Dynamic Probabilistic Approach for Long-Term Pavement Restoration Program with Added User Cost

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Dynamic Probabilistic Approach for Long-Term Pavement Restoration Program with Added User Cost

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A dynamic probabilistic-based approach has been developed for generating a long-term pavement restoration program for a given pavement system. The probabilistic approach applies the basic principles of stochastic processes to predict pavement conditions. Initial state probabilities and transition probabilities are the two main parameters required to develop the future state probability functions used in formulating an effective optimum decision policy. The future state probabilities are only functions of the restoration variables representing potential restoration actions. An optimum decision policy is deployed to yield a pavement restoration program comprising (*n*) restoration plans corresponding to (*n*) transitions. The derived pavement restoration program takes into consideration the long-term transitional performance and budget requirements. The applied decision policy is based on either maximizing the expected system condition ratings subjected to budget constraints or minimizing the net system costs subjected to desired expected system condition ratings. The net system cost may include restoration cost, deteriorated pavement added user cost, and work-zone added user cost. The two decision policy options are subjected to other constraints placing limits on the state probability functions and restoration variables. The restoration variables as applied to state probability functions result in optimum models that are sequentially solved to maintain their linearity. Sample results from a case study have indicated the usefulness of the developed dynamic probabilistic approach in yielding potential long-term pavement restoration programs.

Development of a long-term pavement restoration program requires both an effective pavement performance prediction model and a reliable optimum decision policy. Pavement performance has long been recognized as being probabilistic, which implies that future pavement conditions cannot be estimated with certainty. Several researchers have applied the principles of stochastic processes to model the probabilistic deterioration of pavements (*1–5*). Other researchers have incorporated probabilistic performance prediction models in developing optimum long-term pavement management models (*6–11*). The major challenge facing most pavement management models was solving the resulting optimum models, which generally tend to be nonlinear constrained programs (*6, 7, 10, 12, 13*). This is especially true when the corresponding optimum models involve a large number of restoration options to be represented by an equal number of restoration variables.

The Markov model has been used extensively in modeling pavement performance and management (*1, 2, 4–7*). The Markov model is a basic law in stochastic processes with state and transition probabilities as its main parameters. The state probabilities after a number of transitions (*n*) can be estimated from multiplying the initial state probability vector by the transition matrix raised to power (*n*). Previous research work has integrated the maintenance and rehabilitation (restoration) variables as part of the transition matrix to represent pavement improvement rates as defined by the matrix entries below the main diagonal (*1, 6, 7*). Matrix entries above the main diagonal represent pavement deterioration rates (i.e., transition probabilities) from present states to worse states, whereas entries along the main diagonal represent the transition probabilities of remaining in the same states after a discrete time interval (transition). The Markov model was then used to predict the future state probabilities after (*n*) transitions in the presence of the restoration variables.

The resulting future state probabilities (functions) were nonlinear polynomials with (*n*) degree when the restoration variables had been integrated into the transition matrix (*1, 6, 7*). Solution of a corresponding optimum constrained nonlinear model is not readily available, and solving limitations increase exponentially with the increase in the number of deployed transitions (*n*), which hinders the model usefulness as a long-term pavement management tool. The increase in the number of deployed restoration variables can add further complications to any applicable nonlinear optimization method. In addition, the resulting optimum solution is considered static, as the basic Markov model implies the use of the same solution plan for each time interval (transition) within a study period of (*n*) transitions (*1, 6, 7*). These main disadvantages can be overcome when the restoration variables are integrated into the state probabilities, as proposed in this paper. The restoration variables once integrated into the state probabilities (functions) result in optimum models that are sequentially solved as linear models. An optimum dynamic management model is one that can generate a different restoration plan for each transition and allows for making effective transitional updates on the future pavement conditions.

METHODOLOGY

The developed optimum dynamic probabilistic approach applies the basic principles of stochastic processes to estimate future pavement conditions. The estimated future pavement conditions are used to generate a restoration program that can meet the long-term per-

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formance and budget requirements for a given pavement system. The long-term restoration program comprises (n) restoration plans wherein each transition within a specified analysis period can receive its own restoration plan.

Dynamic Probabilistic Approach Development

The pavement system is divided into a specified number of pavement condition states (m) with state 1 representing the best state and state m denoting the worst. The state probabilities, representing pavement proportions in the various deployed condition states, are used to assess the pavement system conditions at any given time. Deterioration of pavement will change the state probabilities over time. It is assumed that deterioration in one time interval (transition) can result only in the pavement either remaining in its present state or transiting to the next worse state. Therefore, a particular condition state i can have its state probability increased as a result of a deteriorating portion entering (DE_i) the state from the preceding better state and decreased when a deteriorating portion leaving (DL_i) the state itself, as indicated by Equation 1.

$$SA_i^{(1)} = SB_i^{(0)} + DE_i^{(1)} - DL_i^{(1)} + RE_i^{(1)} - RL_i^{(1)} \tag{1}$$

Restoration of pavement in a particular condition state can take place through the application of potential maintenance and rehabilitation actions. A particular condition state represented by its state probability can have a portion entering it (RE_i) from worse states as a result of restoration and a portion leaving it (RL_i) to better states as a result of restoration to the state itself. Therefore, the state probabilities just after one time interval ($SA_i^{(1)}$) can be related to the initial state probabilities just before restoration ($SB_i^{(0)}$), as indicated by Equation 1. It is assumed that restoration work will be carried out during a specified transition with superscripts designating the transition number. Equation 1 simply defines the general trend of deterioration and restoration mechanisms associated with a particular state i with a detailed development to be presented in Equation 2.

