Deconfinement in the Quark Meson Coupling Model

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Deconfinement in the quark meson coupling model

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The quark meson coupling model which describes nuclear matter as a collection of nonoverlapping MIT bags interacting by the self-consistent exchange of scalar and vector mesons is used to study nuclear matter at finite temperature. In its modified version, the density dependence of the bag constant is introduced by a direct coupling between the bag constant and the scalar mean field. In the present work, the coupling of the scalar mean field with the quarks is considered exactly through the solution of the Dirac equation. Our results show that a phase transition takes place at a critical temperature around 200 MeV in which the scalar mean field takes a nonzero value at zero baryon density. Furthermore, it is found that the bag constant decreases significantly when the temperature increases above this critical temperature indicating the onset of quark deconfinement. [S0556-2813(99)08102-9]

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I. INTRODUCTION

The quark meson coupling model (QMC), initially proposed by Guichon [1], describes nuclear matter as a collection of nonoverlapping MIT bags interacting by the self-consistent exchange of scalar σ and vector ω mesons in the mean-field approximation [2–4]. The scalar σ meson is supposed to simulate the exchange of correlated pairs of pions and may represent a very broad resonance observed in $\pi\pi$ scattering, while the vector ω meson is identified with the actual meson having a mass of about 780 MeV. The QMC model is a simple extension of the Walecka model [5–7] except that the meson fields are coupled directly to the quarks themselves rather than to the nucleons as in the Walecka model. This simple model was later refined by including the nucleonic fermi motion and the center-of-mass correction [8] to the bag energy and applied to a variety of problems [9,10]. The QMC model thus incorporates explicitly the quark degrees of freedom and this has nontrivial consequences [9].

In the modified QMC model (MQMC) it has been suggested by Jin and Jennings [3,4] that a reduction of the bag constant in nuclear matter relative to its free-space value may be essential for the success of relativistic nuclear phenomenology and that it may play an important role in low and medium-energy nuclear physics such as understanding the EMC effect [11]. The density dependence of the bag constant is introduced in the present work by coupling it to the scalar meson field as suggested in Ref. [3] where this coupling is motivated by invoking the nontopological soliton model for the nucleon. In this model a scalar soliton field provides the confinement of the quarks. This effect of the soliton field is, roughly speaking, mimicked by the introduction of the bag constant in the bag model. When a nucleon soliton is placed in a nuclear environment, the scalar soliton field interacts with the scalar mean field. It is thus reasonable to couple the bag constant to the scalar mean field.

It was found by Jin and Jennings that when the bag constant is significantly reduced in the nuclear medium with respect to its free-space value, large cancelling isoscalar Lorentz scalar and vector potentials for the nucleon in nuclear matter emerge naturally. Such potentials are comparable to those suggested by relativistic nuclear phenomenology and finite density QCD sum rules [12]. Moreover, Müller and Jennings [13] have shown that the MQMC energy functional is equivalent to that obtained in purely hadronic models with a general nonlinear scalar potential and that it provides a realistic description of nuclear matter and finite nuclei.

Recently, Panda *et al.* [14] have studied nuclear matter at finite temperature using the MQMC model. They determined the scalar mean field by minimizing the grand thermodynamical potential with respect to the σ field and using the self-consistency condition which relates the vector mean field to the baryon density. They found that the nucleon properties at finite temperature and/or nonzero baryon density are appreciably different from their zero-temperature vacuum values.

Our present work is essentially an extension of the work of Panda *et al.* [14]. We intend to use the MQMC model at finite temperature and to take the medium dependence of the bag parameters into account. We, however, will attempt to solve the self-consistency condition for the σ field exactly by taking into consideration the full coupling of the scalar mean field to the internal quark structure by means of the solution of the pointlike Dirac equation with the required boundary condition of confinement at the surface of the bag as suggested by Refs. [2,3]. This was not done exactly in the solution of the self-consistency condition for the σ field by Panda *et al.*

The outline of the paper is as follows. In Sec. II, we present the MQMC model for nuclear matter at finite temperature, together with the details of the self-consistency condition for the scalar mean field. In Sec. III, we discuss our results and present our conclusions.

