Iterative Linear Approach for Nonlinear Nonhomogenous Stochastic Pavement Management Models

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Iterative Linear Approach for Nonlinear Nonhomogenous Stochastic Pavement Management Models

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Abstract: An iterative linear stochastic pavement management model is proposed that deploys a nonhomogenous discrete-time Markov chain for predicting the future pavement conditions for a given pavement network. A nonhomogenous transition matrix is constructed to incorporate both the pavement deterioration rates and improvement rates. The pavement deterioration rates are simply the transition probabilities associated with the deployed pavement states. The improvement rates are mainly the maintenance and rehabilitation variables representing the deployed maintenance and rehabilitation actions. A decision policy is formulated to identify the optimal set of maintenance and rehabilitation actions and their respective timings, and to provide the optimal level of maintenance and rehabilitation funding over an analysis period. The nonhomogenous Markov chain allows for a distinct maintenance and rehabilitation plan (matrix) for each time interval (transition). However, the total number of maintenance and rehabilitation variables will substantially increase depending on the length of the deployed analysis period. The resulting optimum model is associated with a nonlinearity order that is equal to the number of time intervals within the specified analysis period. Solving a nonlinear model with a large number of variables is a very complex task. Alternatively, instead of solving a single nonlinear problem, a series of linear problems are formulated and iteratively solved wherein the optimal solution for one problem becomes the input for the next one. The sample results obtained from the iterative linear approach indicate the effectiveness of the proposed stochastic management model in predicting future pavement conditions.

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Introduction

Several pavement management models have been developed in the last two decades with the main objective of yielding an optimum maintenance and rehabilitation (M&R) plan for a given pavement network. There are two main components for any pavement management model to be reliable, namely, an effective performance prediction model and an efficient decision-making policy. The pavement management problem is then formulated as an optimization program with the M&R variables representing the various deployed M&R actions. The optimal solution defines the amount and type of M&R works to be applied to a given pavement network. Several developed pavement management models have defined the M&R variables to represent proportions of pavement improvements (Grivas et al. 1993; Chen et al. 1996; Liu and Wang 1996; Abaza and Ashur 1999; Abaza et al. 2004). While several of the developed pavement management models applied different approaches to achieve similar objectives, the main obstacle remains the ability to efficiently solve the formulated optimum model which becomes more challenging as the pavement network size increases (Haas et al. 1994; Shahin 1994; Abaza and Ashur 1999; Pilson et al. 1999; Abaza et al. 2004). Several advanced optimization methods have been used in an attempt to solve the pavement management problem (Harper and Majidzadeh 1991; Tavakoli et al. 1992; Mbwana and Tumquist 1996; Abaza and Ashur 1999; Pilson et al. 1999; Ferreira et al. 2002; Abaza et al. 2004). However, the application of advanced optimization methods has its own drawbacks and does not necessarily provide a convenient tool for the practicing pavement engineer.

Pavement performance has long been recognized as being probabilistic which implies that future pavement conditions can never be estimated with certainty. The stochastic model that has successfully been used in modeling pavement performance is the Markov model (Way et al. 1982; Butt et al. 1987; Li et al. 1996; Abaza and Ashur 1999; Hong and Wang 2003; Abaza et al. 2004). Another major advantage of using the Markov model is that it facilitates the integration of both pavement deterioration rates and M&R improvement variables into a single entity called the transition matrix. The transition matrix can then be used as the main parameter in the formulation of an efficient decision policy. Researchers have used both homogenous and nonhomogenous Markov chains in modeling both pavement performance and pavement management (Way et al. 1982; Butt et al. 1987; Li et al. 1996; Abaza and Ashur 1999; Hong and Wang 2003; Abaza et al. 2004). A homogenous chain implies that the transition matrix remains unchanged during the analysis period while a nonhomogenous chain requires a different matrix for each transition. Markovian-based models have also been used in similar civil engineering works such as in bridge maintenance and management.

A nonlinear optimization program results when applying the Markov model to provide a long-term M&R program for a specified analysis period. Researchers have successfully solved

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the nonlinear program for the case of a homogenous discrete-time Markov chain as the associated number of M&R variables can reasonably be managed using available optimization methods (Abaza and Ashur 1999; Abaza et al. 2004). The optimum M&R plan in the homogenous case is the same during each transition (time interval) within a specified analysis period. The nonhomogenous chain applies a different matrix for each transition resulting in a different M&R plan, which is an advantage compared to the homogenous solution. However, the associated number of M&R variables will be equal to the number used in the homogenous case multiplied by the number of deployed transitions. Therefore the resulting nonlinear program for even a limited number of transitions will be very difficult to formulate and solve considering the nonhomogenous chain.

Problem Statement and Research Objectives

It is proposed to develop a stochastic pavement management model (SPMM) that can be applied to a given pavement network. The pavement network is considered to consist of a number of pavement systems. A pavement system represents all pavements with similar pavement structures and loading conditions. Pavements in a given system are assigned to a number of condition states such that all pavements in the same state have similar distress ratings. It is required to derive an optimum M&R plan that defines the amount and type of M&R works that should be done on the pavement network during each transition (year). This can be accomplished by formulating an efficient decision-making policy that can be expressed in terms of the deployed M&R variables representing the percentages of pavements in the various applicable states to be treated by the corresponding M&R actions. An efficient M&R decision-making policy is a critical component in any pavement management model as it decisively impacts the quality of obtained optimal M&R plans. The methodology section will present the relevant mathematical formulations using a nonhomogenous Markov chain incorporating both the M&R variables and the deterioration rates represented by the transition probabilities. The formulations are presented considering a pavement system, which is similar to a project-level problem, however, an expansion to the network level is provided in a later section.

There are three main objectives for this paper: (1) to develop a long-term, network-based pavement management model that deploys a nonhomogenous discrete Markov chain; (2) to formulate an efficient decision policy that can yield an optimal set of M&R actions and their respective timings, and provide optimal M&R fund allocations over a specified analysis period; and (3) to develop a modeling mechanism that converts the nonlinear program associated with the nonhomogenous chain into a number of linear programs that are iteratively solved. A linear program results when the analysis period is comprised of only one transition. Therefore it is required to formulate and iteratively solve (n) linear programs for an analysis period of (n) transitions. The first linear program is treated as if the analysis period consists of only one transition. The subsequent linear programs are similarly formulated but with the exception that the optimal solution obtained from a particular program is used as an input for the next one.

The paper also presents two types of nonlinear pavement management models formulated based on the nonhomogenous Markov chain. The first model, called single target nonlinear model, is shown to be incompatible to the iterative linear model as it applies a different decision policy approach. The second model, called multiple targets nonlinear model, is shown to be highly compatible to the iterative linear model as it applies the same decision policy approach. The effectiveness of the developed iterative approach is investigated by comparing optimal solutions obtained from the iterative linear model to those obtained from the compatible nonlinear model. Also, a sensitivity analysis is performed to demonstrate how the iterative linear model can be used to generate optimal solutions similar to the true optimal solutions obtained from the incompatible nonlinear model.

Methodology

The proposed stochastic pavement management model (SPMM) applies a decision-making policy that is formulated using a nonhomogenous discrete Markov chain for a specified analysis period comprised of (n) transitions. The resulting nonlinear model is optimized iteratively as a series of linear models. The optimum linear models are formulated as constrained programs using the two traditionally deployed decision policy options (Harper and Majidzadeh 1991; Haas et al. 1994; Shahin 1994; Abaza and Ashur 1999; Abaza et al. 2004). The first option is designed to maximize the expected pavement condition rating for a given pavement system subjected to budget constraints that specify the anticipated budget for each transition. The second option is designed to minimize the system M&R cost subjected to performance requirement constraints that specify the desired pavement condition rating for each transition. The expected pavement condition rating for a given pavement system is estimated as the mean of a probability density function defined using the state probabilities derived from the Markov model.

