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Fokker-Planck and linear transport solutions to collective flow in heavy ion collisions

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Collective flow effects associated with bounceoff and side splash recently seen in Nb + Nb collisions are investigated. The nuclear bounceoff effect is studied using a folded optical potential. The velocity dependence of the two-body interaction is found to be important for bounceoff. The side-splash effect is studied using linear transport theory. Based on a relaxation time approximation to the Boltzmann equation, we are able to explicitly isolate two separate contributions to the kinetic flow tensor. Specifically, the kinetic flow tensor is related to the collective velocity field and the pressure tensor. In turn, the pressure tensor is related to the nuclear viscosity. The two contributions to kinetic flow interfere destructively. Assuming laminar flow, the viscous term is found to be important in the nuclear side-splash effect.

I. INTRODUCTION

One of the main motivations for performing experiments with heavy ions at medium and high energies is to produce nuclear matter at high density and excitation energy. By producing a nuclear system which is far from the ground state or a state produced by low energy light ion experiments, new regions in energy and density are explored. One immediate goal is to investigate the nuclear equation of state and a hope is to see some unusual phenomena.

The most extensive type of experiments (done at the Bevalac) were inclusive measurements of protons, pions, kaons, and light composite nuclei.[1] These experiments showed nuclei being produced at high excitation energy. However, by the very nature of inclusive experiments, much information is lost and therefore the extraction of the nuclear equation of state has been somewhat illusive. Recently, exclusive experiments have been carried out using a 4π detection scheme. Event by event the momenta of all charged particles are measured.[2] Emission patterns and event shapes are then analyzed in terms of a kinetic flow tensor[3] discussed in Sec. II. In such an analysis the experimental data show strong sidewise peaking[2] or side splash. The side splash effect is associated with particles in the central rapidity region coming from semicentral or intermediate impact parameter collisions. The other collective effect, a nuclear bounceoff, is seen as a transverse momentum given to particles near the beam rapidity. Numerical studies in a hydrodynamic model[4] can account for some features of the data.

In this paper we investigate this bounceoff effect and side-splash effect. Our primary purpose is to explore what properties of the nuclear system are sufficient to give nuclear bounceoff and side splash. In a limit when the mean free path λ is much less than the linear size L of the system, linear transport theory[5] becomes a useful tool which we will use to investigate the side-splash effect. Using such an approach, we will analytically and explicitly relate properties of the kinetic flow tensor to the nuclear viscosity, heat conductivity, and longitudinal momentum degradation. The nuclear bounceoff effect will be studied using a folded optical potential.

II. TRANSPORT APPROACH TO KINETIC FLOW AND SIDE SPLASH

The kinetic flow tensor[3] is

$$F_{ij}=\sum_{\nu=1}^{N} P_i(\nu)P_j(\nu)/[2M(\nu)] , \tag{2.1}$$

where i,j are components of the momentum of particle ν. In an exclusive experiment the momentum of particles $1,\ldots,N$ are measured for each event and F_{ij} is determined. The flow tensor, in general, has diagonal and off diagonal elements. An event shape in the nucleus-nucleus c.m. system can be characterized by an ellipsoid by diagonalizing the F_{ij} matrix. If f_n are the eigenvalues and e_n are the corresponding orthonormal column vectors, then

$$F=f_1e_1e_1^{+}+f_2e_2e_2^{+}+f_3e_3e_3^{+} . \tag{2.2}$$

In polar coordinates

$$f_n=(f_n,\theta_n,\phi_n) ,$$
$$e_n^{+}=(\sin\theta_n\cos\phi_n,\sin\theta_n\sin\phi_n,\cos\theta_n) .$$

When the f_n's are ordered $f_1>f_2>f_3$, then $\theta_1\equiv\theta_F$, where the flow angle θ_F is the polar angle of the maximum kinetic flow.

We now consider various models of the collision to evaluate F_{ij}. The simplest is to assume complete thermal equilibrium. The momentum space density is

$$f(\vec{P},t)=\frac{N}{(2\pi MkT)^{3/2}}e^{-\vec{P}^{2}/(2MkT)} . \tag{2.3}$$

Then $F_{ij}=\delta_{ij}NkT/2$ so that the event shape is a sphere.