II. MQMC MODEL FOR NUCLEAR MATTER AT FINITE TEMPERATURE

The MQMC model at finite temperature is described in detail in Ref. [14]. We give here the essential equations necessary for the present calculations. The quark field $\psi_q(\vec{r},t)$ inside the bag satisfies the Dirac equation

$$[i\gamma^\mu\partial_\mu-(m_q^0-g_\sigma^q\sigma)-g_\omega^q\omega\beta]\psi_q(\vec{r},t)=0. \quad (2.1)$$

Here the single-particle quark and antiquark energies in units of R^{-1} are given as

$$\epsilon_\pm^{n\kappa}=\Omega^{n\kappa}\pm g_\omega^q\omega R, \quad (2.2)$$

where

$$\Omega^{n\kappa}=\sqrt{x_{n\kappa}^2+R^2m_q^{*2}}, \quad (2.3)$$

and $m_q^*=m_q^0-g_\sigma^q\sigma$ is the effective quark mass. The boundary condition at the bag surface is given by

$$i\gamma\cdot\hat{n}\,\psi_q^{n\kappa}=\psi_q^{n\kappa}, \quad (2.4)$$

which determines the quark momentum $x_{n\kappa}$ in the state characterized by specific values of n and κ. For given values of the bag radius R and the scalar field σ, the quark momentum $x_{n\kappa}$ is obtained from Eq. (2.4). The quark chemical potential μ_q assuming that there are three quarks in the nucleon bag is determined through

$$n_q=3=3\sum_{n\kappa}\left[\frac{1}{e^{(\epsilon_+^{n\kappa}/R-\mu_q)/T}+1}-\frac{1}{e^{(\epsilon_-^{n\kappa}/R+\mu_q)/T}+1}\right]. \quad (2.5)$$

The total energy from the quarks and antiquarks is

$$E_{\rm tot}=3\sum_{n\kappa}\frac{\Omega^{n\kappa}}{R}\left[\frac{1}{e^{(\epsilon_+^{n\kappa}/R-\mu_q)/T}+1}+\frac{1}{e^{(\epsilon_-^{n\kappa}/R+\mu_q)/T}+1}\right]. \quad (2.6)$$

The bag energy is given by

$$E_{\rm bag}=E_{\rm tot}-\frac{Z}{R}+\frac{4\pi}{3}R^3B(\sigma), \quad (2.7)$$

where $B(\sigma)$ is the bag parameter. In the simple QMC model, the bag parameter B is taken as B_0 corresponding to its value for a free nucleon. The medium effects are taken into account in the MQMC model by the following ansatz for the bag parameter [14,3]:

$$B=B_0\exp\left(-\frac{4g_\sigma^B\sigma}{M_N}\right) \quad (2.8)$$

with g_σ^B as an additional parameter. The spurious center-of-mass momentum in the bag is subtracted to obtain the effective nucleon mass [8]

$$M_N^*=\sqrt{E_{\rm bag}^2-\langle p_{\rm c.m.}^2\rangle} \quad (2.9)$$

where

$$\langle p_{\rm c.m.}^2\rangle=\frac{\langle x^2\rangle}{R^2} \quad (2.10)$$

and

$$\langle x^2\rangle=3\sum_{n\kappa}x_{n\kappa}^2\left[\frac{1}{e^{(\epsilon_+^{n\kappa}/R-\mu_q)/T}+1}+\frac{1}{e^{(\epsilon_-^{n\kappa}/R+\mu_q)/T}+1}\right]. \quad (2.11)$$

