Sub-barrier fusion of heavy ions

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Sub-barrier fusion of heavy ions

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Sub-barrier fusion of heavy ions is studied in a model that uses an l-dependent nuclear potential to reproduce the experimentally observed enhancement of the fusion cross sections. The parameters of the potential are found to scale with the reduced radii of the systems investigated, which points to dynamic deformations and/or neck formation as being the possible cause of the enhancement.

[NUCLEAR REACTIONS Sub-barrier fusion, l-dependent nuclear potential.]

Measurements of heavy-ion fusion cross sections at sub-barrier energies have generated a lot of interest over the last few years because of various suggestions as to what they can reveal about the nucleus-nucleus interaction at these energies. These measurements indicate a considerable and universal enhancement of these fusion cross sections over the values predicted by the traditional model of penetration through a one-dimensional energy-independent local potential barrier, although this model is successful in accounting for the cross sections at higher energies. Reference 1 provides a review of the experimental results and the barrier penetration model, while more recent measurements are given in Refs. 2 and 3. Several explanations have been put forward for the above-mentioned enhancement which, at the lowest energies, can be as large as three orders of magnitude. This enhancement has been attributed to static deformations of the nuclei involved,[4] zero-point fluctuations of the nuclear surfaces,[5] dynamic deformations,[6] nucleon transfer,[7] and neck formation.[1] It is still an open question, however, as to which of the above effects (or which combination of effects) can account for the total magnitude of the enhancement.

Although the one-dimensional barrier penetration model is definitely inadequate,[1,8] the enhanced cross sections can be adequately described by penetration through an l-dependent barrier corresponding to an l-dependent nuclear potential. This l dependence can be regarded as mere phenomenology but it tends to support the idea that the enhancement is due to dynamical deformations and neck formation (see below). In the following, we will atempt to fit the experimental cross sections published in Refs. 2 and 3 with such an l-dependent potential.

The effective potential for the radial motion of the lth partial wave of a system of two nuclei is then taken to be

$$V^l_{\mathrm{eff}}(r)=\frac{Z_1Z_2e^2}{r}+\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}+V^l_{\mathrm{nuc}}(r)\ , \qquad (1)$$

where the l-dependent nucleus-nucleus interaction potential is parametrized as follows:

$$V^l_{\mathrm{nuc}}(r)=V_N\Big/\left[1+\exp\frac{r-R_V-\Delta R_l}{a_V}\right]\ , \qquad (2)$$

with

$$V_N=-67\ \mathrm{MeV}\ ,$$
$$a_V=0.574\ \mathrm{fm}\ , \qquad (3)$$
$$R_V=r_V(A_1^{1/3}+A_2^{1/3})\ ,$$

and ΔR_l is an l-dependent shift in the range of $V^l_{\mathrm{nuc}}(r)$ to be determined by fitting the data. For each dinuclear system, r_V is chosen such that the effective potential (with $\Delta R_l=0$) reproduces the height V_B of the "classical" barrier required to fit the measured fusion cross sections for energies above the barrier. These data are well described by the classical sharp cutoff formula

$$\sigma_f=\pi R_B^2(1-V_B/E)\ , \qquad (4)$$

from which the height of the s-wave barrier V_B can be accurately determined.[2] Here R_B is the position of the barrier. There is no need to vary V_N in Eq. (2) because at the values of r needed in the present calculations the r dependence of $V^l_{\mathrm{nuc}}(r)$ is essentially exponential [i.e., the exponential in the denominator of Eq. (2) is $\gg 1$]. No attempt was made, however, at a simultaneous reproduction of the barrier position R_B (this will entail the simultaneous variation of two parameters, e.g., r_V and a_V, for each system) because of the large experimental uncertainty in the position of the barrier due to a 20% systematic error in determining the absolute value of the cross section.[2] The fact that the barrier height, but not its position, is reproduced means that the calculated cross section will differ from the reported experimental cross section by the trivial normalization factor