The "temperature" T is determined by $M\langle \vec{v}^2 \rangle = 3kT$. Inclusive data can be reasonably well described by this thermal Maxwell-Boltzmann distribution.[6] However, exclusive data show some degree of departure from a sphere and also a collective kinetic flow (rotation of the ellipsoid).[2,7] We therefore next consider refinements to the thermal distribution.

As a second approximation we consider a Fokker-Planck distribution[8] as a possible time evolution from two incident nuclei each with $N/2$ particles with momentum $\vec{P}_0$ and $-\vec{P}_0$, respectively, at $t=0$ to a Maxwell-Boltzmann distribution at $t=\infty$.

$$f(\vec{P},t)=\frac{N/2}{[2\pi MkT(1-e^{-2\beta t})]^{3/2}} \times \left(e^{-|\vec{P}-\vec{P}_0 e^{-\beta t}|^2/2MkT(1-e^{-2\beta t})} + e^{-|\vec{P}+\vec{P}_0 e^{-\beta t}|^2/2MkT(1-e^{-2\beta t})}\right) . \quad (2.4)$$

At $t=0$, Eq. (2.4) reduces to

$$f(\vec{P},t=0)=\frac{N}{2}[\delta(\vec{P}-\vec{P}_0)+\delta(\vec{P}+\vec{P}_0)] . \quad (2.5)$$

The centroid momentum is $\pm\vec{P}_0 e^{-\beta t}$, which is just the behavior of a particle started with $\pm\vec{P}_0$ and subject to a frictional retarding force $-\beta\vec{P}$. The variance of the momentum spreads with time as $kT(1-e^{-2\beta t})$. The $1/\beta$ is the collision or relaxation time. For the Fokker-Planck distribution, the F_{ij} matrix is still diagonal, but the elements are no longer equal; specifically,

$$F_{11}=F_{22}=\frac{NkT}{2}(1-e^{-2\beta t}) ,$$
$$F_{33}=\frac{NkT}{2}(1-e^{-2\beta t})+\frac{NP_0}{2M}e^{-2\beta t} . \quad (2.6)$$

For finite t the event shape is now a cigar shaped ellipse. The flow angle is $\theta_F=0$. The time t is cut off at the point when collisions cease. A related concept used extensively in thermal models is the freeze-out density.[9] The cut off in time gives a picture based on incomplete equilibrium.[10]

We next find what is sufficient to give a collective flow angle. When a system is near local equilibrium a good zero-order approximation is a local Maxwell-Boltzmann distribution. For two symmetric colliding nuclei it is natural to try a phase space distribution function

$$f^{(0)}(\vec{r},\vec{v},t)=n(\vec{r},t)\left[\frac{m}{2\pi\theta(\vec{r},t)}\right]^{3/2} \times \left(e^{-[m/2\theta(\vec{r},t)][\vec{v}-\vec{u}(\vec{r},t)]^2} + e^{-[m/2\theta(\vec{r},t)][\vec{v}+\vec{u}(\vec{r},t)]^2}\right) . \quad (2.7)$$

The

$$\vec{u}(\vec{r},t)=\langle \vec{v} \rangle ;\ \ 3\theta(\vec{r},t)=m\langle |\vec{v}-\vec{u}|^2 \rangle ,$$
$$n(\vec{r},t)=\int d^3v\, f(\vec{r},\vec{v},t) , \quad (2.8)$$

where expectation values are taken with $f^{(0)}$. This equation is a generalization of the Fokker-Planck result

$$kT(1-e^{-2\beta t})\rightarrow\theta(\vec{r},t);\ \ \frac{\vec{P}_0}{m}e^{-\beta t}\rightarrow\vec{u}(\vec{r},t) . \quad (2.9)$$

To relate F_{ij} to transport theory, we consider

$$g(\vec{r},\vec{v},t)\equiv f(\vec{r},\vec{v},t)-f^{(0)}(\vec{r},\vec{v},t) , \quad (2.10)$$

where f is the exact distribution function which satisfies Boltzmann's equation

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}_r+\frac{\vec{F}}{m}\cdot\vec{\nabla}_v\right]f=\left[\frac{\partial f}{\partial t}\right]_{\text{coll}} . \quad (2.11)$$

The $\vec{F}$ is the force from an external field or Hartree-Fock field and the $(\partial f/\partial t)_{\text{coll}}$ can be related to the collision cross section. We will look for solutions of Boltzmann's equation in the relaxation time approximation

$$\left[\frac{\partial f}{\partial t}\right]_{\text{coll}}=-(f-f^{(0)})/\tau_R . \quad (2.12)$$

