) assigned to each pavement section has been estimated using Equation [10]. Sample distress rating calculations are provided in Table 1 for a selected number of pavement sections. The i^{th} defected area (A_i) within a particular pavement section is estimated as a rectangular area with length (L_i) and width (W_i). A defected area can either be counted as cracked (C) or deformed (D) area with the corresponding severity factor (F_i) is assigned 1, 2 or 3 for low, medium or high severity rating, respectively.

Each surveyed pavement section is assigned to a pavement condition state based on its distress rating. The number of pavement sections $(N_i^{(k)})$ assigned to each condition state is then determined considering two consecutive cycles of pavement distress assessment. The two cycles were separated by one-year time interval representing the length of the deployed transition. Table 2 provides the numbers of pavement sections needed to estimate the sample initial $(P_{1,2})$ and terminal $(P_{9,10})$ transition probabilities as defined in Equations [14b] and [15b], respectively.

The estimated initial $(P_{1,2})$ and terminal $(P_{9,10})$ transition probabilities are provided in Table 2 for the 12 roadways under investigation. Examination of the tabulated transition probabilities reveals that the increasing deterioration rate model as represented by Equation [5] is valid when the initial probability $(P_{1,2})$ is smaller than the terminal probability $(P_{9,10})$. Similarly, the decreasing deterioration rate model as defined by Equation [8] is applicable when the initial probability $(P_{1,2})$ is larger than the terminal probability $(P_{9,10})$. Then, the performance curves (models) are developed using the predicted distress ratings $(DR^{(k)})$ determined for each roadway as indicated by Equation [3]. A terminal service life (T) of 20 years has been used which is equivalent to 20 transitions (n) as one discrete-time interval (d) is equal to 1 year.

Table 1. *Sample pavement distress rating calculations*

Station (m)		Defect type (i)	Defected area details					Distress Rating (DR)
From	To		L_i (m)	W_i (m)	A_i (m^2)	F_i	$\sum_i A_i F_i$	
90	100	C	9.45	3.00	28.35	3	85.05	5.50
130	140	C C D	1.40 1.05 0.25	2.55 1.65 2.80	3.57 1.73 0.70	2 2 3	12.70	85.89
230	240	C C	8.00 1.65	3.00 0.55	24.00 0.91	3 2	73.82	17.98
450	460	C D D C	5.30 2.25 0.90 1.55	3.00 2.40 1.25 0.85	15.90 5.40 1.12 1.32	3 3 2 3	70.10	22.11
690	700	C D	2.85 1.30	1.15 1.00	3.28 1.30	1 2	5.88	93.47
970	980	C C C D D	5.15 1.20 1.35 0.85 1.50	2.60 2.85 3.00 0.75 1.10	13.39 3.42 4.05 0.64 1.65	3 2 3 1 3	64.75	28.06

The predicted pavement distress ratings have been used to generate two long-term performance indicators. The first long-term performance indicator is the area falling under the performance curve (A_{UPC}) corresponding to 20 years service life. The larger the area under the performance curve, the better is the long-term performance of pavement (Abaza and Abu-Eisheh, 2003; Huang, 2004; Abaza, 2005). The area under the performance curve is determined from the predicted distress ratings ($DR^{(k)}$) using the Trapezoidal rule as indicated by Equation [16]. The second long-term performance indicator is the project average distress rating ($\overline{DR}$) estimated over the same deployed service life. Similarly, the higher the average distress rating, the better is the long-term performance of pavement. The project average distress rating is also determined from the predicted distress ratings as presented in Equation [17]. Table 3 provides the values of these two long-term performance indicators for the roadway sample under investigation.

Table 2. *Estimation of sample initial and terminal transition probabilities*

Road No.	$N_1^{(1)}$	$N_1^{(2)}$	$N_9^{(1)}$	$N_{10}^{(1)}$	$N_{10}^{(2)}$	$P_{1,2}$	$P_{9,10}$
1	50	41	32	10	22	0.18	0.38
2	48	36	28	17	27	0.25	0.36
3	43	32	45	20	51	0.26	0.69
4	55	39	39	26	46	0.29	0.51
5	37	25	53	33	72	0.32	0.74
6	40	26	71	16	38	0.35	0.31
7	34	21	45	27	49	0.38	0.49
8	41	25	68	29	80	0.39	0.75
9	59	31	33	41	69	0.47	0.85
10	54	26	51	18	43	0.52	0.49
11	45	19	46	22	46	0.58	0.52
12	66	24	32	37	53	0.64	0.50

$$A_{UPC} = \frac{d}{2}\left(DR^{(0)} + DR^{(n)} + 2\sum_{j=1}^{n-1} DR^{(j)} \right) \qquad [16]$$

$$\overline{DR} = \frac{\sum_{j=0}^{n} DR^{(j)}}{n+1} \qquad [17]$$

In addition, Table 3 provides the initial Distress Rating (DR_o) and terminal Distress Rating (DR_t). It can be concluded from Table 3 that the two outlined long-term performance indicators, namely (A_{UPC}) and $(\overline{DR})$, are highly compatible whereas the initial and terminal distress ratings are totally incompatible. It can also be concluded that a good long-term pavement performance is generally associated with low initial and terminal transition probabilities. A good long-term pavement performance is also associated with high (A_{UPC}) and $(\overline{DR})$ values. Therefore, these two long-term performance indicators will be used in developing sample empirical stochastic-based models for the design of flexible pavement.