$$N=\left[R_B(\mathrm{theor})/R_B(\mathrm{expt.})\right]^2\ , \qquad (5)$$

which is the square of the ratio of the theoretical and experimental estimates for the position of the classical barrier. Apart from this, our choice of r_V will ensure that the cross section above the barrier is correctly reproduced. The range R_V of the nuclear potential is then shifted by a value ΔR_l for each partial wave according to the ansatz

$$\Delta R_l=2\Delta R_0\Big/\left[1+\exp\frac{l}{\delta}\right]\ . \qquad (6)$$

The shifted potentials for a given pair of ΔR_0 and δ are used to calculate the barrier height V_{Bl} and its position R_{Bl} for each partial wave and then to calculate the fusion cross section via the Hill-Wheeler penetrability formula[9]

$$\sigma_f=\pi\lambda\!\!\!^{-2}\sum_{l=0}^{\infty}(2l+1)\left[1+\exp\frac{2\pi(V_{Bl}-E)}{\hbar\omega_l}\right]^{-1}\ , \qquad (7)$$

where $\hbar\omega_l$ is related to the curvature of the effective poten-

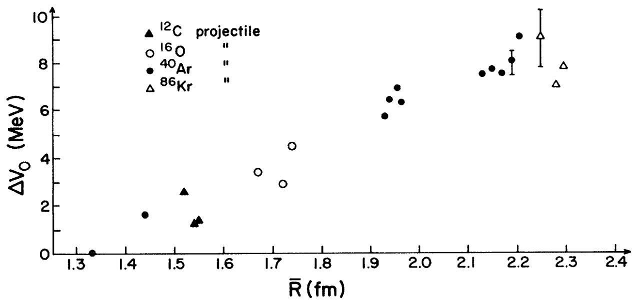

FIG. 1. ΔV_0 vs $\bar{R}$ for the various systems considered which are (arranged according to increasing $\bar{R}$): ^{40}Ar on ^{12}C and ^{16}O; ^{12}C on ^{110}Pd, ^{124}Sn, and ^{130}Te; ^{16}O on ^{110}Pd, ^{144}Sm, and ^{154}Sm; ^{40}Ar on ^{58}Ni, ^{60}Ni, ^{62}Ni, ^{64}Ni, ^{104}Pd, ^{110}Pd, ^{118}Sn, ^{124}Sn, and ^{130}Te; ^{86}Kr on ^{58}Ni, ^{62}Ni, and ^{64}Ni. The error bars for ^{40}Ar + ^{124}Sn and ^{86}Kr + ^{58}Ni reflect the experimental (statistical) uncertainty in the data. The errors for the other points are not shown but are of similar magnitude.

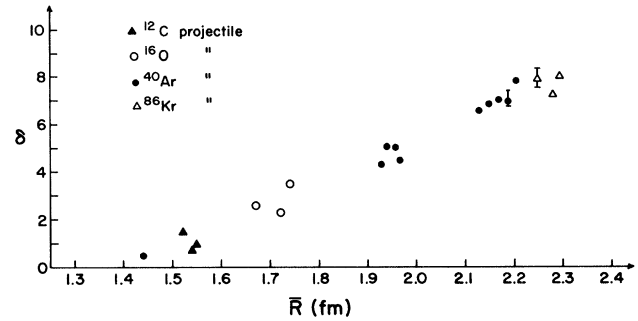

FIG. 2. δ vs $\bar{R}$ for the same systems as shown in Fig. 1. Note that since $\Delta V_0 = 0$ for ^{40}Ar + ^{12}C it is not meaningful to specify a δ for such a system.

tial at the barrier

$$\hbar\omega_l = \left| \frac{\hbar^2}{\mu} \frac{d^2 V_{\rm eff}^l(r)}{dr^2} \right|_{r=R_{Bl}} . \quad (8)$$

The values of ΔR_0 and δ are then varied until the best fit is obtained for the sub-barrier experimental cross section.