The correction g is small compared to $f^{(0)}$ when $\lambda \ll L$ since $g/f^{(0)}\sim\lambda/L$. Under these conditions

$$g=-\tau_R\left[\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}_r+\frac{\vec{F}}{m}\cdot\vec{\nabla}_v\right]f^{(0)} , \quad (2.13)$$

and for the choice $f^{(0)}$ of Eq. (2.7) the

$$g=-\tau_R\left[\frac{1}{\theta}\frac{\partial\theta}{\partial x_i}U_i\left[\frac{m}{2\theta}U^2-\tfrac{5}{2}\right] + \frac{1}{\theta}\Lambda_{ij}(U_iU_j-\tfrac{1}{3}\delta_{ij}U^2)\right]f^{(0)} , \quad (2.14)$$

where

$$\Lambda_{ij}=\tfrac{1}{2}m\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right] \quad (2.15)$$

and

$$\vec{U}=\vec{v}-\vec{u}(\vec{r},t) . \quad (2.16)$$

Repeated indices are summed over. Note that the external force no longer appears in Eq. (2.14). The pressure tensor

$$P_{ij}=mn(\vec{r},t)\langle (v_i-u_i)(v_j-u_j) \rangle \quad (2.17)$$

is

$$P_{ij}=P\delta_{ij}+P'_{ij} , \quad (2.18)$$

where

$$P=n\theta , \quad (2.19)$$

the ideal gas law, and

$$P'_{ij}=-\frac{2\mu}{m}\left[\Lambda_{ij}-\frac{m}{3}\delta_{ij}\vec{\nabla}\cdot\vec{u}\right] \tag{2.20}$$

with P'_{ij} traceless. The μ is a constant which can be identified with the coefficient of shear viscosity and its value is given by $\mu=\rho\tau_R\theta$ with ρ the number density. When the first-order result for $f=f^{(0)}+g$ is taken, the equations of viscous hydrodynamics follow. Specifically, the continuity equation, Navier-Stokes equation, and heat conduction equation can be obtained.

We now make an important identification relating the kinetic flow tensor to the pressure tensor and the collective velocity

$$F_{ij}=\tfrac{1}{2}m\int n(\vec{r},t)u_i(\vec{r},t)u_j(\vec{r},t)d^3r+\tfrac{1}{2}\int P_{ij}d^3r$$
$$\equiv F_{ij}(u)+F'_{ij}(P') . \tag{2.21}$$

The part of F'_{ij} coming from the pressure tensor $F'_{ij}(P')=\int d^3r\,P'_{ij}$ can be written as

$$F'_{ij}(P')=-\frac{\mu}{m}\Lambda_{ij}\Omega_v , \tag{2.22}$$

where Ω_v is the volume of the system and Λ_{ij} is given in Eq. (2.15). Note, only viscosity and not thermal conductivity contributes to the off diagonal part of $F_{ij}(P)$ to first order in f. A sufficient condition for collective flow is then $\Lambda_{ij}\neq 0$ and/or $u_iu_j\neq 0$. Specifically, taking $\vec{u}$ along the beam direction $\vec{u}=u\vec{e}_z=u\vec{e}_3$, then if $\partial u_z/\partial x$, $\partial u_z/\partial y$, or u_x,u_y exists a collective sidewise flow exists.

Collective flow has been analyzed in terms of the collective velocity part, coming from $F_{ij}(u)$.[11] We note here another important explicit contribution coming from viscous flow. We expect $\partial u_3/\partial x$ to reflect laminar flow rather than turbulence motion since the Reynolds number Re (Ref. 12),

$$\mathrm{Re}=\frac{lu\rho}{\mu}\sim\frac{l}{\lambda}\frac{u}{v} , \tag{2.23}$$

is much less than 10^3. For Reynolds numbers above 10^3 turbulence sets in.[12] The l is a characteristic length $\sim R$ (the nuclear radius), λ a characteristic mean free path, and u/v a ratio of collective to thermal velocities. In Eq. (2.23) we have used for $\mu\simeq kTN/\Omega_v\tau_c$. Higher order corrections to μ increase this estimate by a factor[13] from 1.18 at $\rho=\rho_0$ to 2.29 at $\rho=4\rho_0$. For laminar flow, $\partial u_z/\partial x\sim V_{\rm beam}/R$. Since $\partial u_z/\partial x$ is positive the results of Eqs. (2.22) and (2.21) show that the collective velocity term and the pressure tensor term have opposite sign and therefore interfere destructively.