The bag radius R is obtained through the minimization of the nucleon mass with respect to the bag radius

$$\frac{\partial M_N^*}{\partial R}=0. \quad (2.12)$$

The total-energy density at finite temperature T and at finite baryon density ρ_B is

$$\epsilon=\frac{\gamma}{(2\pi)^3}\int d^3k\sqrt{k^2+M_N^{*2}}(f_B+\bar{f}_B)+\frac{g_\omega^2}{2m_\omega^2}\rho_B^2+\frac{1}{2}m_\sigma^2\sigma^2, \quad (2.13)$$

where $\gamma=4$ is the spin-isospin degeneracy factor and f_B and $\bar{f}_B$ are the Fermi-Dirac distribution functions for the baryons and antibaryons

$$f_B=\frac{1}{e^{(\epsilon^*-\mu_B^*)/T}+1}, \quad (2.14)$$

and

$$\bar{f}_B=\frac{1}{e^{(\epsilon^*+\mu_B^*)/T}+1}, \quad (2.15)$$

with $\epsilon^*=\sqrt{k^2+M_N^{*2}}$ the effective nucleon energy and $\mu_B^*=\mu_B-g_\omega\omega$ the effective baryon chemical potential. The chemical potential for a given density ρ_B is determined by

$$\rho_B=\frac{\gamma}{(2\pi)^3}\int d^3k(f_B-\bar{f}_B), \quad (2.16)$$

where

$$\omega=\frac{g_\omega}{m_\omega^2}\rho_B. \quad (2.17)$$

The pressure is the negative of the grand thermodynamic potential density and is given by

$$P=\frac{1}{3}\frac{\gamma}{(2\pi)^3}\int d^3k\frac{k^2}{\epsilon^*}(f_B+\bar{f}_B)+\frac{1}{2}m_\omega^2\omega^2-\frac{1}{2}m_\sigma^2\sigma^2. \quad (2.18)$$

The scalar mean field σ is determined through the minimization of the thermodynamic potential or the maximizing of the pressure $\partial P/\partial\sigma=0$ [14]. The pressure depends explicitly on the scalar mean field σ through the last term in Eq. (2.18). It also depends on the nucleon effective mass M_N^* which in turn also depends on σ. If we write the pressure as a function of M_N^* and σ [3,2], the extremization of $P(M_N^*,\sigma)$ with respect to the scalar mean field σ can be written as

$$\frac{\partial P}{\partial \sigma} = \left(\frac{\partial P}{\partial M_N^*}\right)_{\mu_B, T} \frac{\partial M_N^*}{\partial \sigma} + \left(\frac{\partial P}{\partial \sigma}\right)_{M_N^*} = 0, \qquad (2.19)$$

where

and

$$\left(\frac{\partial P}{\partial \sigma}\right)_{M_N^*} = -m_\sigma^2 \sigma, \qquad (2.20)$$

$$\left(\frac{\partial P}{\partial M_N^*}\right)_{\mu_B, T} = -\frac{\gamma}{3}\frac{1}{(2\pi)^3}\int d^3k \frac{k^2}{\epsilon^{*2}}\frac{M_N^*}{\epsilon^*}[f_B + \bar{f}_B] - \frac{\gamma}{3}\frac{1}{(2\pi)^3}\frac{1}{T}\int d^3k \frac{k^2}{\epsilon^*}\frac{M_N^*}{\epsilon^*}[f_B(1-f_B) + \bar{f}_B(1-\bar{f}_B)]$$

$$-\frac{\gamma}{3}\frac{1}{(2\pi)^3}\frac{1}{T}g_\omega\left(\frac{\partial\omega}{\partial M_N^*}\right)_{\mu_B,T}\int d^3k \frac{k^2}{\epsilon^*}[f_B(1-f_B) - \bar{f}_B(1-\bar{f}_B)] + m_\omega^2\omega\left(\frac{\partial\omega}{\partial M_N^*}\right)_{\mu_B,T}. \qquad (2.21)$$

Since the baryon chemical potential μ_B and temperature are treated as input parameters, the variation of the vector mean field ω with respect to the effective nucleon mass M_N^* at a given value of the baryon density ρ_B reads

$$\left(\frac{\partial\omega}{\partial M_N^*}\right)_{\mu_B,T} = -\frac{g_\omega/m_\omega^2[\gamma/(2\pi)^3](1/T)\int d^3k(M_N^*/\epsilon^*)[f_B(1-f_B) - \bar{f}_B(1-\bar{f}_B)]}{1+g_\omega^2/m_\omega^2[\gamma/(2\pi)^3](1/T)\int d^3k[f_B(1-f_B) + \bar{f}_B(1-\bar{f}_B)]}. \qquad (2.22)$$