The resulting best-fit parameters are shown in Figs. 1 and 2 for several systems whose cross sections have been reported in Ref. 2. In both figures, the parameters are plotted versus the mean curvature or "reduced" radius:[10] $\bar{R} = C_1 C_2/(C_1 + C_2)$ of the dinuclear system. Instead of giving the values of ΔR_0, we show in Fig. 1 the value of ΔV_0 which is the amount by which the *s*-wave potential barrier is lowered when the range of the nuclear potential is shifted by ΔR_0. The change in the height of the barrier is more meaningful since it is the barrier height and not its position that goes into the Hill-Wheeler formula. Moreover, as noted earlier, the position of the barrier is not well determined. In Fig. 2, we show the values of δ for the various systems investigated. The magnitude of δ is a measure of the range of partial waves for which the barrier is lowered (as compared with the classical barrier). It is this lowering of the barrier for small l that leads to the enhancement of the cross section at sub-barrier energies. The cross sections at energies above the barrier remain unaffected, however, because then the magnitude of the cross section is mainly determined by the large contribution from high angular momentum partial waves.

Figures 1 and 2 indicate an almost linear dependence of ΔV_0 and δ on $\bar{R}$. The fact that the reduced radius, which reflects the size of the opposing surfaces, provides a good scaling for both parameters indicates the presence of shape distortions or neck formation, as already noted in Ref. 2. This point of view is supported further by the observation that the values of ΔV_0 shown in Fig. 1 are in agreement with those obtained in extended-liquid-drop model calculations.[11] The l dependence of the nuclear potential would then come about because the fast tangential motion for high l prevents the formation of the neck.

In Fig. 3, we show the fusion excitation function for the system ^{40}Ar + ^{122}Sn. The data points are from Ref. 3 and they provide a further test for the present model since they include very low energy measurements well below the classical barrier where the fusion cross section is in the microbarn level. In contrast, the measurements reported in Ref. 2 are limited to cross sections $\gtrsim 1$ mb. The procedure followed in fitting the data is the same as outlined above except that only data points corresponding to $\sigma_f > 10$ mb are used in obtaining the best-fit parameters. It is then found that these parameters are able to reproduce the experimental cross section down to the microbarn level.

In conclusion, we have demonstrated that an l-dependent nuclear potential is able, within the barrier penetration picture, to reproduce all the interesting features of sub-barrier fusion excitation functions for 20 different dinuclear systems. The reduced radius $\bar{R}$ is found to provide a good scaling to the parameters determined by fitting these excitation functions. This is interpreted as indicating dynamic de-

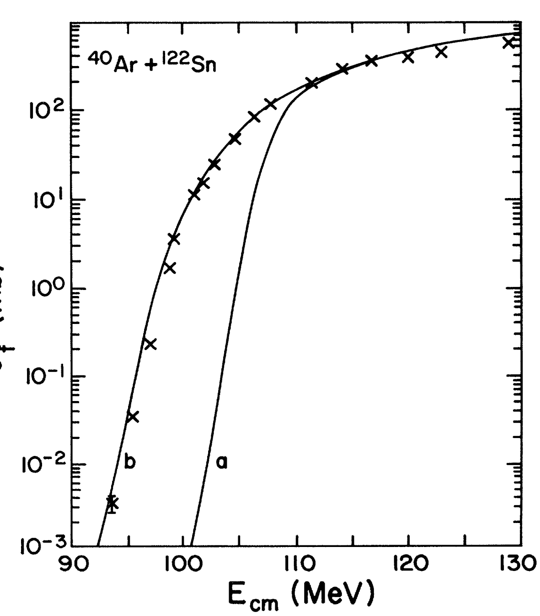

FIG. 3. The fusion excitation function for ^{40}Ar + ^{122}Sn. The data and curve a are from Ref. 3. Curve a is the result of a one-dimensional barrier-penetration calculation. Curve b is obtained from the present model with the parameters determined by fitting the excitation function for $\sigma_f > 10$ mb.

formations or neck formation. This idea is supported by the "global"[2] character of the enhancement in the fusion cross section.

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