Homogenous Discrete Markov Chain

The main components of the discrete-time Markov chain are the transition matrix, a number of condition states defining the size of the transition matrix, and a number of transitions (n) defining the length of the analysis period with the transition time being the time interval separating any two successive transitions. The deployed states are used to define the pavement conditions using an appropriate pavement condition indicator. The proposed SPMM is presented using five condition states labeled **1**, **2**, **3**, **4**, and **5**, which designate pavement condition ratings of very good, good, fair, poor, and bad, respectively. The transition matrix (R) in the absence of M&R works takes on the form indicated by Eq. (1)

$$R = \begin{pmatrix} R_{11} & R_{12} & 0 & 0 & 0 \\ 0 & R_{22} & R_{23} & 0 & 0 \\ 0 & 0 & R_{33} & R_{34} & 0 \\ 0 & 0 & 0 & R_{44} & R_{45} \\ 0 & 0 & 0 & 0 & 1.0 \end{pmatrix} \tag{1}$$

The matrix only contains the transition probabilities ($R_{i,i}$ and $R_{i,i+1}$) representing the deteriorating rates in the absence of any M&R works as declared by the zero entries below the main diagonal. It has typically been assumed for a moderate size of transition matrix that the only possible outcome for a pavement

section in state (**i**) after one transition (time interval) is either remaining in the same state (**i**) with probability ($R_{i,i}$) or transiting to the next worst state (**i+1**) with probability ($R_{i,i+1}$) (Way et al. 1982; Butt et al. 1987; Abaza and Ashur 1999; Abaza et al. 2004). The transition matrix for the homogenous Markov chain in the presence of M&R works is presented in Eq. (2) with the M&R variables (X_{ij}) representing the improvement percentages from origin state (**i**) to the better state (**j**) in one transition. There are a total of 10 M&R variables that can be incorporated into the transition matrix with each variable representing a particular M&R action. M&R works can only be applied to States **2**, **3**, **4**, and **5**. Also, the matrix framework allows only one M&R action for each M&R variable. Therefore consideration must be given to selecting the most cost-effective M&R action for each M&R variable using the cost-effectiveness ratio defined in the sample presentation section

$$P = \begin{pmatrix} P_{11} & P_{12} & 0 & 0 & 0 \\ X_{21} & P_{22} & P_{23} & 0 & 0 \\ X_{31} & X_{32} & P_{33} & P_{34} & 0 \\ X_{41} & X_{42} & X_{43} & P_{44} & P_{45} \\ X_{51} & X_{52} & X_{53} & X_{54} & P_{55} \end{pmatrix} \tag{2}$$

The transition probabilities ($P_{i,i}$ and $P_{i,i+1}$) can be related to the transition probabilities ($R_{i,i}$ and $R_{i,i+1}$) and the M&R variables (X_{ij}) as outlined in Eq. (3). The assumption made in deriving Eq. (3) is that the ratio of the two transition probabilities in the presence of M&R works ($P_{i,i}$ and $P_{i,i+1}$) will be equal to the ratio of the two corresponding transition probabilities in the absence of M&R works ($R_{i,i}$ and $R_{i,i+1}$) (Abaza and Ashur 1999; Abaza et al. 2004). It is to be noted that the sum of any row in a transition matrix must add up to 1

$$P_{i,i} = \left(1 - \sum_{j=1}^{i-1} X_{ij}\right) R_{i,i} \quad (i = 2,3,4,5) \tag{3a}$$

$$P_{i,i+1} = \left(1 - \sum_{j=1}^{i-1} X_{ij}\right) R_{i,i+1} \quad (i = 2,3,4) \tag{3b}$$

The state probabilities are defined as the proportions of pavement that exist in the various states at any given time. The basic Markov model for the homogenous discrete-time chain is presented in Eq. (4). The state probabilities after (n) transitions ($S_i^{(n)}$) are obtained from multiplying the row vector representing the initial state probabilities ($S_i^{(0)}$) by the transition matrix multiplied (n) times. Therefore the state probabilities after (n) transitions are derived based on the initial state probabilities, the transition probabilities ($R_{i,i}$ and $R_{i,i+1}$), and the M&R variables (X_{ij}). It is to be pointed out that the transition probabilities ($P_{i,i}$ and $P_{i,i+1}$) are expressed in terms of the transition probabilities ($R_{i,i}$ and $R_{i,i+1}$) as defined in Eq. (3). The state probabilities ($S_i^{(n)}$) as obtained from Eq. (4) become only a function of the M&R variables raised to power (n) as the transition matrix is multiplied (n) times

$$\mathbf{S^{(n)} = S^{(0)} P^{(n)}} \tag{4}$$

where

$$S^{(n)} = (S_1^{(n)}, S_2^{(n)}, S_3^{(n)}, S_4^{(n)}, S_5^{(n)})$$

$$S^{(0)} = (S_1^{(0)}, S_2^{(0)}, S_3^{(0)}, S_4^{(0)}, S_5^{(0)})$$

The transition matrix (P) for the homogenous chain is used to represent each transition within a specified analysis period. Therefore the M&R variables remain the same.

Optimum Decision Policy Formulation

The state probability functions ($S_i^{(n)}$) are used in the formulation of an efficient decision policy that aims to yielding optimum pavement conditions. There are two main options used in the formulation of an efficient optimum decision policy. The first option for the homogenous Markov chain is associated with an objective function that maximizes the expected pavement condition rating ($\mathrm{PCR}_s^{(n)}$) for a particular pavement system as presented in Eq. (5). The expected system ($\mathrm{PCR}_s^{(n)}$) value is estimated after (n) transitions as the mean of a probability density function. The deployed pavement condition states are defined by placing upper and lower PCR limits, for example, State **1** is defined using PCR_1 and PCR_2 with the average of these two limits (b_1) representing the mean PCR value for State **1**

$$\mathrm{PCR}_s^{(n)} = b_1 S_1^{(n)} + b_2 S_2^{(n)} + b_3 S_3^{(n)} + b_4 S_4^{(n)} + b_5 S_5^{(n)} \tag{5}$$

where

$$b_1 = \frac{\mathrm{PCR}_1 + \mathrm{PCR}_2}{2}, \quad b_2 = \frac{\mathrm{PCR}_2 + \mathrm{PCR}_3}{2}, \quad \ldots$$

$$b_5 = \frac{\mathrm{PCR}_5 + \mathrm{PCR}_6}{2}$$

The second option for the homogenous Markov chain has an objective function that minimizes the total system M&R cost ($C_s^{(n)}$) after (n) transitions as presented in Eq. (6). The M&R cost for the kth transition is obtained as the product sum of the M&R variables (X_{ij}), the state probabilities for the preceding transition ($S_i^{(k-1)}$), the corresponding M&R cost rates per square meter for the kth transition ($C_{ij}^{(k)}$), and the pavement system surface area in square meter (A_s). Therefore the M&R variables denote the percentages of pavement in the applicable states as represented by the state proportions that exist at the end of the ($k-1$) transition

$$C_s^{(n)} = (A_s) \sum_{k=1}^{n} \sum_{i=2}^{5} \sum_{j=1}^{i-1} C_{ij}^{(k)} S_i^{(k-1)} X_{ij} \tag{6}$$

The resulting objective functions from both options are nonlinear in form with (n) degree, and it becomes progressively more difficult to perform the optimization as the number of transitions increases. The optimal solution obtained from the homogenous approach is considered a "static" one as it implies the same M&R plan to be applied during each transition (Abaza and Ashur 1999; Abaza et al. 2004).