III. MODEL CALCULATIONS FOR COLLECTIVE FLOW

A. Nuclear bounceoff

We now try to estimate the transverse impulse that can be imparted to each of the colliding nuclei in a relativistic heavy ion collision (the so-called bounceoff effect). We use a Skyrme-type interaction of the form

$$v(\vec{r}_{12})=t_0\delta(\vec{r}_{12})+\tfrac{1}{2}t_1[(k')^2\delta(\vec{r}_{12})+\delta(\vec{r}_{12})k^2]$$
$$+\frac{t_3}{6}\rho\left[\frac{\vec{r}_1+\vec{r}_2}{2}\right]\delta(\vec{r}_{12}) , \tag{3.1}$$

where $\vec{k}$ ($\vec{k}'$) is the relative momentum operator acting to the right (left). Since the nuclei are expected to become highly excited during the collision we use Gaussian density distributions of the form $\rho_0e^{-\nu r^2}$ rather than the usual Fermi density distributions that are appropriate for cold nuclei. If we neglect the internal momentum of the nucleons, then the interaction potential energy at a separation $\vec{r}$ between two identical nuclei is given by[14]

$$V(\vec{r})=\left[t_0+t_1\frac{p_z^2}{\hbar^2}\right]\left[\frac{\pi}{2\nu}\right]^{3/2}\rho_0^2e^{-(\nu r^2/2)}$$
$$+2\frac{t_3}{6}\left[\frac{\pi}{3\nu}\right]^{3/2}\rho_0^3e^{-(2\nu r^2/3)} , \tag{3.2}$$

where p_z is the momentum per nucleon of either nucleus in the center of mass system. If we assume that the projectile nucleus is incident with an impact parameter b and that all motion of projectile and target is in a straight line parallel to the z axis, then the transverse momentum imparted to each nucleus is

$$P_\perp=\int F_\perp(\vec{b}+z\hat{k})dt=-\int\left[\frac{\partial V(\vec{b}+z\hat{k})}{\partial b}\right]dt . \tag{3.3}$$

We take the origin to be at the center of mass and assume that each nucleus is a sphere of radius R. We stay in the center of momentum reference frame and take the time t to be zero when the nuclei touch, i.e., when the projectile is at

$$z_0=-\left[R^2-\frac{b^2}{4}\right]^{1/2} .$$

If we assume that the projectile is incident with velocity u_0 and that for $t>0$ its velocity decreases exponentially,

$$u=u_0e^{-\beta t} \tag{3.4}$$

as suggested by the Fokker-Planck distribution, then its position for $t>0$ is given by

$$z=\frac{u_0}{\beta}(1-e^{-\beta t})-\left[R^2-\frac{b^2}{4}\right]^{1/2} . \tag{3.5}$$

The nuclei separate when

$$z=+\left[R^2-\frac{b^2}{4}\right]^{1/2}$$

which therefore provides us with the following relation for the collision time τ:

$$e^{-\beta\tau}=1-\frac{2\beta}{u_0}\left[R^2-\frac{b^2}{4}\right]^{1/2} \tag{3.6}$$

and

$$P_\perp=-\int_0^\tau\left[\frac{\partial V(\vec{b}+z\hat{k})}{\partial b}\right]dt . \tag{3.7}$$

TABLE I. The total transverse momentum in MeV/c imparted to each nucleus in Nb + Nb at 400 MeV (lab) for various impact parameters using the Skyrme II and III interactions.

b (fm)	Skyrme II ρ_0=0.17 fm^{-3}	0.25	0.30	Skyrme III 0.17	0.25	0.30
5	6517	11453	14708	2851	7204	10442
6	6662	10350	12430	2916	6039	8049
7	6389	8730	9758	2856	4792	5800
8	5740	6893	7161	2671	3654	3998
9	4748	5026	4858	2337	2659	2642
10	3345	3139	2833	1765	1714	1564