The coupling of the scalar mean field σ with the quarks in the nonoverlapping MIT bag through the solution of the pointlike Dirac equation should be taken into account to satisfy the self-consistency condition. This constraint is essential to obtain the correct solution of the scalar mean field σ. The differentiation of the effective nucleon mass M_N^* with respect to σ gives

$$\frac{\partial M_N^*}{\partial \sigma} = \frac{E_{\text{bag}}(\partial E_{\text{bag}}/\partial\sigma) - 1/2(1/R^2)(\partial\langle x^2\rangle/\partial\sigma)}{M_N^*}, \qquad (2.23)$$

where

$$\frac{\partial E_{\text{bag}}}{\partial \sigma} = \sum_{n\kappa}\frac{\partial E_{\text{bag}}}{\partial\Omega_q^{n\kappa}}\frac{\partial\Omega_q^{n\kappa}}{\partial\sigma} + \left(\frac{\partial E_{\text{bag}}}{\partial\sigma}\right)_{\{\Omega_q^{n\kappa}\}}, \qquad (2.24)$$

and

$$\frac{\partial\langle x^2\rangle}{\partial \sigma} = \sum_{n\kappa}\frac{\partial\langle x^2\rangle}{\partial\Omega_q^{n\kappa}}\frac{\partial\Omega_q^{n\kappa}}{\partial\sigma} + \left(\frac{\partial\langle x^2\rangle}{\partial\sigma}\right)_{\{\Omega_q^{n\kappa}\}}. \qquad (2.25)$$

The $\partial\Omega_q^*/\partial\sigma$ depends on $x_{n\kappa}$. Its evaluation can be obtained from the solutions of the pointlike Dirac equation for the quarks which satisfy the required boundary condition on the surface of the bag [2,3]. In our case it reads [2,3]

$$\left(\frac{\partial\Omega_q^{n\kappa}}{\partial\sigma}\right) = -g_\sigma^q R\langle\bar{\psi}^{n\kappa}|\psi^{n\kappa}\rangle. \qquad (2.26)$$

III. RESULTS AND DISCUSSIONS

We have studied nuclear matter at finite temperature using the modified quark meson coupling model which takes the medium dependence of the bag into account. We choose a direct coupling of the bag constant to the scalar mean field σ in the form given in Eq. (2.8). The bag parameters are taken as those adopted by Jin and Jennings [3] where $B_0^{1/4} = 188.1$ MeV and $Z_0 = 2.03$ are chosen to reproduce the free nucleon mass M_N at its experimental value 939 MeV and bag radius $R_0 = 0.60$ fm. The current quark mass m_q is taken equal to zero. For $g_\sigma^q = 1$, the values of the vector meson coupling and the parameter g_σ^B as fitted from the saturation properties of nuclear matter, are given as $g_\omega^2/4\pi = 5.24$ and $g_\sigma^{B2}/4\pi = 3.69$.

We first solve Eqs. (2.16) and (2.17) for given values of temperature and density ρ_B to determine the baryon chemical potential μ_B. This constraint is given in terms of the effective nucleon mass M_N^* which depends on the bag radius R, the quark chemical potential μ_q, and the mean field σ. For given values of σ and ω the bag radius and the quark chemical potential μ_q are obtained using the self-consistency conditions Eqs. (2.12) and (2.5), respectively. The pressure is evaluated for specific values of temperature and μ_B which now becomes an input parameter. We then determine the value of σ by using the maximization condition given in Eq. (2.19). This method is analogous to the method used by Saito [2] and Jin [3] for the zero-temperature case. It takes into account the coupling of the quarks with the scalar mean field in the frame of the solution of the pointlike Dirac equation exactly. It differs from the minimization method used in Ref. [14].