Nonhomogenous Discrete Markov Chain

The nonhomogenous Markov chain differs from the homogenous one wherein a different transition matrix is used to represent each transition. This allows for the incorporation of a different M&R plan for each transition (time interval). The state probability functions ($S_i^{(n)}$) after (n) transitions for the nonhomogenous case are derived from Eq. (7a). The transition matrix ($P^{(k)}$) for the kth transition is presented in Eq. (7b) wherein the corresponding transition probabilities and M&R variables can be different for each transition.

$$\mathbf{S^{(n)}} = \mathbf{S^{(0)}}\left(\prod_{\mathbf{k=1}}^{\mathbf{n}} \mathbf{P^{(k)}}\right) \tag{7a}$$

where

$$P^{(k)} = \begin{pmatrix} P_{11}^{(k)} & P_{12}^{(k)} & 0 & 0 & 0 \\ X_{21}^{(k)} & P_{22}^{(k)} & P_{23}^{(k)} & 0 & 0 \\ X_{31}^{(k)} & X_{32}^{(k)} & P_{33}^{(k)} & P_{34}^{(k)} & 0 \\ X_{41}^{(k)} & X_{42}^{(k)} & X_{43}^{(k)} & P_{44}^{(k)} & P_{45}^{(k)} \\ X_{51}^{(k)} & X_{52}^{(k)} & X_{53}^{(k)} & X_{54}^{(k)} & P_{55}^{(k)} \end{pmatrix} \tag{7b}$$

Similar to the assumption made in the homogenous case, the transition probabilities ($P_{i,i}^{(k)}$ and $P_{i,i+1}^{(k)}$) for the kth transition can be related to the corresponding M&R variables ($X_{i,j}^{(k)}$) and the transition probabilities ($R_{i,i}$ and $R_{i,i+1}$) as indicated by Eq. (8). The total number of M&R variables for the nonhomogenous chain becomes equal to the number used in the homogenous case multiplied by the number of deployed transitions (n). Therefore the optimization of the resulting nonlinear program becomes more complicated as the number of transitions increases. However, the corresponding optimal solution is classified as a "dynamic" one as it can provide a different M&R plan for each transition within a specified analysis period. The transition probabilities ($R_{i,i}$ and $R_{i,i+1}$) can also be incorporated as nonhomogenous indicating different deterioration rates for each deployed transition

$$P_{i,i}^{(k)} = \left(1 - \sum_{j=1}^{i-1} X_{ij}^{(k)}\right) R_{i,i} \quad (i = 2,3,4,5)$$

$$P_{i,i+1}^{(k)} = \left(1 - \sum_{j=1}^{i-1} X_{ij}^{(k)}\right) R_{i,i+1} \quad (i = 2,3,4)$$

$$P_{11}^{(k)} = R_{11}; \quad P_{12}^{(k)} = R_{12} \tag{8}$$

Iterative Nonhomogenous Model Formulation

The nonlinear model associated with the nonhomogenous Markov chain can iteratively be solved as a number of linear programs that is equal to the number of deployed transitions. This is a vital alternative to solving a compatible nonlinear model that tends to be very complicated even for a limited number of transitions. In addition, an absolute optimal solution is not always guaranteed when optimizing the associated nonlinear program, which was evidenced from the results obtained using the homogenous Markov chain (Abaza and Ashur 1999; Abaza et al. 2004). The state probability functions ($S_i^{(1)}$) for the first iteration are obtained from multiplying the initial state probabilities by the transition matrix for the first transition as indicated by Eq. (9a). The resulting objective functions, as defined in the outlined decision policy options, are linear as the corresponding state probability functions are linear. The derived optimal solution ($X_{ij}'^{(1)}$) is then used in Eq. (9b) to obtain the associated optimal state probabilities ($S_i'^{(1)}$) that will be used as the initial state probabilities for the second iteration

$$\mathbf{S^{(1)}} = \mathbf{S^{(0)}}\mathbf{P^{(1)}} \tag{9a}$$

$$\mathbf{S'^{(1)}} = \mathbf{S^{(0)}}\mathbf{P'^{(1)}} \tag{9b}$$

where

$$P_{i,i}'^{(1)} = \left(1 - \sum_{j=1}^{i-1} X_{ij}'^{(1)}\right) R_{i,i}, \quad P_{i,i+1}'^{(1)} = \left(1 - \sum_{j=1}^{i-1} X_{ij}'^{(1)}\right) R_{i,i+1}$$

Similarly, the state probability functions ($S_i^{(2)}$) for the second iteration are derived from Eq. (10a) based on the initial state probabilities obtained from the first iteration and the transition matrix for the second transition. Eq. (10b) is then used to obtain the optimal state probabilities ($S_i'^{(2)}$) based on the derived optimal solution ($X_{ij}'^{(2)}$)

$$\mathbf{S^{(2)}} = \mathbf{S'^{(1)}}\mathbf{P^{(2)}} \tag{10a}$$

$$\mathbf{S'^{(2)}} = \mathbf{S'^{(1)}}\mathbf{P'^{(2)}} \tag{10b}$$

where

$$P_{i,i}'^{(2)} = \left(1 - \sum_{j=1}^{i-1} X_{ij}'^{(2)}\right) R_{i,i}, \quad P_{i,i+1}'^{(2)} = \left(1 - \sum_{j=1}^{i-1} X_{ij}'^{(2)}\right) R_{i,i+1}$$

The general iterative nonhomogenous linear model is presented in Eq. (11). It is required to solve (n) linear models for an analysis period of (n) transitions. The optimal solution associated with one model is used to obtain the corresponding optimal state probabilities that are used as the initial state probabilities for formulating the subsequent model. The state probability functions ($S_i^{(k)}$) for the kth transition are provided in Fig. 1, which are obtained from multiplying the initial state probability row vector ($S^{(k-1)}$) by the transition matrix ($P^{(k)}$). The state probability functions are to be used in the two previously outlined decision policy options for yielding optimum pavement conditions. It is to be reminded that the state probability functions are only a function of the M&R variables

$$\mathbf{S^{(k)}} = \mathbf{S^{(k-1)}}\mathbf{P^{(k)}}, \quad \mathbf{S'^{(k)}} = \mathbf{S^{(k-1)}}\mathbf{P'^{(k)}} \quad (k = 1)$$

$$\mathbf{S^{(k)}} = \mathbf{S'^{(k-1)}}\mathbf{P^{(k)}}, \quad \mathbf{S'^{(k)}} = \mathbf{S'^{(k-1)}}\mathbf{P'^{(k)}} \quad (k = 2,3,\ldots,n) \tag{11}$$

where

$$P_{i,i}'^{(k)} = \left(1 - \sum_{j=1}^{i-1} X_{ij}'^{(k)}\right) R_{i,i}, \quad P_{i,i+1}'^{(k)} = \left(1 - \sum_{j=1}^{i-1} X_{ij}'^{(k)}\right) R_{i,i+1}$$

Stochastic Pavement Management Model Formulation

The decision-making policy associated with the proposed SPMM for the nonhomogenous Markov chain can be formulated as a constrained maximization linear program as presented in Eq. (12). The objective function is designed to maximize the expected pavement condition rating (PCR) for a given pavement system estimated as the mean of a probability density function defined using the state probability functions ($S_i^{(k)}$). The maximization linear program is subjected to a budget constraint requiring the M&R cost associated with the kth transition to be less than or equal to the system budget ($B_s^{(k)}$) available during the kth transition. The other two constraints are used to place upper and lower limits on the M&R variables. The M&R variables ($X_{ij}^{(k)}$) in the budget constraint are multiplied by the system surface area in square meter (A_s), the M&R cost rates per square meter for the kth transition ($C_{ij}^{(k)}$), and the initial state probabilities ($S_i^{(k-1)}$). Therefore an M&R variable ($X_{ij}^{(k)}$) represents the percentage of the initial state proportion ($S_i^{(k-1)}$) that should be treated by

$$S_1^{(k)} = R_{11}\, S_1^{(k-1)} + S_2^{(k-1)}\, X_{21}^{(k)} + S_3^{(k-1)}\, X_{31}^{(k)} + S_4^{(k-1)}\, X_{41}^{(k)} + S_5^{(k-1)}\, X_{51}^{(k)}$$
$$S_2^{(k)} = R_{12}\, S_1^{(k-1)} + S_2^{(k-1)}\, P_{22}^{(k)} + S_3^{(k-1)}\, X_{32}^{(k)} + S_4^{(k-1)}\, X_{42}^{(k)} + S_5^{(k-1)}\, X_{52}^{(k)}$$
$$\text{where: } P_{22}^{(k)} = \left(1 - X_{21}^{(k)}\right) R_{22}$$