Pavement state improvements can take place as a result of applying potential restoration actions. Each restoration action is to be represented by a corresponding restoration variable ($X_{i,k}$). The restoration variable ($X_{i,k}$) represents a portion of the initial state probability associated with state i to be improved to the kth state during a specified time interval. All pavement condition states, with the exception of state 1 and worst-case state m, can have portions leaving and entering as a result of deterioration and restoration, respectively, as indicated by Equation 2. State 1 can have only a portion leaving it as a result of deterioration ($PD_{1,2}$) and portions entering it from worse states as a result of restoration. State 1 is assumed to require no improvement. State m can have only a portion entering it from the deterioration of the preceding worse state ($PD_{m-1,m}$) and portions leaving it to better states as a result of restoration. Equation 2 incorporates the deteriorating portions and restoration variables into the initial state probabilities ($SB_i^{(0)}$) just before restoration to yield the corresponding state probabilities ($SA_i^{(1)}$) just after one time interval. Figure 1 is a flowchart representing the state deterioration and restoration plan as defined by Equation 2. The boxes in Figure 1 contain the initial state probabilities (proportions) ($SB_i^{(0)}$) whereas the branches represent the deterioration and restoration portions, ($PD_{i,i+1}$) and ($X_{i,k}$), respectively.

$$SA_i^{(1)} = SB_i^{(0)} - PD_{1,2}^{(1)} + \sum_{j=2}^{m} X_{j,1}^{(1)} \qquad (i = 1)$$

$$SA_i^{(1)} = SB_i^{(0)} + \left(PD_{i-1,i}^{(1)} - PD_{i,i+1}^{(1)}\right) + \sum_{j=i+1}^{m} X_{j,i}^{(1)} - \sum_{k=1}^{i-1} X_{i,k}^{(1)} \qquad (i = 2,3,\ldots,m-1) \tag{2}$$

$$SA_m^{(1)} = SB_m^{(0)} + PD_{m-1,m}^{(1)} - \sum_{k=1}^{i=1} X_{m,k}^{(1)} \qquad (i = m)$$

The deteriorating portions ($PD_{i,i+1}$) after a discrete time interval can be estimated on the basis of the initial state probabilities ($SB_i^{(0)}$) and transition probabilities ($P_{i,i+1}$) as defined in Equation 3. The transition probability ($P_{i,i+1}$) is defined as the probability of a pavement section in state (i) that will transit to state (i + 1) during the same deployed discrete time interval. State and transition probabilities are typically estimated from historical records of pavement distress (*1, 2, 6, 7*). One cycle of field survey is required to estimate the state

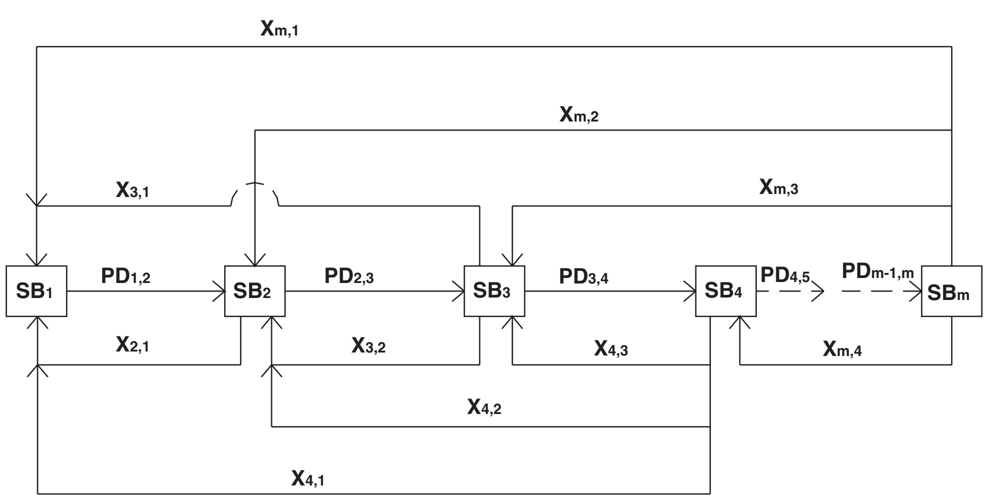

FIGURE 1 Proposed state deterioration and restoration plan.

probabilities, and a minimum of two cycles are required to estimate the transition probabilities.

$$\mathrm{PD}^{(1)}_{i,i+1} = \mathrm{SB}^{(0)}_i \mathrm{P}_{i,i+1} \qquad (i = 1, 2, \ldots, m-1) \tag{3}$$

Now, since restoration work is assumed to occur during a specified time interval, the deteriorating portions derived from Equation 3 are overestimated because some portions of the initial state probabilities ($\mathrm{SB}^{(0)}_i$) would have been restored to better condition states during the early stages of that time interval. This problem can be overcome by considering that pavement proportions available for restoration are the average values of the initial state probabilities just before restoration ($\mathrm{SB}^{(0)}_i$) and the corresponding values just after one transition ($\mathrm{SB}^{(1)}_i$) in the absence of any restoration. Therefore, the average state proportions ($\overline{S}^{(0,1)}_i$) available for restoration during the first time interval are as defined in Equation 4. The state probabilities ($\mathrm{SB}^{(1)}_i$) are obtained from multiplying the initial state probability row vector ($\mathrm{SB}^{(0)}_i$) by the transition matrix. It is to be emphasized that any row in the deployed transition matrix is assumed to contain only the two transition probabilities ($P_{i,i}$ and $P_{i,i+1}$) with their sum being equal to one, which is a valid assumption for a pavement system with a moderate number of condition states (*1, 2, 6, 7*).

$$\overline{S}^{(0,1)}_1 = \frac{\mathrm{SB}^{(0)}_1 + \mathrm{SB}^{(1)}_1}{2} = \mathrm{SB}^{(0)}_1 - 0.5\mathrm{PD}^{(1)}_{1,2} \qquad (i = 1)$$

$$\overline{S}^{(0,1)}_1 = \frac{\mathrm{SB}^{(0)}_i + \mathrm{SB}^{(1)}_i}{2} = \mathrm{SB}^{(0)}_i + 0.5\left(\mathrm{PD}^{(1)}_{i-1,i} - \mathrm{PD}^{(1)}_{i,i+1}\right)$$
$$(i = 2, 3, \ldots, m-1) \tag{4}$$

$$\overline{S}^{(0,1)}_m = \frac{\mathrm{SB}^{(0)}_i + \mathrm{SB}^{1}_i}{2} = \mathrm{SB}^{(0)}_m + 0.5\mathrm{PD}^{(1)}_{m-1,m} \qquad (i = m)$$