Figure 1 displays various pressure isotherms vs the baryon density. The pressure has the usual trend of increas-
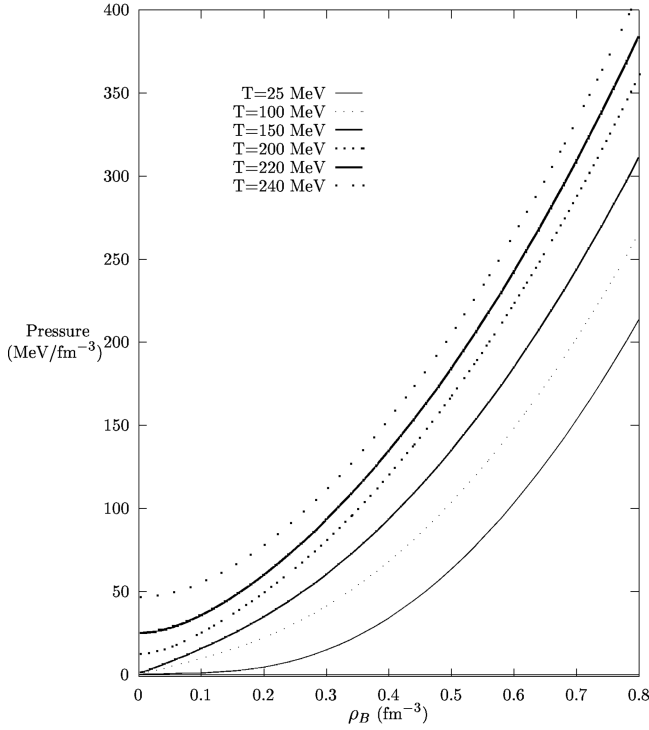
FIG. 1. The pressure for nuclear matter as a function of the baryon density ρ_B for various values of temperature.

ing with temperature and density. It is important to note that the pressure attains a nonzero value at zero baryon density above a critical temperature $T_c \simeq 200$ MeV. This occurs because the scalar mean field σ attains a nonzero value at zero baryon density $\rho_B = 0$ (see discussion concerning Fig. 2 below), as was also observed in the Walecka [7] model where it leads to a sharp fall in the effective nucleon mass at T_c. This rapid fall of M_N^* with increasing temperature resembles a phase transition when the system becomes a dilute gas of baryons in a sea of mesons and baryon-antibaryon pairs.

In Fig. 2, we display the scalar mean field σ as a function of the baryon density for various temperatures. Figure 2(a) indicates that the value of σ initially decreases with increasing temperature for temperatures less than 200 MeV. However, by the time the temperature reaches 150 MeV there are indications of an increase in σ at very low baryon densities with a nonzero value at $\rho_B = 0$. For still higher temperatures, as can be seen in Fig. 2(b), the situation is more dramatic with the value of σ increasing with temperature for all values of ρ_B. This is an indication of a phase transition to a system of baryon-antibaryon pairs at very low densities as mentioned above.