$$S_3^{(k)} = S_2^{(k-1)}\, P_{23}^{(k)} + S_3^{(k-1)}\, P_{33}^{(k)} + S_4^{(k-1)}\, X_{43}^{(k)} + S_5^{(k-1)}\, X_{53}^{(k)}$$
$$\text{where: } P_{23}^{(k)} = \left(1 - X_{21}^{(k)}\right) R_{23};\ \ P_{33}^{(k)} = \left(1 - X_{31}^{(k)} - X_{32}^{(k)}\right) R_{33}$$

$$S_4^{(k)} = S_3^{(k-1)}\, P_{34}^{(k)} + S_4^{(k-1)}\, P_{44}^{(k)} + S_5^{(k-1)}\, X_{54}^{(k)}$$
$$\text{where: } P_{34}^{(k)} = \left(1 - X_{31}^{(k)} - X_{32}^{(k)}\right) R_{34};\ \ P_{44}^{(k)} = \left(1 - X_{41}^{(k)} - X_{42}^{(k)} - X_{43}^{(k)}\right) R_{44}$$

$$S_5^{(k)} = S_4^{(k-1)}\, P_{45}^{(k)} + S_5^{(k-1)}\, P_{55}^{(k)}$$
$$\text{where: } P_{45}^{(k)} = \left(1 - X_{41}^{(k)} - X_{42}^{(k)} - X_{43}^{(k)}\right) R_{45};\ \ P_{55}^{(k)} = \left(1 - X_{51}^{(k)} - X_{52}^{(k)} - X_{53}^{(k)} - X_{54}^{(k)}\right) 1.0$$

Fig. 1. Iterative state probability functions used in the presented SPMM model

the corresponding M&R action during the kth transition (year) to improve the pavement from its current state (**i**) to a better state (**j**). For example, a value of 1 indicates that all pavements in state (**i**) must be treated. Intervention timings are defined by the superscript (k) placed on the M&R variables indicating the amount and type of M&R works to be done during the kth transition (year).

Maximize

$$\mathrm{PCR}_s^{(k)} = \sum_{i=1}^{5} b_i S_i^{(k)} \tag{12}$$

Subject to

1.

$$(A_s) \sum_{i=2}^{5} \sum_{j=1}^{i-1} C_{ij}^{(k)} S_i^{(k-1)} X_{ij}^{(k)} \leqslant B_s^{(k)}$$

2.

$$\sum_{j=1}^{i-1} X_{ij}^{(k)} \leqslant 1.0 \quad (i = 2,3,4,5)$$

3.

$$X_{ij}^{(k)} \geqslant 0.0$$

The second option deployed by the proposed SPMM decision policy is designed with the objective of minimizing the system M&R cost ($C_s^{(k)}$) as indicated by Eq. (13). The objective function is the same as the budget constraint used in the maximization model excluding the budget parameter. The first constraint is used to enforce a specified desired pavement condition rating ($\mathrm{SPCR}_s^{(k)}$) at the end of the kth transition for the pavement system under consideration. The objective function used in the maximization model is set equal to or greater than a specified desired pavement condition rating. The specified PCR value must be higher than the expected one obtained in the absence of M&R works.

Minimize

$$C_s^{(k)} = (A_s) \sum_{i=2}^{5} \sum_{j=1}^{i-1} C_{ij}^{(k)} S_i^{(k-1)} X_{ij}^{(k)} \tag{13}$$

Subject to

1.

$$\sum_{i=1}^{5} b_i S_i^{(k)} \geqslant \mathrm{SPCR}_s^{(k)}$$

2.

$$\sum_{j=1}^{i-1} X_{ij}^{(k)} \leqslant 1.0 \quad (i = 2,3,4,5)$$

3.

$$X_{ij}^{(k)} \geqslant 0.0$$

The two outlined SPMM decision policy options are to be iteratively solved for a specified analysis period. The number of required iterations is equal to the number of transitions (n) determined from dividing the analysis period by the time interval between successive transitions. The optimal solution derived from the kth transition is used as formerly outlined to obtain the corresponding state probabilities, which become the initial state probabilities for the subsequent iteration.

Estimation of Transition Probabilities

A major input requirement for the use of any Markovian model is the reliable estimation of the corresponding transition probabilities. The transition probabilities are estimated based on pavement distress records that are typically obtained from conducting a field survey of pavement defects. Two cycles of field survey conducted annually or biennially are required as a minimum to obtain estimates of the transition probabilities (Abaza and Ashur 1999; Abaza et al. 2004). The two transition probabilities ($R_{i,i}$ and $R_{i,i+1}$), required for the application of the presented optimum

SPMM, can be estimated using Eq. (14). The initial number of pavement sections ($N_i^{(0)}$) rated as state (**i**) in the first cycle is compared, using the direct definition of transition probabilities, to the number of pavement sections ($N_i^{(1)}$) that are still rated as state (**i**) in the second cycle

$$R_{i,i+1} = \frac{N_i^{(0)} - N_i^{(1)}}{N_i^{(0)}}$$
$$R_{i,i} = 1 - R_{i,i+1} \tag{14}$$

Another method for estimating the transition probabilities is developed by researchers at the United States Army Construction Engineering Research Laboratory (CERL) (Butt et al. 1987). It is based on minimizing the residuals defined as the difference between the observed pavement condition index (PCI) values and their corresponding predicted ones as derived from the Markov model. This method requires extensive historical distress records to be effective.

Network-Level SPMM Formulation

The optimum SPMM iterative linear approach can be expanded to the network level assuming the network is comprised of a number of pavement systems. The objective functions and main constraints associated with the two outlined SPMM decision policy options can simply be expanded to include the sum of all systems to be considered in a given network. An example of that is provided in Eq. (15), which is equivalent to the iterative linear maximization model presented in Eq. (12). The objective is to maximize the average network pavement condition rating ($\overline{\mathrm{PCR}}_N^{(k)}$) defined as the average of the pavement condition ratings associated with (s) pavement systems. The network iterative linear model requires a single projected transitional budget ($B_N^{(k)}$) for global optimization. The network model formulation has required using a third subscript (l).
Maximize

$$\overline{\mathrm{PCR}}_N^{(k)} = \frac{1}{s}\sum_{l=1}^{s}\sum_{i=1}^{5} b_i S_{il}^{(k)} \tag{15}$$

Subject to
1.

$$\sum_{l=1}^{s}\sum_{i=2}^{5}\sum_{j=1}^{i-1} A_l C_{ijl}^{(k)} S_{il}^{(k-1)} X_{ijl}^{(k)} \leqslant B_N^{(k)}$$

2.

$$\sum_{j=1}^{i-1} X_{ijl}^{(k)} \leqslant 1.0 \quad (i = 2,3,4,5; l = 1,2,\ldots,s)$$

3.

$$X_{ijl}^{(k)} \geqslant 0.0$$

Each pavement system is to be represented by its own non-homogenous Markov chain that reflects its own deterioration and improvement rates. Therefore the resulting number of M&R variables becomes equal to the sum of all variables associated with the various deployed systems. The presented iterative linear approach can be applied to any network-level problem regardless of its size as linear programming software packages can handle a very large number of variables and produce very reliable results. A pavement system is typically defined as one with similar pavement structures and loading conditions.

Nonlinear Optimum Pavement Management Models

There are two different nonlinear pavement management models that can be formulated based on the two presented optimum decision policy options. The first nonlinear model is designed to meet a single target at the end of a specified analysis period whereas the second model is designed to meet multiple targets within an analysis period. Detailed descriptions of these two models are presented below.