We can also estimate $e^{-\beta\tau}$ from

$$e^{-\beta\tau}=\frac{u_f}{u_0}=\frac{0.127}{0.455}=0.28\ , \tag{3.8}$$

where u_f is the final velocity of the projectile in the center of momentum system and the numerical values used previously correspond to the experimental situation in the 400 MeV Nb + Nb experiment.[2] Using the preceding results we can also estimate β and τ separately,

$$\beta=9\times10^{21}/\text{sec}\ ,\qquad \tau=1.4\times10^{-22}\ \text{sec}\ . \tag{3.9}$$

The total transverse momenta calculated in this manner are shown in Table I using parameters t_0, t_1, and t_3 corresponding to the Skyrme II and III interactions.

Since most of the transverse momentum will be imparted to the spectators rather than the whole nucleus, we note that the results given in Table I are compatible with the experimental observation that particles detected with projectile rapidity have a transverse momentum ≈ 50 MeV/c per nucleon. We also like to emphasize the important role played by the momentum dependent term in the Skyrme interaction in producing these momenta. This term accounts for the larger part of the difference between the results obtained with the Skyrme II and III interactions which have $t_1=585.6$ and 395.0 MeV fm^5, respectively. The repulsion produced by the density-dependent t_3 term previously mentioned is not able to produce such an effect. Compression effects are simulated by choosing different values for ρ_0 (with ν adjusted to ensure particle number conservation). It is evident that compression effects can be very important for intermediate impact parameters but here the model itself may not be very reliable.

B. Nuclear side splash

We now turn to estimating the flow angle (the side-splash effect). We assume that the particles that are detected come from four sources.

(1) The projectile spectators that continue to move with the projectile velocity $u_Z=u_0$ and acquire in addition a transverse velocity component $\vec{u}_\perp$ as a result of the bounceoff;

(2) the target spectators that move with $\vec{u}=-u_0\hat{k}-\vec{u}_\perp$, mirroring the projectile spectators;

(3) the projectile participants that move at different velocities with the nucleons close to the spectators moving essentially with a velocity $\vec{u}=u_0\hat{k}$ while those on the other end moving with $\vec{u}=u_0e^{-\beta\tau}\hat{k}$, thus leading to a velocity gradient

$$\frac{\partial u_3}{\partial x_1}\ \text{or}\ \frac{\partial u_3}{\partial x_2}\sim\frac{u_0-u_0e^{-\beta\tau}}{R-\frac{b}{2}}\ ; \tag{3.10}$$

(4) the target participants that mirror the projectile participants.

The number of participants (N_{part}) from each nucleus for the case of two identical nuclei can be determined from the approximate formula,[15]

$$N_{\text{part}}=A\left[\frac{3}{\sqrt{2}}(1-\gamma)^2-\left[\frac{3}{\sqrt{2}}-1\right](1-\gamma)^3\right]\ , \tag{3.11}$$

where A is the mass number of the nucleus, and $\gamma=b/2R$. Here b is the impact parameter and R is the radius of the nucleus. It is now possible to explicitly evaluate the kinetic flow tensor. Neglecting relativistic effects for the time being we get

$$\begin{aligned}F_{ij}&=\tfrac{1}{2}m\int d^3\vec{r}\int d^3\vec{v}\,v_iv_jf(\vec{r},\vec{v},t)\\&=\tfrac{1}{2}m\int d^3\vec{r}\int d^3\vec{v}(v_i-u_i)(v_j-u_j)f(\vec{r},\vec{v},t)\\&\quad+\tfrac{1}{2}m\int d^2\vec{r}\int d^3\vec{v}\,u_i(\vec{r},t)u_j(\vec{r},t)f(\vec{r},\vec{v},t)\ ,\end{aligned} \tag{3.12}$$

using

$$\begin{aligned}f(\vec{r},\vec{v},t)&=n(\vec{r},t)[\tilde{f}^{(0)}(\vec{v}-\vec{u},\vec{r},t)+\tilde{g}(\vec{v}-\vec{u},\vec{r},t)]\\&=n(\vec{r},t)[\tilde{f}^{(0)}(\vec{U},\vec{r},t)+\tilde{g}(\vec{U},\vec{r},t)]\ ;\\f^{(0)}&=n(\vec{r},t)\tilde{f}^{(0)};\quad g=n(\vec{r},t)\tilde{g}\ .\end{aligned} \tag{3.13}$$