The density and temperature dependence of the baryon effective mass M_N^* is shown in Fig. 3. For low baryon density ρ_B, as the temperature is increased, M_N^* first increases slightly and then decreases rapidly for $T = 200$ MeV for densities less than about 0.2 fm^{-3}. This rapid decrease of M_N^* with increasing temperature resembles a phase transition at a high temperature and low density, when the system becomes a dilute gas of baryons in a sea of baryon-antibaryon pairs [7]. This behavior is consistent with the Walecka model [7] and resolves the contradiction that appeared in the earlier calculations. Below the critical temperature the effective
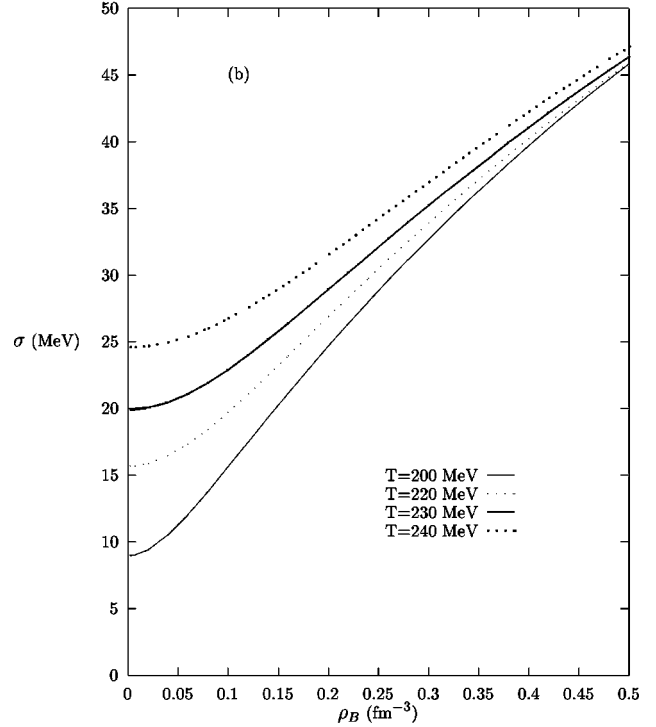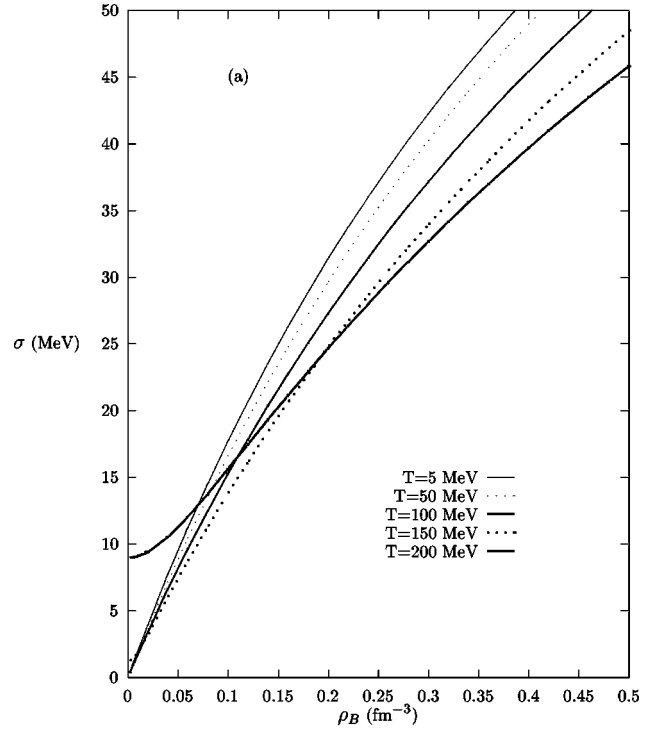

FIG. 2. The mean scalar field σ as a function of the baryon density ρ_B, (a) for different temperatures up to $T = 200$ MeV, (b) for different temperatures above $T = 200$ MeV.

mass grows with temperature. Above the critical temperature, σ increases with temperature thus reducing the nucleon effective mass M_N^*.