Single Target Nonlinear Model

The single target nonlinear model can be formulated as a maximization model with the objective of maximizing the average system pavement condition rating ($\overline{\mathrm{PCR}}_s^{(n)}$) over ($n$) transitions subjected to a single budget constraint. The single budget constraint provides complete accessibility to the total system budget ($\mathrm{TB}_s^{(n)}$) to be used in any feasible schedule during the analysis period. Eq. (16) outlines this model, which is similar to the model presented in Eq. (12). The single budget constraint includes the total M&R costs associated with (n) transitions. The corresponding objective function and budget constraint are both multivariable nonlinear polynomials with (n) degree. The model contains a total number of variables equal to the sum of M&R variables incorporated into all deployed transition matrices.
Maximize

$$\overline{\mathrm{PCR}}_s^{(n)} = \frac{1}{n}\sum_{k=1}^{n}\sum_{i=1}^{5} b_i S_i^{(k)} \tag{16}$$

Subject to
1.

$$(A_s)\sum_{k=1}^{n}\sum_{i=2}^{5}\sum_{j=1}^{i-1} C_{ij}^{(k)} S_i^{(k-1)} X_{ij}^{(k)} \leqslant \mathrm{TB}_s^{(n)}$$

2.

$$\sum_{j=1}^{i-1} X_{ij}^{(k)} \leqslant 1.0 \quad (i = 2,3,4,5)$$

3.

$$X_{ij}^{(k)} \geqslant 0.0$$

Similarly, the equivalent nonlinear model associated with minimizing the total M&R costs over (n) transitions is presented in Eq. (17). The single target is meeting a specified desired average system rating ($\overline{\mathrm{SPCR}}_s^{(n)}$) over ($n$) transitions. Alternatively, the single target can be meeting a desired system PCR value after (n) transitions. The objective function associated with Eq. (16) can also be formulated to maximize the system PCR value after (n) transitions ($\mathrm{PCR}_s^{(n)}$).

Minimize

$$C_s^{(n)} = (A_s)\sum_{k=1}^{n}\sum_{i=2}^{5}\sum_{j=1}^{i-1} C_{ij}^{(k)} S_i^{(k-1)} X_{ij}^{(k)} \tag{17}$$

Subject to

1.

$$\frac{1}{n}\sum_{k=1}^{n}\sum_{i=1}^{5} b_i S_i^{(k)} \geqslant \overline{\mathrm{SPCR}}_s^{(n)}$$

2.

$$\sum_{j=1}^{i-1} X_{ij}^{(k)} \leqslant 1.0 \quad (i = 2,3,4,5)$$

3.

$$X_{ij}^{(k)} \geqslant 0.0$$

The presented single target nonlinear model can generate an optimum M&R cost schedule that is not within the budgeting limits of a given highway agency, which makes its application very limited in practice as M&R funding is mostly available on an annual basis. The iterative linear model and the single target nonlinear model thus provide different decision policy approaches, which make them incompatible.

Multiple Targets Nonlinear Model

The multiple targets nonlinear model is mainly designed to meet the transitional performance requirements as M&R budgets are available annually in most cases. The corresponding maximization model is presented in Eq. (18) with its objective function being the same one used in the single target maximization model. However, it is subjected to (n) budget constraints as each transition (year) has it own allocated budget. The objective function includes all M&R variables deployed in the transition matrices representing a given analysis period. The kth budget constraint contains all variables incorporated into the transition matrices up to and including the kth transition matrix. Therefore the first budget constraint is linear while the kth constraint is nonlinear to the kth degree (Abaza and Ashur 1999; Abaza et al. 2004).

Maximize

$$\overline{\mathrm{PCR}}_s^{(n)} = \frac{1}{n}\sum_{k=1}^{n}\sum_{i=1}^{5} b_i S_i^{(k)} \tag{18}$$

Subject to

1.

$$(A_s)\sum_{i=2}^{5}\sum_{j=1}^{i-1} C_{ij}^{(k)} S_i^{(k-1)} X_{ij}^{(k)} \leqslant B_s^{(k)} \quad (k = 1,2,\ldots,n)$$

2.

$$\sum_{j=1}^{i-1} X_{ij}^{(k)} \leqslant 1.0 \quad (i = 2,3,4,5)$$

3.

$$X_{ij}^{(k)} \geqslant 0.0$$

Similarly, the equivalent multiple targets nonlinear model associated with minimizing the total M&R costs over (n) transitions is presented in Eq. (19). The objective function is the same one associated with the minimization model in the case of a single target model. However, it is subjected to (n) performance constraints requiring a specified desired system rating at the end of each transition. The characteristics of the performance constraints are the same as for the budget constraints in the equivalent maximization model.

Minimize

$$C_s^{(n)} = (A_s)\sum_{k=1}^{n}\sum_{i=2}^{5}\sum_{j=1}^{i-1} C_{ij}^{(k)} S_i^{(k-1)} X_{ij}^{(k)} \tag{19}$$

Subject to

1.

$$\sum_{i=1}^{5} b_i S_i^{(k)} \geqslant \mathrm{SPCR}_s^{(k)} \quad (k = 1,2,\ldots,n)$$

2.

$$\sum_{j=1}^{i-1} X_{ij}^{(k)} \leqslant 1.0 \quad (i = 2,3,4,5)$$

3.

$$X_{ij}^{(k)} \geqslant 0.0$$

The iterative linear model presented earlier is fairly compatible to the multiple targets nonlinear model. This compatibility arises from the fact that the transitional budget and performance constraints are the controlling parameters in the optimization process as they must be satisfied in both models. In addition, satisfaction of the first budget constraint in both models can only be achieved using M&R variables from those assigned to the first transition. In the second budget constraint, it is again expected that the nonlinear model will use M&R variables from those associated with the second transition because of their superior impact on performance outcomes compared to their equivalent variables assigned to the first transition. The superior impact stems from the fact that doing the same M&R work in the second transition produces the immediate anticipated improvement compared to doing it in the first transition for it would be affected by pavement deterioration. This trend repeats itself for the sub sequent budget constraints as the kth budget constraint in the nonlinear model seems to be making use of only the M&R variables assigned to the kth transition indicating that the two models are quite compatible. Sample results provided in the sample presentation section support this conclusion.

Requirements for SPMM Application

The presented optimum SPMM can effectively be applied to any pavement system or network with minimal requirements. The main requirements include those related to the pavement system itself, the appropriate structure of the Markovian model, the potential M&R actions, and the computer software for solving the corresponding linear programs. Detailed descriptions of these main requirements are provided as follows.

1. The presented optimum SPMM is formulated for a given pavement system. The pavement system is typically divided into pavement sections of about 50 m lane length for

conducting an accurate field survey of pavement distress. Each section is then assigned a pavement condition rating (PCR) using an appropriate scale. These ratings can be used in estimating the required transition probabilities. Other required system parameters include the total length in lane kilometer and average lane width to be used in estimating the surface area in square meter, and the projected budget for each transition (time interval).

2. The Markovian structure associated with the presented SPMM can be modified. A nonhomogenous matrix with 5×5 has been used in the formulation of the corresponding optimum model options. Also, an assumption has been made to only provide two transition probabilities for each condition state which is a realistic assumption considering the number of deployed states (Way et al. 1982; Butt et al. 1987; Abaza and Ashur 1999; Abaza et al. 2004). However, a larger transition matrix up to 10×10 can be deployed which would probably require adding a third transition probability for each state, a requirement that can be verified from conducting a minimum of two cycles of distress survey.
3. Once an appropriate matrix size is selected, potential maintenance and rehabilitation actions are chosen to represent the M&R variables incorporated into the transition matrix. There are a total of 10 M&R variables that can exist in a 5×5 transition matrix. However, it is not required to use all 10 variables if so deemed necessary. But the major requirement is that each selected M&R action must be able to produce the intended improvement from state (**i**) to state (**j**) as indicated by the corresponding M&R variable ($X_{ij}^{(k)}$). This needs to be determined based on experience and engineering judgment especially for maintenance actions which would not be expected to result in more than one state improvement. However, rehabilitation actions can be expected to improve pavements to State **1** regardless of the origin state considering a 5×5 transition matrix. The cost rate per square meter associated with each M&R action is also required.
4. The presented optimum SPMM is intended for use in developing a long-term M&R program. Therefore an analysis period is required which must be comprised of an integer number of time intervals. The length of time interval is typically taken as 1 or 2 years consistent with the length of time interval separating two successive cycles of pavement distress assessment. The required number of transition (n) to be used in the long-term analysis is obtained from dividing the specified analysis period by the time interval.
5. A number of linear programs, equal to the number of deployed transitions, are to be iteratively solved wherein the optimal solution obtained from one program is used in solving the next one as outlined earlier. There are several commercially available computer software packages that can effectively be applied to solve the resulting linear programs. These software packages are designed to solve the linear programming problem using the simplex method (Phillips et al. 1976; Bazaraa and Shetty 1979), which yields very reliable optimal solutions even for problems with a very large number of M&R variables. A software package called *Maple 8* developed by the Waterloo Maple Inc. has effectively been used in generating optimal solutions for the sample applications presented in the next section.