The F_{ij} is

$$\begin{aligned}F_{ij}=&\tfrac{1}{2}m\int d^3\vec{r}\,n(\vec{r},t)\int d^3\vec{U}\,U_iU_j[\tilde{f}^{(0)}(\vec{U},\vec{r},t)+\tilde{g}(\vec{U},\vec{r},t)]\\&+\tfrac{1}{2}m\int d^3\vec{r}\,n(\vec{r},t)u_i(\vec{r},t)u_j(\vec{r},t)\int d^3\vec{U}[\tilde{f}^{(0)}(\vec{U},\vec{r},t)+\tilde{g}(\vec{U},\vec{r},t)]\ .\end{aligned} \tag{3.14}$$

Noting that

$$\int d^3\vec{U}\,\bar{f}^{(0)}(\vec{U},\vec{r},t)=1\ ,$$
$$\int d^3\vec{U}\,\bar{g}(\vec{U},\vec{r},t)=0\ , \qquad (3.15)$$

and

$$\int d^3U U_i U_j f^{(0)}(\vec{U},\vec{r},t)=\frac{\theta(\vec{r},t)}{m}\delta_{ij}\ , \qquad (3.16)$$

we get

$$F_{ij}=\tfrac{1}{2}m\int d^3\vec{r}\,n(\vec{r},t)\frac{\theta(\vec{r},t)}{m}\delta_{ij}+\tfrac{1}{2}m\int d^3\vec{r}\,n(\vec{r},t)u_i(\vec{r},t)u_j(\vec{r},t)+\tfrac{1}{2}m\int d^3\vec{r}\,n(\vec{r},t)\int d^3\vec{U}\,U_iU_j g(\vec{U},\vec{r},t)\ . \qquad (3.17)$$

The first term in this expression for F_{ij} is of thermal origin. It contributes equally to the three diagonal elements of F_{ij} and does not therefore affect the value of the flow angle. We therefore subtract it out to obtain the nonthermal part which we denote by F'_{ij}. In the third term, the integral over d^3U can be easily evaluated and we get:

(a) for the diagonal elements

$$\int d^3\vec{U}\,U_i^2 g(\vec{U},\vec{r},t)=-\frac{2\theta(\vec{r},t)}{3m^2}\tau_R\left[2\Lambda_{ii}-\sum_{k\neq i}\Lambda_{kk}\right]\ ; \qquad (3.18)$$

(b) for the off-diagonal elements

$$\int d^3\vec{U}\,U_iU_j g(\vec{U},\vec{r},t)=-\frac{2\theta(r,t)}{m^2}\tau_R\Lambda_{ij}\ . \qquad (3.19)$$

We now make some simplifying assumptions. We choose the three axis along the beam direction (i.e., the z axis). We assume that $\partial u_1/\partial x_1$, $\partial u_2/\partial x_2$, $\partial u_1/\partial x_2$, $\partial u_2/\partial x_1$, $\partial u_1/\partial x_3$, and $\partial u_2/\partial x_3$ are all negligible compared with $\partial u_3/\partial x_1$, $\partial u_3/\partial x_2$, and $\partial u_3/\partial x_3$. This allows us to set

$$\Lambda_{11}=\Lambda_{22}=\Lambda_{12}=\Lambda_{21}=0\ ,$$
$$\Lambda_{13}=\Lambda_{31}=\frac{m}{2}\frac{\partial u_3}{\partial x_1}\ ,$$
$$\Lambda_{23}=\Lambda_{32}=\frac{m}{2}\frac{\partial u_3}{\partial x_2}\ .$$

Note that the Λ_{ij}'s are functions of $\vec{r}$ and t; here we will take them to vanish (or to be negligible) in the spectator regions. Similarly

$$\Lambda_{33}=m\frac{\partial u_3}{\partial x_3}$$

is negligible in the spectator regions and has the value

$$m\,\partial u_3/\partial x_3=m\,(du_3/dt)/dx_3/dt=\frac{m}{u_3}\frac{du_3}{dt}=-\beta m$$

in the heart of the participant region where $u_3=u_0e^{-\beta t}$.