The state probabilities just after restoration ($\mathrm{SA}^{(1)}_i$) presented in Equation 2 are revised as indicated by Equation 5 to incorporate the average state proportions ($\overline{S}^{(0,1)}_i$). It can now be noted that the deteriorating portions appearing in Equation 5 are equal to half their corresponding values provided in Equation 2. The revised state probabilities just after restoration are only functions of the restoration variables ($X^{(1)}_{i,k}$), which represent portions to be deducted from the pavement state proportions that exist during the first discrete time interval. The revised state probability functions ($\mathrm{SA}^{(1)}_i$) are to be used in the formulation of a long-term restoration program.

$$\mathrm{SA}^{(1)}_1 = \mathrm{SB}^{(0)}_1 - 0.5\mathrm{PD}^{(1)}_{1,2} + \sum_{j=2}^{m} X^{(1)}_{j,1} \qquad (i = 1)$$

$$\mathrm{SA}^{(1)}_i = \mathrm{SB}^{(0)}_i + 0.5\left(\mathrm{PD}^{(1)}_{i-1,i} - \mathrm{PD}^{(1)}_{i,i+1}\right) + \sum_{j=i+1}^{m} X^{(1)}_{j,i} - \sum_{k=1}^{i-1} X^{(1)}_{i,k}$$
$$(i = 2, 3, \ldots, m-1) \tag{5}$$

$$\mathrm{SA}^{(1)}_m = \mathrm{SB}^{(0)}_m + 0.5\mathrm{PD}^{(1)}_{m-1,m} - \sum_{k=1}^{i-1} X^{(1)}_{m,k} \qquad (i = m)$$

Generally, the resulting total number of restoration variables (N) associated with a pavement system with (m) states is determined by using Equation 6. For example, a total of 15 restoration variables is required for a pavement system with six condition states. Each restoration variable is to represent a potential restoration action. The selected restoration actions must produce the expected state improvements.

$$N = \sum_{k=1}^{m-1} k \tag{6}$$

Pavement condition states can be defined by using any appropriate pavement condition indicator, such as the present serviceability index or pavement condition index (PCI). A pavement condition indicator similar to the PCI is used in the sample presentation to define the deployed condition states and expected state improvements.

Derivation of Future State Probability Functions

The state probability functions estimated after restoration ($\mathrm{SA}^{(1)}_i$) can be used to predict the future state probability functions ($\mathrm{S}^{(r-1)}_i$) after a desired number of discrete time intervals (r) by using the basic law of stochastic processes, Markov law, as indicated by Equation 7. The transition matrix contains the transition probabilities with only two entries in each row ($P_{i,i}$ and $P_{i,i+1}$) as typically assumed for a moderate matrix size (*1, 6, 7*). The last row includes only the entry ($P_{m,m}$) with a value of one.

$$\mathbf{S}^{(r-1)}_{(1\times m)} = \mathbf{SA}^{(1)}_{1\times m}\,\mathbf{P}^{(r-1)}_{(m\times m)} = \left(\mathrm{S}^{(r-1)}_1, \mathrm{S}^{(r-1)}_2, \ldots, \mathrm{S}^{(r-1)}_m\right) \qquad (r \geq 2) \tag{7}$$

where

$\mathbf{S}^{(r-1)}_{1\times m}$ = row vector representing state probability functions after (r) transitions,
$\mathbf{SA}^{(1)}_{1\times m}$ = row vector representing the state probability functions after the first transition, and
$\mathbf{P}^{(r-1)}_{m\times m}$ = transition matrix multiplied ($r - 1$) times.

The expected state probability functions ($\mathrm{S}^{(r-1)}_i$) after (r) transitions consequently become only functions of the restoration variables ($X^{(1)}_{i,k}$) associated with the first transition. Equation 7 estimates the state probabilities after (r) transitions although it is raised to the power ($r - 1$), because the first transition is already incorporated in the estimation of the state probability functions ($\mathrm{SA}^{(1)}_i$). The resulting state probability functions ($\mathrm{S}^{(r-1)}_i$) are linear in form regardless of the deployed number of transitions (r).

LONG-TERM PAVEMENT RESTORATION PROGRAM

The future state probability functions will be used to formulate two decision policy options for generating a long-term pavement restoration program. The two deployed decision policy options as applied to a particular system are based on either maximizing the expected pavement condition ratings subjected to budget constraints or minimizing the net restoration costs subjected to pavement condition rating requirements. A sequential linear approach for generating a long-term pavement restoration program also is presented.

Expected Pavement System Condition Rating

An appropriate pavement condition rating scale is to be used according to the selected pavement condition indicator. Then, each pavement condition state is defined by using an appropriate rating range consisting of upper rating (UR_i) and lower rating (LR_i). The average state rating $\bar{R}_i$ can be estimated as the average of upper and lower ratings. The expected pavement system condition rating $\bar{R}_s^{(r)}$ after (r) transitions consequently can be estimated as the mean of a probability density function, as indicated by Equation 8. The probability density function is defined by using the expected state probability functions ($SA_i^{(1)}$) incorporating the restoration variables. The expected system condition rating represents a weighted average of the average state ratings ($\bar{R}_i$).

$$\bar{R}_s^{(r)} = \begin{cases} \sum_{i=1}^{m} \left(\bar{R}_i\right)\left(\mathrm{SA}_i^{(1)}\right) & r = 1 \\ \sum_{i=1}^{m} \left(\bar{R}_i\right)\left(\mathrm{S}_i^{(r-1)}\right) & r \geq 2 \end{cases} \tag{8}$$

where $\bar{R}_i = \dfrac{\left(\mathrm{UR}_i + \mathrm{LR}_i\right)}{2}$

The expected system condition rating as obtained from Equation 8 will be used in the formulation of the two previously outlined decision policy options. The expected system rating will have a linear form regardless of the deployed number of transitions (n).