Sample Presentation

The presented optimum SPMM has been applied to the arterial roadway system in the city of Nablus, West Bank. The arterial pavement system is mainly comprised of a two-layer flexible pavement structure: a 10 cm asphalt concrete layer and a 30 cm aggregate base layer. The selected system has a length of about 132 lane-km and 3.5 m average lane width serving an overall average daily traffic of about 30,000 vehicles per day. A sample of 650 pavement sections was randomly selected with each being 50 m in lane length. The pavement sections were first surveyed for pavement defects during the month of May 2003 and surveyed a second time during the same month in the year 2004. The surveyed defects mainly included local depression, rutting, transverse and longitudinal cracking, block cracking, and alligator cracking. A pavement condition rating was assigned to each pavement section during each survey using a 100-point scale with the maximum rating designating perfect pavement. Eq. (20) is typically used for determining the pavement condition rating (PCR) based on the defect rating (d_i) and its assigned weight (w_i). The defect rating (d_i) was assigned using a 10-point scale depending on the severity and extent of prevailing defect. The assigned weights for the five mentioned defects were 1.5, 2.5, 1.5, 2, and 2.5, respectively

$$\mathrm{PCR} = 100 - \sum_{i=1}^{5} w_i d_i \tag{20}$$

A nonhomogenous transition matrix with 5×5 is used in the formulation of the sample optimum SPMM models. The corresponding five condition states are defined using 20 points equal to PCR intervals resulting in the 100–80, 80–60, 60–40, 40–20, and 20–0 intervals for States **1**, **2**, **3**, **4**, and **5**, respectively. The initial numbers of pavement sections ($N_i^{(0)}$) that existed in the five states during the first survey were determined based on their assigned PCR ratings and are provided in Table 1. The initial numbers of pavement sections are used to determine the initial state probabilities ($S_i^{(0)}$) considering a total of 650 sections. The transition probabilities ($R_{i,i}$ and $R_{i,i+1}$) are estimated using Eq. (14) based on the initial section numbers and the numbers of pavement sections ($N_i^{(1)}$) that existed in the five states during the second survey as provided in Table 1.

The nonhomogenous matrix is formulated to include only 7 M&R actions consisting of four maintenance actions and three rehabilitation actions. The four maintenance actions include repair works such as crack sealing and pothole patching, surface treatments, and localized reconstruction, and are represented by the variables ($X_{21}^{(k)}, X_{32}^{(k)}, X_{43}^{(k)}, X_{54}^{(k)}$) with subscripts indicating one state improvement. The three rehabilitation actions may consist of plain overlay, skin patch, and major reconstruction, and are represented by the variables ($X_{31}^{(k)}, X_{41}^{(k)}, X_{51}^{(k)}$) with subscripts

Table 1. Estimated Sample Pavement Deterioration Rates (Transition Probabilities)

State i	$N_i^{(0)}$	$N_i^{(1)}$	$S_i^{(0)}$	$R_{i,i}$	$R_{i,i+1}$
1	103	87	0.158	0.845	0.155
2	148	113	0.228	0.764	0.236
3	215	146	0.331	0.679	0.321
4	86	51	0.132	0.593	0.407
5	98	98	0.151	1.000	N/A

Maximize:

$$
\begin{aligned}
PCR_s^{(k)} = {} & 90[0.845S_1^{(k-1)} + S_2^{(k-1)}X_{21}^{(k)} + S_3^{(k-1)}X_{31}^{(k)} + S_4^{(k-1)}X_{41}^{(k)} + S_5^{(k-1)}X_{51}^{(k)}] + \\
& 70[0.155S_1^{(k-1)} + S_2^{(k-1)}(1 - X_{21}^{(k)})0.764 + S_3^{(k-1)}X_{32}^{(k)}] + \\
& 50[S_2^{(k-1)}(1 - X_{21}^{(k)})0.235 + S_3^{(k-1)}(1 - X_{31}^{(k)} - X_{32}^{(k)})0.679 + S_4^{(k-1)}X_{43}^{(k)}] + \\
& 30[S_3^{(k-1)}(1 - X_{31}^{(k)} - X_{32}^{(k)})0.321 + S_4^{(k-1)}(1 - X_{41}^{(k)} - X_{43}^{(k)})0.593 + S_5^{(k-1)}X_{54}^{(k)}] + \\
& 10[S_4^{(k-1)}(1 - X_{41}^{(k)} - X_{43}^{(k)})0.407 + S_5^{(k-1)}(1 - X_{51}^{(k)} - X_{54}^{(k)})]
\end{aligned}
$$

Subject to:

1) $1000(3.5)(132)[3S_2^{(k-1)}X_{21}^{(k)} + S_3^{(k-1)}(12X_{31}^{(k)} + 4X_{32}^{(k)}) + S_4^{(k-1)}(17X_{41}^{(k)} + 5X_{43}^{(k)}) + S_5^{(k-1)}(25X_{51}^{(k)} + 6X_{54}^{(k)})] \le 500{,}000$

2) $X_{21}^{(k)} \le 1.0; \quad X_{31}^{(k)} + X_{32}^{(k)} \le 1.0; \quad X_{41}^{(k)} + X_{43}^{(k)} \le 1.0; \quad X_{51}^{(k)} + X_{54}^{(k)} \le 1.0$

3) $X_{ij}^{(k)} \ge 0.0$

Minimize:

$$
\begin{aligned}
C_s^{(k)} = {} & 1000(3.5)(132)[3S_2^{(k-1)}X_{21}^{(k)} + S_3^{(k-1)}(12X_{31}^{(k)} + 4X_{32}^{(k)}) + S_4^{(k-1)}(17X_{41}^{(k)} + 5X_{43}^{(k)}) \\
& + S_5^{(k-1)}(25X_{51}^{(k)} + 6X_{54}^{(k)})]
\end{aligned}
$$

Subject to:

1) $PCR_s^{(k)}$ (as given above) $\ge SPCR_s^{(k)}$ (as provided in Table 3)

2) $X_{21}^{(k)} \le 1.0; \quad X_{31}^{(k)} + X_{32}^{(k)} \le 1.0; \quad X_{41}^{(k)} + X_{43}^{(k)} \le 1.0; \quad X_{51}^{(k)} + X_{54}^{(k)} \le 1.0$

3) $X_{ij}^{(k)} \ge 0.0$

Fig. 2. Sample iterative optimum SPMM model formulations

denoting improvement from state (**i**) to State **1**. The first year cost rates associated with the four maintenance actions as applied to States **2**, **3**, **4**, and **5**, are locally estimated to be \$3, \$4, \$5, and \$6 per square meter, respectively. Whereas the first year cost rates for the three rehabilitation actions as applied to States **3**, **4**, and **5**, are locally estimated to be \$12, \$17, and \$25, respectively. These cost rates include only the cost of M&R works, but can be adjusted to include added work-zone user costs especially if work is to be done during the daytime. Added work-zone user costs are mainly a function of service traffic volumes and required highway closures with the latter being different for various M&R actions. Also, the cost rates are assumed to be constant for the entire analysis period, however, the first year cost rates ($C_{ij}^{(1)}$) can be adjusted to account for interest and inflation rates. The adjusted cost rates ($C_{ij}^{(k)}$) for the kth year can be obtained from the basic present worth formula using an annual discount rate (r) as indicated by Eq. (21)

$$C_{ij}^{(k)} = C_{ij}^{(1)}(1 + r)^k \tag{21}$$

The municipality of Nablus, West Bank, is seeking to obtain financial aid from donor countries through the World Bank in an effort to rebuild the city's infrastructure system. The presented optimum SPMM will be used in this sample presentation to generate an optimum M&R program considering a 5-year analysis period to provide the cost justification typically required by the World Bank. The required number of transitions (n) is five considering a 1 year time interval, which is the same time interval used between the two conducted cycles of pavement distress survey. Therefore the two maximization and minimization SPMM models will iteratively be solved five times to reach an optimum 5-year M&R program consisting of five annual M&R plans.