Table 3. *Sample long-term pavement performance indicators*

Road No.	$P_{1,2}$	$P_{9,10}$	A_{UPC}	$\overline{DR}$	DR_o	DR_t
1	0.18	0.38	1473.4	73.6	95	48.9
2	0.25	0.36	1358.1	67.9	95	38.8
3	0.26	0.69	1191.8	59.5	95	20.2
4	0.29	0.51	1224.2	61.2	95	25.0
5	0.32	0.74	1066.7	53.4	95	12.6
6	0.35	0.31	1223.3	61.2	95	29.4
7	0.38	0.49	1083.4	54.2	95	16.2
8	0.39	0.75	956.6	48.0	95	8.3
9	0.47	0.85	828.7	41.9	95	5.6
10	0.52	0.49	919.6	46.3	95	9.1
11	0.58	0.52	855.4	43.2	95	7.1
12	0.64	0.50	812.0	41.1	95	6.5

6.2. *Sample empirical stochastic-based flexible pavement design models*

Table 4 provides the original design data for the roadway sample under investigation. The pavement structures associated with these roads were originally designed as two-layer flexible pavements using the Caltrans design method (Caltrans, 2008). The original design data includes the asphalt layer thickness (h_a), granular-base layer thickness (h_b), subgrade resistance value (R), and 80kN (18K) equivalent single axle load applications (W_{80}). Table 4 also provides a proposed design factor related to the roadway drainage condition. The indicated qualitative drainage condition rating is assigned according to the condition of the existing roadway drainage system, quality of roadway transverse and longitudinal profiles, and type of surrounding terrain (level, rolling, or mountainous).

It can be noticed from Table 4 that the R-value is highly correlated to the drainage condition rating. This is because the quality of the subgrade can be considered highly correlated to the surrounding terrain type wherein good quality subgrade is generally encountered underneath roads built in a mountainous topography and poor quality subgrade is usually prevailing under roads built in a level terrain. In addition, the developing countries don't typically spend much money on providing a good drainage system especially for low volume roads and rely mostly on the natural topography to help draining the roadway system. In this sense, the subgrade quality will also be highly correlated to the quality of the existing drainage system which is more efficient in a mountainous topography compared to a level one.

Table 4. *Sample original flexible pavement design parameters*

Road No.	$P_{1,2}$	$P_{9,10}$	h_a (cm)	h_b (cm)	h_p (cm)	R-value	W_{80} ($x10^3$)	Drainage Condition
1	0.18	0.38	8	25	33	46	470	Good
2	0.25	0.36	8	25	33	41	320	Good
3	0.26	0.69	8	30	38	30	350	Fair
4	0.29	0.51	7	30	37	27	180	Fair
5	0.32	0.74	7	35	42	24	250	Fair
6	0.35	0.31	8	30	38	40	520	Good
7	0.38	0.49	8	35	43	28	650	Fair
8	0.39	0.75	8	40	48	21	700	Fair
9	0.47	0.85	9	45	54	10	620	Poor
10	0.52	0.49	8	35	43	26	580	Fair
11	0.58	0.52	9	40	49	16	660	Poor
12	0.64	0.50	9	45	54	13	750	Poor

The data provided in Tables 3 and 4 has been used in developing sample empirical stochastic-based models. The sample empirical models have been developed using regression techniques with nonlinear transformations. The first set of empirical predictive models is developed to estimate the initial and terminal transition probabilities from related design variables. Equation [18] predicts the initial transition probability ($P_{1,2}$) using the four design variables presented in Table 4, namely, the total pavement thickness (h_p), subgrade resistance value (R), design ESAL (W_{80}), and drainage condition rating (D). The drainage condition rating (D) is assigned 1, 2 and 3 for good, fair and poor condition ratings, respectively. The generated predictive model is significant at 99.82% confidence level as the corresponding F-statistic is equal to 13.77. The variable coefficients are also significant at more than 94.86% as the t-statistics for the three coefficients are 2.25, 2.78 and 3.31. The model has 75.37% determination coefficient (R square) and 0.077 standard error of estimate.

$$P_{1,2} = 0.424 + 0.017\ Log W_{80}^{3} - 0.048\ Log\left(\frac{h_p R}{D}\right)^2 \qquad [18]$$