Optimum Decision Policy Development

An effective decision policy for yielding an optimum restoration plan typically is based on either maximizing pavement conditions subject to budget constraints or minimizing restoration costs subject to pavement condition requirement constraints (*1, 6, 7, 12–14*). The first deployed decision policy is aimed at maximizing the expected system condition rating ($\bar{R}_s^{(r)}$) for each transition by using the state probability functions ($SA_i^{(r)}$). The resulting optimum model is presented in Equation 9 with the objective function and constraints in linear form. A long-term pavement restoration program comprised of (n) restoration plans are to be sequential derived as a series of linear programming models.

$$\text{Maximize} \quad \bar{R}_s^{(r)} = \sum_{i=1}^{m} \left(\bar{R}_i\right) \mathrm{SA}_i^{(r)} \qquad \left(r = 1, 2, \ldots, n\right) \tag{9}$$

subject to

$$(1) \quad A_s \sum_{i=2}^{m} \sum_{k=1}^{i-1} \left(\mathrm{RC}_{i,k}\right)\left(X_{i,k}^{(r)}\right) \leq B_s^{(r)}$$

$$(2) \quad 0.0 \leq \mathrm{SA}_i^{(r)} \leq 1.0 \qquad \left(i = 1, 2, \ldots, m\right)$$

$$(3) \quad \sum_{i=1}^{m} \mathrm{SA}_i^{(r)} = 1.0$$

$$(4) \quad \sum_{k=1}^{i-1} X_{i,k}^{(r)} \leq \bar{S}_i^{(r-1,r)} \qquad \left(i = 2, 3, \ldots, m\right)$$

$$(5) \quad X_{i,k}^{(r)} \geq 0.0$$

The first constraint requires that the net cost associated with the rth restoration plan be less than or equal to the available system budget ($B_i^{(r)}$). The net restoration cost is estimated from multiplying the pavement system surface area (A_s) in square meters by the sum product of restoration portions as represented by the restoration variables ($X_{i,k}^{(r)}$) and their corresponding restoration cost rates ($RC_{i,k}$) per square meter. The other constraints are used to place upper and lower limits on the state probabilities and restoration variables and recognizing the sum of state probabilities must add to one. Pavement in condition state 1 is considered to be in a very good condition and no restoration work is required.

Alternatively, a second decision policy is based on minimizing the net restoration cost ($RC^{(r)}{}_s$) for each transition. The resulting model is provided in Equation 10 with the objective function and constraints remain linear in form. The associated objective function is the net restoration cost constraint associated with the maximization model. The first constraint is similar to the objective function used in the maximization model, but the expected system condition rating is set equal to or greater than a desired average system rating ($\overline{\mathrm{DR}}_s^{(r)}$) for the rth restoration plan.

$$\text{Minimize} \quad RC_s^{(r)} = A_s \sum_{i=2}^{m} \sum_{k=1}^{i-1} \left(\mathrm{RC}_{i,k}\right)\left(\mathrm{X}_{i,k}^{(r)}\right) \qquad \left(r = 1, 2, \ldots, n\right) \tag{10}$$

subject to

$$(1) \quad \sum_{i=1}^{m} \left(\bar{R}_i\right)\left(SA_i^{(r)}\right) \geq \overline{\mathrm{DR}}_s^{(r)}$$

$$(2) \quad 0.0 \leq \mathrm{SA}_i^{(r)} \leq 1.0 \qquad \left(i = 1, 2, \ldots, m\right)$$

$$(3) \quad \sum_{i=1}^{m} \mathrm{SA}_i^{(r)} = 1.0$$

$$(4) \quad \sum_{k=1}^{i-1} X_{i,k}^{(r)} \leq \bar{S}_i^{(r-1,r)} \qquad \left(i = 2, 3, \ldots, m\right)$$

$$(5) \quad X_{i,k}^{(r)} \geq 0.0$$

The optimum models presented in Equations 9 and 10 can be extended to a network comprising a number of pavement systems. The objective in this case will be to optimize the expected network condition rating determined as a weighted average of the expected system ratings (*7*). The resulting number of restoration variables will equal the number of deployed systems multiplied by the number of variables used in a given system, and the number of constraints will increase in a similar way. However, the formulated network models can be optimized efficiently by using available linear programming software packages regardless of the model size.

Long-Term Optimal Dynamic Sequential Approach

The optimal solution for the proposed long-term restoration program is sought through a sequential approach that requires solving (n) linear models. The sequential approach is indicated by Equation 11, based on Equations 3 through 5. The sequential approach for generating a long-term restoration program for an analysis period of (n) discrete time intervals requires the formulation of (n) linear models as outlined in Equation 11. The optimal solution derived for the (r – 1) restoration plan model becomes the input used in formulating the rth restoration plan model. This is achieved by requiring that the initial state probabilities just before a particular restoration plan be equal to the optimal state probabilities associated with the preceding restoration solution plan. Figure 2 is a flowchart indicating the major steps required for generating a long-term restoration program comprising (n) restoration plans.

$$\mathrm{SB}_i^{(r-1)} = \mathrm{S}_i^{(r-1)} \qquad (r = 1) \tag{11a}$$

$$\mathrm{SB}_i^{(r-1)} = \mathrm{SA}_i'^{(r-1)} \qquad (r = 2, 3, \ldots, n) \tag{11b}$$

$$\mathrm{PD}_{i,i+1}^{(r)} = \mathrm{SB}_i^{(r-1)}\mathrm{P}_{i,i+1} \tag{11c}$$