The iterative maximization model is formulated as outlined in Eq. (12) using an annual budget of \$500,000. The model is shown in Fig. 2 with the objective function defined as in Eq. (5) using the state probability functions provided in Fig. 1 multiplied by the middle ratings of the state PCR intervals. The corresponding minimization model is also shown in Fig. 2 wherein the budget constraint for the maximization model becomes mainly the objective function and the objective function associated with the maximization model is used as the performance requirement constraint. The desired system PCR value for each year is set equal to the same optimum PCR value obtained from the maximization model. The initial system PCR rating is estimated from the initial state probabilities and found to be 52.2.

Table 2. Sample Optimum M&R Plans for Maximizing System Pavement Condition Ratings over 5 Year Analysis Period

M&R variable	M&R plan for the kth year				
	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$X'^{(k)}_{21}$	1.000	1.000	1.000	1.000	1.000
$X'^{(k)}_{31}$	0.000	0.000	0.000	0.000	0.000
$X'^{(k)}_{32}$	0.301	1.000	1.000	1.000	0.000
$X'^{(k)}_{41}$	0.000	0.000	0.000	1.000	0.198
$X'^{(k)}_{43}$	0.000	0.108	0.938	0.000	0.802
$X'^{(k)}_{51}$	0.000	0.000	0.000	0.000	0.000
$X'^{(k)}_{54}$	0.000	0.000	0.000	0.305	0.000
$\mathrm{PCR}'^{(k)}_s$	55.72	59.46	64.18	67.36	71.10

Table 3. Sample Optimum M&R Plans for Minimizing System M&R Costs over 5 Year Analysis Period

M&R variable	Specified system pavement condition rating ($\mathrm{SPCR}^{(k)}_s$) ($k=1,2,\ldots,5$)				
	55.72	59.46	64.18	67.36	71.10
$X'^{(k)}_{21}$	1.000	1.000	1.000	1.000	1.000
$X'^{(k)}_{31}$	0.000	0.000	0.000	0.000	0.000
$X'^{(k)}_{32}$	0.303	1.000	1.000	1.000	1.000
$X'^{(k)}_{41}$	0.000	0.000	0.000	1.000	0.179
$X'^{(k)}_{43}$	0.000	0.107	0.953	0.000	0.821
$X'^{(k)}_{51}$	0.000	0.000	0.000	0.000	0.000
$X'^{(k)}_{54}$	0.000	0.000	0.000	0.314	0.000
$C'^{(k)}_s$($)	0.50 million	0.50 million	0.51 million	0.49 million	0.50 milliom

Table 2 provides the optimal linear solutions for the five attempted iterations considering the maximization model. It is to be noted that only a few M&R variables have contributed to the optimal solutions. Also, maintenance variables have only been used in the first three iterations and one rehabilitation variable has only contributed to the solutions in the last two iterations. The system ($\mathrm{PCR}'^{(k)}_s$) rating associated with the optimal solutions has improved from an initial value of 52.20 to 71.10 at the end of the fifth transition at a total projected cost of 2.5 million US dollars. Table 3 provides the optimal solutions derived from the minimization model that required the same annual system PCR values obtained from the maximization model. The optimal solutions are similar to those obtained from the maximization model at the same total cost of $2.5 million. Therefore it can be concluded that both models are quite compatible as they also generated the same average PCR value of 63.56 over the 5-year analysis period. The optimal solutions from both models have used the same M&R variables, however, there are small differences in the values of certain variables.

The maintenance variables have dominated the presented optimal solutions from both models. This can be expected as the maintenance actions result in an average improvement of 20 PCR points, which makes them more cost-effective than the rehabilitation actions. Also, the maintenance action represented by the variable ($X^{(k)}_{21}$) is the most cost-effective amongst the seven deployed M&R variables, thus it has dominated all optimal solutions. The rehabilitation variables ($X^{(k)}_{31}, X^{(k)}_{41}, X^{(k)}_{51}$) result in average improvements of 40, 60, 80 PCR points, respectively, which make them less cost-effective compared to their conjugate maintenance variables ($X^{(k)}_{32}, X^{(k)}_{43}, X^{(k)}_{54}$). The comparison is made using a cost-effectiveness ratio determined from dividing the variable cost rate by the corresponding PCR improvement points. The most cost-effective variable is the one associated with the lowest cost-effectiveness ratio.

Interpretation of Sample Results

There are four pavement states considered for M&R works in this sample presentation, namely, States 2, 3, 4, and 5. Application of the derived optimal solutions requires knowing the expected pavement proportions that exist in the four states at the beginning of each transition. The derived optimal solution as represented by the M&R variables for a given year defines the amount of work to be done on each state during that year. The value of an optimum M&R variable ($X'^{(k)}_{ij}$) represents the percentage of the state proportion ($S'^{(k-1)}_i$) that should be treated by the corresponding M&R action. A value of 0.50 implies 50% of pavements in the corresponding state shall be treated. Therefore the amount of work in lane-kilometer to be done during the kth year on the pavements that exist in state (**i**) for improvement to state (**j**) is estimated from multiplying the applicable M&R variable value ($X'^{(k)}_{ij}$) by the corresponding state proportion for the ($k-1$) transition. Table 4 provides the state probabilities (proportions) associated with the derived optimal solutions from both models.

For example, Table 2 shows three optimal variable values associated with the second year M&R plan, namely, $X'^{(2)}_{21}=1.00$, $X'^{(2)}_{32}=1.00$, and $X'^{(2)}_{43}=0.108$. Therefore, according to this plan, M&R work needs to be done on States 2, 3, and 4 for improvements to States 1, 2, and 3, respectively. Table 4 shows that the optimal state proportions expected at the end of the first transition to be 0.124, 0.157, and 0.152 for States 2, 3, and 4, respectively, considering the maximization model. The amount of M&R work to be done on States 2, 3, and 4 during the second year are calculated to be 16.37, 20.72, and 2.17 lane-km, respectively. The estimated work amounts represent all pavements in States 2 and 3, and 10.8% of the pavements in State 4 considering a 132 lane-km total system length. It is assumed that all pavements in the same state have similar pavement distress, therefore, selection can randomly be done. However, selecting the worst pavements in the same state will definitely be an advantage.