Equation [19] predicts the terminal transition probability ($P_{9,10}$) using three design variables, namely, the total pavement thickness (h_p), design ESAL (W_{80}), and drainage condition rating (D). The developed model is significant at 97.87% confidence level as the model F-statistic is equal to 6.09. The variable coefficients are also significant at more than 94.69% as the corresponding t-statistics are 2.53, 3.16, and 2.23. The model has 57.51% determination coefficient (R square) and 0.098 standard error of estimate. Equations [18] and [19] provide a new empirical approach to predict the initial and terminal transition probabilities directly from related pavement design variables.

$$Log\ P_{9,10} = -0.935 + 0.174\sqrt{h_p} - 0.074\left(Log\left(\frac{W_{80}}{\sqrt{D}}\right)\right)^2 \qquad [19]$$

The second set of empirical predictive models is developed to estimate the total pavement thickness (h_p) from related design variables including the stochastic-based design factors. Equation [20] predicts the total design thickness using five major design factors including the initial and terminal transition probabilities. The initial and terminal transition probabilities provide a new mechanism to control the long-term performance of pavement. The empirical predictive model presented in Equation [20] is significant at 99.99% confidence level as the corresponding F-statistic is equal to 67.88. The variable coefficients are also significant at more than 97.52% as the t-statistics for the three coefficients are 4.31, 3.17, and 2.69. The model has 93.78% determination coefficient (R square) and 0.020 standard error of estimate.

$$Log\ h_p = 1.186 + 0.371\left(\sqrt{Log\left(W_{80} D^{1.5}\right)}\right) - 0.072\ Log\left(\frac{P_{9,10} R^2}{P_{1,2}}\right) \qquad [20]$$

The two long-term performance indicators that are determined based on the initial and terminal transition probabilities have been also used in developing empirical predictive design models. Equation [21] is developed using four main design variables including the area under the performance curve (A_{UPC}). The area under the performance curve has replaced both the initial and terminal transition probabilities. The empirical model presented in Equation [21] is significant at 99.99% confidence level as the model F-statistic is equal to 103.79. The variable coefficients are also significant at more than 98.49% as the corresponding t-statistics are 5.43, 3.91 and 3.00. The model has 95.84% determination coefficient (R square) and 0.017 standard error of estimate.

$$Log\ h_p = 1.607 - 0.087\ Log\left(A_{UPC}R^2\right) + 0.299\left(\sqrt{Log\left(W_{80}D^{1.5}\right)}\right) \qquad [21]$$

Equation [22] presents the last sample empirical design model which is similar to the one indicated by Equation [21] except that the area under the performance curve (A_{UPC}) is replaced by the project average distress rating ($\overline{DR}$). Equations [21] and [22] are very much similar in format and are associated with similar statistics. Table 5 provides a summary of statistics for all presented sample predictive models. The strong similarity between Equation [21] and Equation [22] indicates that the two long-term performance indicators, namely, (A_{UPC}) and ($\overline{DR}$), are highly compatible.

$$Log\ h_p = 1.494 - 0.088\ Log\left(\overline{DR}R^2\right) + 0.300\left(\sqrt{Log\left(W_{80}D^{1.5}\right)}\right) \qquad [22]$$

The negative logarithmic term in Equations [21] and [22] contains the product of two parameters, namely, the area under the performance curve/average distress rating and the R-value raised to the second power. A higher area under the performance curve/average distress rating requires a higher pavement thickness whereas a higher R-value will result in a lower pavement thickness. Therefore, the two parameters used inside the negative logarithmic term have an inverse effect on pavement thickness which means that the negative sign associated with the logarithmic term is based on the net impact of the two parameters. A higher R-value will also help provide a larger area under the performance curve/average distress rating considering the same pavement thickness.

The deployed long-term performance indicators are directly related to the initial and terminal transition probabilities as evidenced from Table 3. Therefore, sample empirical models can be developed to predict the two long-term performance indicators using only the initial and terminal transition probabilities as indicated by Equations [23] and [24]. Equations [23] and [24] are both linear in form and are associated with highly significant statistics as provided in Table 5. The designer can estimate the long-term performance indicators from Equations [23] and [24] using desired initial and terminal transition probabilities. The estimated long-term performance indicators can then be used to obtain the required total pavement thickness from Equations [21] and [22]. Alternatively, Equation [20] can be directly solved for the total pavement thickness using desired initial and terminal transition probabilities.