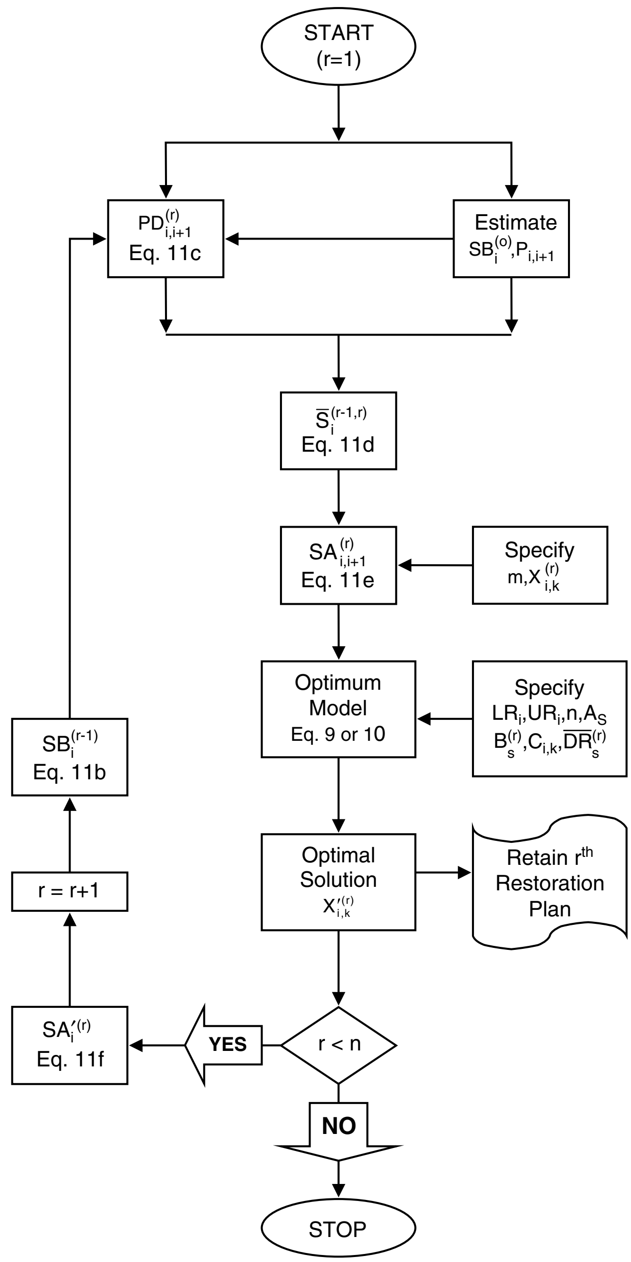

FIGURE 2 Flowchart of sequential approach for generating a long-term restoration program.

$$\bar{S}_i^{(r-1,r)} = \mathrm{SB}_i^{(r-1)} + 0.5\left(\mathrm{PD}_{i-1,i}^{(r)} - \mathrm{PD}_{i,i+1}^{(r)}\right) \tag{11d}$$

$$\mathrm{SA}_i^{(r)} = \bar{S}_i^{(r-1,r)} + \sum_{j=i+1}^{m} X_{j,i}^{(r)} - \sum_{k=1}^{i-1} X_{i,k}^{(r)} \tag{11e}$$

$$\mathrm{SA}_i'^{(r)} = \bar{S}_i^{(r-1,r)} + \sum_{j=i+1}^{m} X_{j,i}'^{(r)} - \sum_{k=1}^{i-1} X_{i,k}'^{(r)} \tag{11f}$$

For example, the derivation of the first restoration plan (r = 1) to be applied during the first time interval is initiated by requiring the state probabilities just before restoration ($\mathrm{SB}_i^{(0)}$) to be equal to the initial (present) state probabilities ($\mathrm{S}_i^{(0)}$). The state probability functions just after restoration ($\mathrm{SA}_i^{(1)}$) are then determined, as outlined in Equation 11*e*. The optimal state probabilities just after restoration ($\mathrm{SA}_i'^{(1)}$) are obtained once an optimal solution ($X_{i,k}'^{(1)}$) is reached according to the presented two decision policy options. The generation of the second restoration plan (r = 2) requires use of the optimal solution ($X_{i,k}'^{(1)}$) associated with the first restoration plan. Therefore, the initial state probabilities ($\mathrm{SB}_i^{(1)}$) just before implementing the second restoration plan are set equal to the optimal state probabilities ($\mathrm{SA}_i'^{(1)}$). The initial state probabilities ($\mathrm{SB}_i^{(1)}$) are used to define the state probability functions ($\mathrm{SA}_i^{(2)}$) for the second restoration plan.

A dynamic feature of the presented optimal sequential approach is that it allows for making effective transitional updates on the future pavement conditions as represented by the state probabilities. The expected state probabilities after each transition can be compared to the observed values obtained from conducting field distress assessment. It is expected that there will be differences between the expected and observed values; therefore, the observed values can be used for generating the subsequent restoration plan. In essence, this provides another major justification for using the sequential linear approach.

Deteriorated Pavement Added User Cost

Deteriorated pavements can greatly affect user cost, especially when traveling on pavements in the worst condition states. The added user cost caused by pavement deterioration is not typically accounted for in solving the pavement management problem. Restoration plans consist of maintenance and rehabilitation actions with highly variable restoration cost rates and performance outcomes. The generated optimal solutions usually favor the maintenance actions for their restoration effectiveness. Therefore, the rehabilitation actions as applied to the worst condition states receive limited use (*1, 6, 7*). The outcome is an additional cost to roadway users, which is not accounted for in the optimization modeling.

The deteriorated pavement added user cost ($\mathrm{DC}_s^{(r)}$) as defined in Equation 12 optionally can be added to the objective function provided in Equation 10 that minimizes the net restoration cost. The added user cost during a given time interval (ΔT) in years can be determined on the basis of the system average daily traffic ($\overline{\mathrm{ADT}}_s$) system length ($\mathrm{L}_s$) in lane kilometers, added user cost rate (DC_i) per thousand vehicle kilometers for state i, and corresponding state probability function ($\mathrm{SA}_i^{(r)}$). The specified time interval (ΔT) is the time used between two successive transitions.

$$\mathrm{DC}_s^{(r)} = 365\left(\Delta\mathrm{T}\right)\left(\overline{\mathrm{ADT}}_s\right)\left(\mathrm{L}_s\right)\sum_i\left(\mathrm{DC}_i\right)\left(\mathrm{SA}_i^{(r)}\right) \tag{12}$$