Finally, we display the density dependence of the bag constant for different values of the temperature in Fig. 4. The bag constant as shown in Fig. 4(a), grows with temperature for temperatures less than 200 MeV (except at densities smaller than 0.1 fm^{-3} where B starts to decrease for tem-
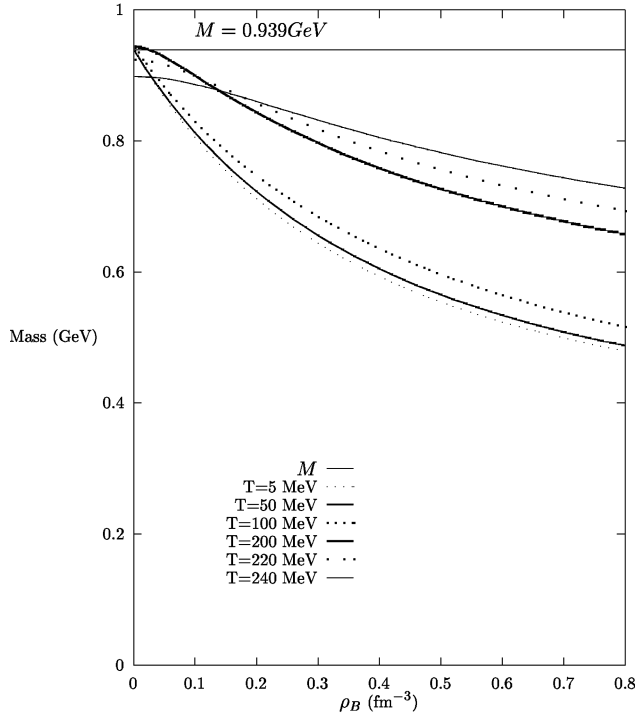
FIG. 3. The effective nucleon mass M_N^* for nuclear matter as a function of the baryon density ρ_B for different values of temperature.

peratures greater than 150 MeV). However, the situation is completely reversed after the phase transition takes place. This is displayed in Fig. 4(b), where the bag constant displays a dramatic decrease with temperature for all densities at temperatures greater than 200 MeV. This indicates the onset of quark deconfinement above the critical temperature: at high enough temperature and/or baryon density there is a phase transition from the baryon-meson phase to the quark-gluon phase. These results are qualitatively similar to those obtained by Serot and Walecka [6] who, however, have to utilize different equations of state for the two phases. For the baryon-meson phase, they use a quantum hadrodynamics (i.e., Walecka Model) equation of state, while for the quark-gluon phase they use the equation of an ideal relativistic gas of massless quarks and gluons inside a bag. They neglect interactions in the quark-gluon phase in order to be consistent with asymptotic freedom. The boundary between the two phases is determined by requiring them to be in thermal, mechanical, and chemical equilibrium. Serot and Walecka obtained a transition temperature of 125.6 MeV at zero baryon density. It is interesting that in the present work the bag constant at zero baryon density starts to decrease with temperature for temperatures greater than about 150 MeV, as can be seen by examining Fig. 4(b). Lattice QCD results [15] also indicate that the phase transition at $\rho_B = 0$ occurs at a temperature of about 140 MeV.

In conclusion, because the MQMC model uses the quark degrees of freedom explicitly it has allowed us, despite its limitations and shortcomings, to investigate the quark deconfiment phase transition using only one equation of state which would not have been possible if only the nucleonic degrees of freedom are used. It remains to be seen if these calculations can also be carried out with more sophisticated
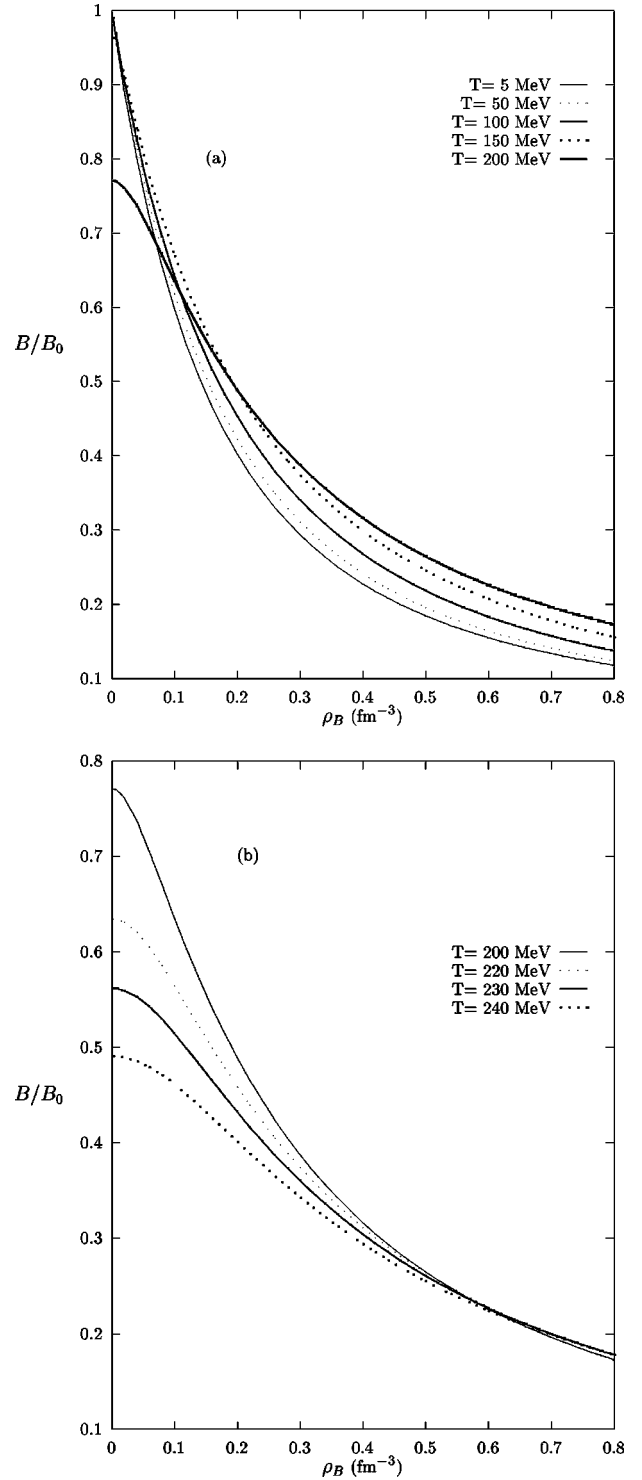