$$A_{UPC} = 1860.35 - 1298.30\ P_{1,2} - 503.79\ P_{9,10} \qquad [23]$$

$$\overline{DR} = 92.52 - 63.51\ P_{1,2} - 25.02\ P_{9,10} \qquad [24]$$

Table 5. *Summary of statistics for sample empirical predictive models*

Predictive Model	Model R-Square	Model Standard Error	Model F-Statistic	Model Coefficients	Coefficient t-Statistic	Confidence Level
Equation [18]	75.37%	0.077	13.77 (99.82%)#	0.424 0.017 -0.048	2.25 2.78 3.31	94.86% 97.87% 99.10%
Equation [19]	57.51%	0.098	6.09 (97.87%)	-0.935 0.174 -0.074	2.53 3.16 2.23	96.78% 98.84% 94.69%
Equation [20]	93.78%	0.020	67.88 (99.99%)	1.186 0.371 -0.072	4.31 3.17 2.69	99.80% 98.86% 97.52%
Equation [21]	95.84%	0.017	103.79 (99.99%)	1.607 -0.087 0.299	5.43 3.91 3.00	99.96% 99.64% 98.49%
Equation [22]	95.84%	0.017	103.70 (99.99%)	1.494 -0.088 0.300	5.57 3.91 3.01	99.96% 99.64% 98.52%
Equation [23]	98.85%	25.61	386.19 (99.99%)	1860.35 -1298.30 -503.79	59.68 23.41 10.99	99.99% 99.99% 99.99%
Equation [24]	98.76%	1.31	357.81 (99.99%)	92.52 -63.51 -25.02	58.21 22.45 10.71	99.99% 99.99% 99.99%
Equation [25]	94.20%	0.023	73.06 (99.99%)	1.701 -0.124 0.320	4.48 4.73 2.25	99.85% 99.89% 94.89%

F-Statistic Confidence Level.

In addition, sample empirical design models similar to those developed to predict the total pavement thickness can be generated to yield the granular-base layer thickness (h_b). Equation [25] provides a sample empirical model to predict the granular-base layer thickness from related design variables including the area under the performance curve. The statistics for Equation [25] are highly significant as provided in Table 5. The asphalt layer thickness (h_a) is then estimated as the difference between the total pavement thickness (h_p) and the granular-base layer thickness (h_b). Alternatively, minimum asphalt layer thickness can be used as typically required for low volume roads with light traffic loading.

$$Log\ h_b = 1.701 - 0.124\ Log\left(A_{UPC}R^2\right) + 0.320\left(\sqrt{Log\left(W_{80}D\right)}\right) \qquad [25]$$

The presented sample empirical models are mainly developed to introduce the stochastic-based approach for the design of flexible pavement. The presented sample empirical models are developed using a limited sample of low volume roadways due to the time and effort involved in conducting the pavement distress assessment in two separate survey cycles. However, it is believed that the presented stochastic-based approach can help pavement designers save time and money once empirical design models become available. Empirical predictive models can then be applied to pavement design conditions similar to those used in their development. Predictive design models should be developed for a family of pavements with similar traffic and climate conditions just the same way as in the case of performance prediction models.

7. Conclusions and recommendations

An empirical stochastic-based approach has been presented for the design of flexible pavement. The presented empirical approach deploys the same typical design factors but it includes new stochastic-based design factors that can provide an effective control-mechanism for the highly desirable long-term pavement performance. The new design factors mainly include the project initial and terminal transition probabilities which are stochastically estimated from historical records of pavement distress. Two survey cycles of pavement distress are required as a minimum to obtain estimates of the transition probabilities using a "back-calculation" of the Markov model. The estimated initial and terminal transition probabilities have been exclusively used to predict the future pavement distress ratings using the Markov model for a given service life. The predicted distress ratings are then used to yield two long-term performance indicators, namely the area under the performance curve and the corresponding average distress rating, which are in turn used as new stochastic-based design factors. The new deployed stochastic-based design factors will provide the designer with new design alternatives that can better control the long-term performance of pavement, thus, resulting in substantial savings to roadway users.

The sample empirical predictive models are mainly developed to demonstrate the potential use of the proposed stochastic-based approach in the design of flexible pavement. The presented sample empirical models have all statistically indicated the substantial significance of the new deployed stochastic-based design factors. In particular, the models incorporating the two long-term performance indicators, namely the area under the performance curve and the corresponding average distress rating, have resulted in the highest levels of confidence. These two long-term performance indicators have straightforward interpretation and can be easily used by pavement designers. The development of similar empirical models only requires two cycles of pavement distress assessment and the original pavement design data. Most highway agencies have established databases for storing pavement distress records and pavement design data to be mainly used in pavement management applications. These databases can now be retrieved to develop empirical stochastic-based design models for flexible pavement using the approach presented in this paper. The presented sample predictive models are mainly developed for low volume roads in a developing country with moderate climate. Therefore, it is recommended that different sets of empirical design models be established for roadway classes with different traffic loading and climate conditions. The stochastic approach is expected to provide the highway agencies with predictive design models that can better suit their local conditions.

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