Added User Cost of Restoration Work Zone

Added user cost due to travel through restoration work zones also may be considered. This cost component is incurred as a consequence of traffic flow interruptions and delays associated with the restoration site. It is affected by time of day, traffic volume, diverted traffic, number of travel lanes, number of closed lanes, and lane closure duration. Benz et al. used computer models to simulate the travel delay during and after construction with delay difference converted into user cost (*15*). Therefore, it is suggested that the average system delay difference ($\Delta\bar{\mathrm{D}}_s$) in hours per vehicle be estimated for a particular lane closure arrangement. The work zone added user cost ($\mathrm{WC}_s^{(r)}$) as indicated by Equation 13 depends on the average system hourly volume ($\overline{\mathrm{HV}}_s$) and expected time period (T_k) in hours required for restoring a lane kilometer by using the kth restoration action. This cost component can be added to the net restoration cost outlined in Equation 10.

$$\mathrm{WC}_s^{(r)} = (\mathrm{WC})\left(\Delta\bar{\mathrm{D}}_s\right)\left(\overline{\mathrm{HV}}_s\right)\sum_{i=2}^{m}\sum_{k=1}^{i-1}(T_k)(L_s)\left(X_{i,k}^{(r)}\right) \quad (13)$$

The work zone user cost rate (WC) per hour should include the time value and vehicle operating costs. Different average delay differences must be established for the time of day and for different highway systems and lane closure arrangements.

SAMPLE PRESENTATION

The use of the presented optimum models for developing a long-term restoration program is illustrated by using a case study involving the secondary highway system in the District of Nablus, West Bank. The secondary highway system is mainly a two-lane rural highway connecting the city of Nablus with surrounding major towns. The corresponding pavement system is made of flexible pavement consisting of two layers: 7 cm asphalt layer and 25 cm aggregate base layer. The pavement system is also subjected to similar loading conditions with an average daily traffic of about 7,500 vehicles. The studied pavement system is 147.36 lane kilometers long and has a 3.4-m average lane width.

Pavement Condition Rating Estimation

The pavement system was surveyed for major pavement defects twice during a period of 2 years for the purpose of estimating the transition probabilities as a minimum of two cycles of pavement survey is required (*1, 6, 7*). Pavement sections of 50 m were used, resulting in 2,947 surveyed sections. The major pavement defects considered in the surveys were cracking and deformation. The crack width and deformation depth were used to measure severity, as presented in Equation 14. A pavement condition rating (R) is estimated based on the section surface area (A), cracked area (A_c), deformed area (A_d), cracking severity factor (F_c), and deformation severity factor (F_d). Severity factors are assigned the values of 1, 2, and 3 for low, medium, and high levels of severity, respectively. Equation 14 yields for each surveyed section a condition rating on a scale of 100 points with high ratings indicating better pavements. A defected area can be counted only as cracked or deformed.

$$R = \left(\frac{3A - F_c A_c - F_d A_d}{3A}\right) \times 100\% \quad (14)$$

Pavement sections are then assigned to six condition states to be deployed in this sample presentation. The six deployed condition states are defined by using upper and lower condition ratings, as provided in Table 1. States 1 through 4 are defined by using 15-point rating ranges, whereas states 5 and 6 are defined by using 20-point ranges. State 1 contains the best pavements, and state 6 includes the worst pavements. The average state rating ($\bar{\mathrm{R}}_i$) is the average of upper and lower state condition ratings. A 10-point range is recommended if 10 condition states are to be used.

Stochastic Parameters and Restoration Variables

The transition probabilities ($P_{i,i+1}$) representing the deterioration rates are estimated from the numbers of pavement sections assigned to the six condition states in the two survey cycles (*1, 6, 7*). The initial (present) state probabilities ($\mathrm{SB}_i^{(0)}$) provided in Table 1 are estimated on the basis of the numbers of sections assigned from the second cycle.

A total of nine restoration variables are used to represent nine restoration actions. The nine introduced variables comprise five one-state restoration variables ($X_{i,i-1}^{(r)}$) and four multiple-state restoration variables ($X_{i,1}^{(r)}$). The one-state restoration variables represent pavement portions improved to a better condition state ($i - 1$) from a worse state (i) by using repair works such as crack sealing, pothole patching, and surface treatment with different intensities as applied to different states. The four multiple-state restoration variables represent pavement portions improved to the best condition state (1) from a worse state (i = 3, 4, 5, 6) by using rehabilitation actions such as thin plain overlay, thicker plain overlay, skin patch, and reconstruction. The restoration cost rates ($\mathrm{RC}_{i,k}$) associated with the nine restoration variables in U.S. dollars per square meter are provided in Table 2.

Restoration Effectiveness Ratio

A restoration effectiveness ratio ($\mathrm{RE}_{i,k}$) for a particular restoration variable is defined as the ratio of restoration cost rate to the average change in pavement condition rating ($\Delta\bar{R}_{i,k}$) as defined in Equation 15. The parameter $\Delta\bar{R}_{i,k}$ represents the difference in the expected average state rating ($\bar{R}_k$) and the current average state rating ($\bar{R}_i$) as condition state (i) represents the current state and condition state (k) denotes the state expected from restoration.

$$\mathrm{RE}_{i,k} = \frac{RC_{i,k}}{\Delta\bar{R}_{i,k}} = \frac{\mathrm{RC}_{i,k}}{\bar{R}_k - \bar{R}_i} \quad (15)$$

TABLE 1 Condition State Rating System and Basic Stochastic Parameters

State i	UR_i	LR_i	$\bar{R}_i$	$\mathrm{SB}_i^{(0)}$	$P_{i,\,i+1}$	$\mathrm{PD}_{i,\,i+1}^{(1)}$	$\bar{S}_i^{(0,1)}$
1	100	85	92.5	0.147	0.268	0.039	0.1275
2	85	70	77.5	0.214	0.319	0.068	0.1995
3	70	55	62.5	0.178	0.414	0.074	0.1750
4	55	40	47.5	0.236	0.481	0.114	0.2160
5	40	20	30	0.104	0.566	0.059	0.1315
6	20	00	10	0.121	N/A	N/A	0.1505

TABLE 2 Sample Restoration Effectiveness Ratio Calculations

$X_{i,k}$	$\bar{R}_i$	$\bar{R}_k$	$\Delta\bar{R}_{i,k}$	$RC_{i,k}$	$RE_{i,k}$
$X_{2,1}$	77.5	92.5	15	2.5	0.167
$X_{3,2}$	62.5	77.5	15	3.5	0.233
$X_{4,3}$	47.5	62.5	15	4.5	0.300
$X_{5,4}$	30	47.5	17.5	6.0	0.343
$X_{6,5}$	10	30	20	8.0	0.400
$X_{3,1}$	62.5	92.5	30	10.0	0.333
$X_{4,1}$	47.5	92.5	45	15.0	0.333
$X_{5,1}$	30	92.5	62.5	22.0	0.352
$X_{6,1}$	10	92.5	82.5	30.0	0.364

Table 2 provides the restoration effectiveness ratios associated with the nine deployed restoration variables. It is believed that this ratio has a major impact on the optimal solutions obtained from solving the corresponding linear programs.