FIG. 4. The bag constant versus the baryon density ρ_B, (a) for different temperatures up to $T = 200$ MeV, (b) for different temperatures above $T = 200$ MeV.

nucleonic models and ultimately corroborated by lattice QCD calculations.

[1] P. A. M. Guichon, Phys. Lett. B **200**, 235 (1988).

[2] K. Saito and A. W. Thomas, Phys. Lett. B **327**, 9 (1994).

[3] X. Jin and B. K. Jennings, Phys. Rev. C **54**, 1427 (1996).

[4] X. Jin and B. K. Jennings, Phys. Lett. B **374**, 13 (1996).

[5] J. D. Walecka, Ann. Phys. (N.Y.) **83**, 491 (1974); Phys. Lett. B **59**, 109 (1975).

[6] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).

[7] R. J. Furnstahl and B. D. Serot, Phys. Rev. C **41**, 262 (1990).

[8] S. Fleck, W. Bentz, K. Shimizu, and K. Yazaki, Nucl. Phys. **A510**, 731 (1990).

[9] K. Saito and A. W. Thomas, Phys. Lett. B **335**, 17 (1994); **363**, 157 (1995); Phys. Rev. C **51**, 2757 (1995); **52**, 2789 (1995); K. Saito, K. Tsushima, and A. W. Thomas, Nucl. Phys. **A609**, 339 (1996).

[10] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. Lett. **67**, 961 (1991).

[11] EMC Collaboration, J. J. Aubert *et al.*, Phys. Lett. **123B**, 275 (1983); M. Arneodo, Phys. Rep. **240**, 301 (1994).

[12] T. D. Cohen, R. J. Furnstahl, D. K. Griegel, and X. Jin, Prog. Part. Nucl. Phys. **35**, 221 (1995).

[13] H. Müller and B. K. Jennings, Nucl. Phys. **A626**, 966 (1997); nucl-th/9807031.

[14] P. K. Panda, A. Mishra, J. M. Eisenberg, and W. Greiner, Phys. Rev. C **56**, 3134 (1997).

[15] S. Gottlieb, U. M. Heller, A. D. Kennedy, S. Kim, J. B. Kogut, C. Liu, R. L. Renken, D. K. Sinclair, R. L. Sugar, D. Toussaint, and K. C. Wang, Phys. Rev. D **55**, 6852 (1997).