We are now in a position to evaluate F_{ij} explicitly in the spectators-participants model outlined previously,

$$F'_{11}=F'_{22}=\frac{\bar{\theta}}{3m}\tau_R N_{\text{part}}\bar{\Lambda}_{33}\ ,$$
$$F'_{33}=-\frac{2\bar{\theta}\tau_R}{3m}N_{\text{part}}\bar{\Lambda}_{33}+\tfrac{1}{2}mN_{\text{part}}\overline{\left[\frac{u_3^2}{1-\frac{u_3^2}{c^2}}\right]}+\tfrac{1}{2}m\frac{(A-N_{\text{part}})}{1-\frac{u_0^2}{c^2}}u_0^2\ , \qquad (3.20)$$
$$F_{12}=0\ ,$$
$$F_{13}=F_{31}=F_{23}=F_{32}=-\frac{\bar{\theta}}{m}\tau_R N_{\text{part}}\bar{\Lambda}_{13}+\tfrac{1}{2}m(A-N_{\text{part}})\frac{u_0u_\perp}{\sqrt{2}[1-(u_0^2/c^2)]}\ ,$$

where we have introduced the averaged quantities $\bar{\theta}$, $\bar{\Lambda}_{33}$, $\bar{\Lambda}_{13}$, and $\overline{\{u_3^2/[1-(u_3/c)^2]\}}$ with the averaging done over the participants. In the preceding expression the relativistic effects are included explicitly with the approximation that $u_\perp^2/2$ can be neglected as compared with u_0^2. In the following, we take

$$\bar{\theta}=40\text{ MeV}\ ,$$
$$\bar{\Lambda}_{33}=\beta m/2\ ,$$
$$\overline{\left[\frac{u_3^2}{1-(u_3^2/c^2)}\right]}=\frac{\tilde{u}^2}{1-(\tilde{u}/c)^2}\ , \qquad (3.21)$$
$$\tilde{u}=(u_0+u_0e^{-\beta\tau})/2\ ,$$

and we set $\tau_R=\tau$. We also take $u_0/c=0.4548$,

$$P_\perp=\frac{mu_\perp}{\sqrt{1-u_0^2/c^2}}=50\text{ MeV}/c\ ,$$

which are appropriate for the Nb + Nb experiment.

Table II summarizes the results of our calculation for the collective flow angle using the model and results described in this subsection. Experimentally, one finds

TABLE II. The flow angle for different impact parameters calculated in various approximations.

b (fm)	$\overline{\Lambda}_{33}=\beta m/2$ Relativistic corrections	$\overline{\Lambda}_{33}=\beta m/2$ No relativistic corrections	$\overline{\Lambda}_{33}=\beta m$ Relativistic corrections	$\overline{\Lambda}_{33}=\beta m$ No relativistic corrections
1	31.13	32.07	27.70	28.52
2	29.25	30.63	26.29	27.50
3	26.46	28.35	24.03	25.70
4	22.57	25.00	20.76	22.90
5	17.54	20.32	16.38	18.87
6	11.66	14.32	11.10	13.54

collective flow angles of the order of 35°. This angle is a little larger than those listed in Table II for medium impact parameters. At these impact parameters the nuclear viscosity term is important in determining the collective flow.

IV. CONCLUSIONS AND SUMMARY

Collective flow effects in heavy ion collisions were investigated. Two different effects were considered which were the bounceoff and side splash. The bounceoff effect is seen as a transverse momentum given to particles with velocity near the beam velocity. We associated the bounceoff with peripheral collisions and used a folded optical potential obtained from a Skyrme interaction to calculate the transverse momentum given to these particles. The momentum dependent terms in the Skyrme interaction are found to be important for the bounceoff; the repulsion produced by the density-dependent term is not enough to account for the repulsion. Compression effects can also play an important role in the bounceoff.

The side-splash effect is believed to come from semicentral parameters. We use linear transport theory to investigate this effect. Starting from a relaxation time approximation for the collision term in the Boltzmann equation, an analytic expression for the kinetic flow tensor is developed. Specifically, the kinetic flow tensor is related to the collective velocity field and to the pressure tensor. This approach thus allowed us to explicitly isolate two separate contributions to the kinetic flow tensor. The pressure tensor part of the kinetic flow tensor is then related to the nuclear viscosity. The two distinct contributions interfere destructively. A calculation of the Reynolds number shows that the motion is not turbulent. We therefore assume laminar flow to evaluate the side-splash effect. Reasonable agreement with data can be obtained for the collective flow angle.

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