Optimum Long-Term Pavement Restoration Programs

Sample optimum long-term restoration programs are generated for an analysis period of 10 years (n = 5) by using both maximization and minimization linear models presented in Equations 9 and 10, respectively. Five optimum restoration plans are sequentially derived as outlined in Figure 2, wherein the optimal solution obtained for a particular time interval (transition) becomes the input for formulating the linear model for the subsequent transition. Table 3 provides the optimal solution plans associated with maximizing the expected system condition rating using a constant transitional budget ($B_s^{(r)}$) of \$1.0 million. The generated optimal solutions have yielded a consistent improvement in the expected system condition rating with a limited use of the multiple-state restoration variables. The limited use of the multiple-state variables can be verified by using the restoration effectiveness ratios provided in Table 2. The optimization process appears to favor the variables with lowest restoration effectiveness ratios such as ($X_{2,1}'^{(r)}$) and ($X_{3,2}'^{(r)}$) that have dominated all derived optimal solutions, and it disfavors the ones with highest ratios such as the variable ($X_{6,5}'^{(r)}$) that has not been picked up by any of the presented solutions. The optimal variable values indicate portions to be deducted from the average state probabilities ($\bar{S}_i^{(0,1)}$) that exist in the corresponding condition states during each transition. For example, the two one-state restoration variables ($X_{2,1}'^{(r)}$) and ($X_{3,2}'^{(r)}$) provided in Tables 3 and 4 for the first restoration plan have carried on the full values of their corresponding average state probabilities ($\bar{S}_i^{(0,1)}$) provided in Table 1.

Similarly, Table 4 provides the generated sample long-term restoration program for minimizing the net restoration cost. The desired expected system rating assigned for each transition is assumed to be the same rating obtained from the maximization model. The derived optimal solution plans as provided in Table 4 are somewhat different from the corresponding ones in Table 3, with the exception of the first transition plan. The total restoration cost associated with the minimization program is \$5.35 million compared to \$5.00 million for the maximization program considering a 10-year analysis period. Therefore, the two optimum decision policies appear to be not quite compatible when they are applied to generate a long-term restoration program. The maximization model has generated an equivalent restoration program that is a little cheaper than the one obtained from the minimization model. It is therefore recommended that both models be applied to a particular pavement system to select the best restoration program.

Long-Term Restoration Program with Added User Cost

The sample optimal solutions provided in Tables 3 and 4 have deprived severely deteriorated pavements such as those in states 5 and 6 from receiving any restoration funding, especially during the initial transitions. Therefore, added user cost due to traveling on severely deteriorated pavements can be considered as defined in Equation 12. The required input parameters are time interval (ΔT = 2 years), system average daily traffic ($\overline{ADT}_s$ = 7,500 vpd), and system length (L_s = 147.36 lane kilometers). The user cost is assumed to consist only

TABLE 3 Sample Optimum Restoration Program for Maximizing Expected System Condition Rating Using Analysis Period of Five Transitions

	Derived r^{th} Sequential Restoration Plan					
$X_{i,k}'^{(r)}$	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$X_{2,1}'^{(r)}$	0.0000	0.1995	0.1909	0.2184	0.1527	0.1338
$X_{3,2}'^{(r)}$	0.0000	0.1750	0.1838	0.0708	0.0288	0.0046
$X_{4,3}'^{(r)}$	0.0000	0.1966	0.0523	0.0220	0.0000	0.0000
$X_{5,4}'^{(r)}$	0.0000	0.0000	0.0990	0.0238	0.0000	0.0000
$X_{6,5}'^{(r)}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$X_{3,1}'^{(r)}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$X_{4,1}'^{(r)}$	0.0000	0.0000	0.0030	0.0640	0.0227	0.0000
$X_{5,1}'^{(r)}$	0.0000	0.0000	0.000	0.0000	0.0057	0.0000
$X_{6,1}'^{(r)}$	0.0000	0.0000	0.0000	0.0000	0.0349	0.0548
$\bar{R}_s'^{(r)}$	56.85	62.47	67.93	74.21	79.46	84.35
$RC_s'^{(r)}$ USD × 10^6	0.000	1.000	0.999	1.000	1.000	0.999

TABLE 4 Sample Optimum Restoration Program for Minimizing System Restoration Cost Using Analysis Period of Five Transitions

	Desired Expected System Condition Rating ($\overline{\mathrm{DR}}_s^{(r)}$) ($r$ = 0,1,2, . . . ,5)					
$X'^{(r)}_{i,k}$	56.85	62.47	67.93	74.21	79.46	84.35
$X'^{(r)}_{2,1}$	0.0000	0.1995	0.1909	0.2181	0.1578	0.1214
$X'^{(r)}_{3,2}$	0.0000	0.1750	0.1839	0.0732	0.0117	0.0019
$X'^{(r)}_{4,3}$	0.0000	0.1967	0.0553	0.0000	0.0000	0.0000
$X'^{(r)}_{5,4}$	0.0000	0.0000	0.0869	0.0000	0.0000	0.0000
$X'^{(r)}_{6,5}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$X'^{(r)}_{3,1}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$X'^{(r)}_{4,1}$	0.0000	0.0000	0.0000	0.0114	0.0000	0.0000
$X'^{(r)}_{5,1}$	0.0000	0.0000	0.0000	0.0710	0.0000	0.0000
$X'^{(r)}_{6,1}$	0.0000	0.0000	0.0000	0.0073	0.0525	0.0574
$\bar{R}'^{(r)}_s$	56.85	62.47	67.93	74.21	79.46	84.35
$\mathrm{RC}'^{(r)}_s$ USD × 10^6	0.000	1.000	0.948	1.380	1.007	1.018

of vehicle operating cost, locally estimated at $160 per thousand vehicle kilometers, considering good pavements. Deteriorated pavements in condition states 4, 5, and 6 are considered for added user cost with the corresponding added user cost rates (DC_i) determined as percentages of the estimated user cost for traveling on good pavements.

Table 5 provides sample results equivalent to those presented in Table 4 but considering the added user cost rates for condition states 4, 5, and 6 to be only 10%, 20%, and 30%, respectively, of the estimated user cost rate of $160 per thousand vehicle kilometers. The generated optimal solution for the first transition requires the complete restoration of all pavement proportions in condition states 4, 5, and 6 as the utilized restoration variables are taking on the full values of their corresponding average state probabilities($\bar{S}_i^{(0,1)}$) provided in Table 1. Therefore, the first transition optimal solution has totally eliminated the added user cost at a total restoration cost of $3.36 million. The corresponding expected system condition rating is 77.78, which is greater than the 62.47 desired system rating. In the subsequent two transitions, the optimization process has necessitated the restoration of all portions deteriorating to condition state 4 by using the corresponding one-step restoration variable for its effectiveness and to do away with added user cost. In the last two transitions, the optimization process has continued to prevent any pavement portions from reaching condition states 5 and 6 by requiring restoration while in state 4; however, obtaining the desired expected system rating has necessitated additional restoration work performed by using the two most effective one-step variables, namely, ($X'^{(r)}_{2,1}$) and ($X'^{(r)}_{3,2}$).

The generated sample restoration program for minimizing the system restoration cost including added user cost has resulted in a total restoration cost of $5.28 million with an overall expected system rating of 78.66 considering an analysis period of five transitions. In comparison, the equivalent restoration program excluding added user cost caused by severely deteriorated pavements has resulted in a total restoration cost of $5.35 million with an overall expected system rating of 73.68. Therefore, the restoration program that

TABLE 5 Sample Optimum Restoration Program for Minimizing System Restoration Cost with Added User Cost for Analysis Period of Five Transitions

	Desired Expected System Condition Rating ($\overline{\mathrm{DR}}_s^{(r)}$) ($r$ = 0,1,2, . . . ,5)					
$X'^{(r)}_{i,k}$	56.85	62.47	67.93	74.21	79.46	84.35
$X'^{(r)}_{2,1}$	0.0000	0.0000	0.0000	0.0000	0.2384	0.1700
$X'^{(r)}_{3,2}$	0.0000	0.0000	0.0000	0.0000	0.1219	0.2431
$X'^{(r)}_{4,3}$	0.0000	0.2160	0.0810	0.0875	0.0948	0.0774
$X'^{(r)}_{5,4}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$X'^{(r)}_{6,5}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$X'^{(r)}_{3,1}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$X'^{(r)}_{4,1}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$X'^{(r)}_{5,1}$	0.0000	0.1315	0.0000	0.0000	0.0000	0.0000
$X'^{(r)}_{6,1}$	0.0000	0.1505	0.0000	0.0000	0.0000	0.0000
$\bar{R}'^{(r)}_s$	56.85	77.78[a]	76.48[a]	75.23[a]	79.46	84.35
$\mathrm{RC}'^{(r)}_s$ USD × 10^6	0.000	3.360	0.183	0.197	0.726	0.814

[a]Optimum expected system rating higher than the desired expected system rating.

considers added user cost is superior to the one that discards it. However, its only disadvantage is the requirement of a higher initial investment that may prevent its implementation. The restoration work zone added user cost has not been considered; however, its impact is expected to be similar to that of restoration cost because the work zone user cost is directly proportional to the expected time period (T_k) required for restoring a lane kilometer by using the kth restoration action.

CONCLUSIONS AND RECOMMENDATIONS

The presented sample results have indicated the usefulness of the developed optimum dynamic probabilistic approach in yielding potential long-term pavement restoration programs. The deployed probabilistic approach is a simple one with minimal data requirements as the initial state probabilities and transition probabilities are only required. Estimates of these probabilities can be obtained by conducting two cycles of distress survey. The derived state probability functions are effectively integrated into the development of a dynamic optimum decision policy that aims to bring up the pavement system condition rating to desired standards. Formulation of a corresponding optimum decision policy model is relatively straightforward and the optimum solution can be efficiently obtained by using available linear programming software packages.

The presented long-term restoration program is flexible with several vital options. The sample results have been obtained by using only six condition states and nine restoration variables. The number of deployed condition states can practically be increased to 10 without any difficulty in formulating and solving the resulting optimum models. A pavement system with 10 condition states allows for the incorporation of 45 restoration variables, which offers the pavement engineer a large number of options for selecting potential restoration actions. A major requirement is that each potential restoration action shall produce the state improvement outcome as indicated by the subscript associated with the corresponding restoration variable. Pavement condition states, and consequently state improvement outcomes, can be defined by using any appropriate low-cost pavement condition indicator similar to the one used in the presented case study.

It is recommended that a long-term restoration program be generated for each pavement system by using the two outlined decision policy alternatives. A pavement system is one with similar loading conditions and pavement structures, which implies similar pavement deterioration rates as defined by the corresponding transition probabilities. The presented sample results indicated that both the maximization and minimization models are fairly compatible and capable of yielding reliable long-term restoration programs that meet the desired objectives. Sample results have also indicated that the one-step restoration variables are most likely to dominate the optimal solutions when a long-term restoration program excluding added user cost is generated. In addition, sample results obtained from the minimization model with added user cost have emphasized the significant impact of severely deteriorated pavements on a long-term restoration program. Consideration of added user cost has necessitated that all pavements in the worst condition states must be treated as a first priority by using mostly multiple-state restoration variables. However, the corresponding initial investment can be relatively high and unaffordable. An alternative would be to include only the added user cost for at least the worst state.

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