Sensitivity Analysis

The presented two iterative linear models have been tested for the purpose of generating an optimum 5-year M&R program considering variations in budget allocations and performance requirements. The impact of variable budget schedules is tested considering a total 5-year budget of $2.5 million as used in generating the sample results provided in Tables 2 and 3. Six different budget schedules are provided in Table 5 with the first schedule being the same one used before with equal allocations amongst the 5 year analysis period. The corresponding optimal system PCR value is provided below each budget allocation. The best budget schedule is No. 4 yielding a 5-year average system PCR value of 72.33 based on 5-year allocations of 1.2, 0.8, 0.5, 0.0, and 0.0 million with years 1, 2, and 3 indicating the best intervention timings. The true optimal solution may be obtained using the outlined incompatible single target nonlinear model. A

Table 4. State Probabilities Associated with Sample Optimum M&R Plans

State probability	kth year M&R plan									
	Maximization model					Minimization model				
	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
$S_1'^{(k)}$	0.362	0.430	0.576	0.574	0.665	0.362	0.430	0.576	0.571	0.662
$S_2'^{(k)}$	0.124	0.213	0.084	0.164	0.089	0.125	0.212	0.083	0.165	0.092
$S_3'^{(k)}$	0.157	0.017	0.075	0.000	0.064	0.156	0.017	0.077	0.003	0.066
$S_4'^{(k)}$	0.152	0.080	0.003	0.080	0.000	0.152	0.081	0.002	0.081	0.000
$S_5'^{(k)}$	0.205	0.260	0.262	0.182	0.182	0.205	0.260	0.262	0.180	0.180
$\mathrm{PCR}_s'^{(k)}$	55.72	59.46	64.18	67.36	71.10	55.72	59.46	64.18	67.36	71.10

Table 5. Impact of Variable Budget Schedules on System Pavement Condition Ratings as Obtained from the Iterative Maximization Model

Schedule number	Annual system budget in million US dollars					Total budget ($ million)
	$B_s^{(1)}$	$B_s^{(2)}$	$B_s^{(3)}$	$B_s^{(4)}$	$B_s^{(5)}$	
	$\mathrm{PCR}_s'^{(1)}$	$\mathrm{PCR}_s'^{(2)}$	$\mathrm{PCR}_s'^{(3)}$	$\mathrm{PCR}_s'^{(4)}$	$\mathrm{PCR}_s'^{(5)}$	$\overline{\mathrm{PCR}}_s^{(5)}$
1[a]	0.50	0.50	0.50	0.50	0.50	2.5
	55.72	59.46	64.18	67.36	71.10	63.56
2	0.30	0.40	0.50	0.60	0.70	2.5
	51.74	54.54	58.22	63.76	68.78	59.41
3	0.70	0.60	0.50	0.40	0.30	2.5
	58.56	64.32	68.14	70.60	72.06	66.74
4	1.20	0.80	0.50	0.0	0.0	2.50
	65.13	74.17	77.46	74.12	70.76	72.33[b]
5	1.20	0.0	0.80	0.0	0.50	2.50
	65.13	61.51	70.56	67.51	72.56	67.45
6	2.50	0.0	0.0	0.0	0.0	2.50
	74.63	70.26	66.07	61.87	57.61	66.09

[a]The same optimum M&R schedule provided in Table 2.

[b]Best M&R schedule.

Table 6. Impact of Variable System Pavement Condition Rating Schedules on System M&R Costs as Obtained from the Iterative Minimization Model

Schedule number	Specified annual system pavement condition rating					
	$\mathrm{SPCR}_s^{(1)}$	$\mathrm{SPCR}_s^{(2)}$	$\mathrm{SPCR}_s^{(3)}$	$\mathrm{SPCR}_s^{(4)}$	$\mathrm{SPCR}_s^{(5)}$	$\overline{\mathrm{SPCR}}_s^{(5)}$
	$C_s'^{(1)}$	$C_s'^{(2)}$	$C_s'^{(3)}$	$C_s'^{(4)}$	$C_s'^{(5)}$	$TC_s'^{(5)}$[a]
1[b]	55.72	59.46	64.18	67.36	71.10	63.56
	0.50	0.50	0.51	0.49	0.50	2.50
2	63.56	63.56	63.56	63.56	63.56	63.56
	1.07	0.20	0.19	0.18	0.13	1.77[c]
3	61.56	62.56	63.56	64.56	65.56	63.56
	0.91	0.25	0.22	0.24	0.21	1.83
4	65.56	64.56	63.56	62.56	61.56	63.56
	1.24	0.15	0.15	0.15	0.14	1.83
5	59.56	61.56	63.56	65.56	67.56	63.56
	0.77	0.32	0.34	0.26	0.43	2.12
6	67.56	65.56	63.56	61.56	59.56	63.56
	1.51	0.14	0.13	0.11	0.09	1.98

[a]($ million).

[b]The same optimum M&R schedule provided in Table 3.

[c]Best M&R schedule.

Table 7. Sample Results Obtained from the Iterative Linear Model and Nonlinear Models for Maximizing System Pavement Condition Rating over 3 Year Analysis Period

M&R variable	M&R plan for the kth year								
	Iterative linear model			Multiple targets nonlinear model			Single target nonlinear model		
	$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$
$X'^{(k)}_{21}$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.190
$X'^{(k)}_{31}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$X'^{(k)}_{32}$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000
$X'^{(k)}_{41}$	0.000	0.938	0.000	0.000	0.927	0.000	1.000	0.000	0.000
$X'^{(k)}_{43}$	0.237	0.062	0.000	0.235	0.073	0.000	0.000	1.000	0.000
$X'^{(k)}_{51}$	0.000	0.000	0.202	0.000	0.000	0.201	0.000	0.000	0.000
$X'^{(k)}_{54}$	0.000	0.000	0.798	0.000	0.000	0.799	0.706	0.000	0.000
$C'^{(k)}_{s}$ (\$ million)	1.00	1.00	1.00	1.00	1.00	1.00	2.25	0.73	0.02
$\mathrm{PCR}'^{(k)}_{s}$	62.70	72.75	78.51	62.70	72.72	78.46	72.87	80.51	77.45

near optimal solution can be reached by performing an analysis similar to the one presented in Table 5 using the iterative maximization model. The true optimal solution seems to be following the budgeting pattern used in schedule No. 4 wherein 48% of the budget is used in the first year to produce substantial initial improvements and 52% left for the following 2 years to maintain these improvements. Unfortunately, most highway agencies could not afford making substantial initial investments as M&R budgets are annually allocated. Schedules No. 1 and 2 are the more practical ones, however, both have produced the worst overall improvement outcomes.

Similarly, the impact of variable PCR schedules on generating an optimum 5-year M&R program is investigated using the iterative minimization model. Table 6 provides the optimal results obtained from applying six different PCR schedules. The first PCR schedule is the same one shown in Table 3, which is associated with a 5-year average PCR value of 63.56 at a total cost of \$2.5 million. The other five PCR schedules are equivalently selected to produce the same 5-year average PCR value of 63.56. Schedule No. 2 with equal specified annual PCR values offers the best schedule for it is associated with the lowest total M&R cost of \$1.77 million, which is a major advantage compared to schedule No. 1. However, schedule No. 2 requires \$1.07 million as a first year investment.

Iterative Linear Model Validation

The effectiveness of the iterative approach has been verified by comparing sample results derived from the iterative linear model to those obtained from the compatible multiple targets nonlinear model. Nonlinear optimization methods are mostly search methods requiring either derivatives or functional evaluations. Derivatives can only be attained if the nonlinear model is derived in a closed form, which is very tedious work considering the presented nonlinear models even for a limited number of transitions. Therefore the resulting nonlinear model has been solved using a sequential solution of the penalty function method. The penalty function method transforms the constrained model into a sequence of unconstrained problems. The constraints are placed into the objective function via a penalty parameter in such a way that penalizes any violation of the constraints (Bazaraa and Shetty 1979). The method of Hooke and Jeeves has been used to solve the generated sequences of the penalty function method. The method of Hooke and Jeeves applies only functional evaluations in its search for an optimal solution. It performs two types of search: exploratory search and acceleration search using coordinate directions. This nonlinear optimization approach was found to be effective in solving the nonlinear model associated with the homogenous Markov chain using a limited number of variables (Abaza and Ashur 1999; Abaza et al. 2004).

Table 7 provides samples of optimal results obtained using the three models, namely, the iterative linear model, the compatible nonlinear multiple targets model, and the incompatible nonlinear single target model. The maximization model has been used with an equal budget allocation of \$1.0 million for each year considering a 3-year analysis period. Each nonlinear model is therefore associated with a total of 21 M&R variables as there are seven variables incorporated in each transition matrix. The optimal solution
obtained from the nonlinear multiple targets model is found to be highly compatible to the one derived from the iterative linear model. A total of 11 variables have contributed to the optimal solutions obtained from both models. Both models have reached similar optimal system PCR values for the 3-year analysis period with minor differences in the optimal values associated with 5 M&R variables. The linear model and the compatible nonlinear one have reached a 3-year average system PCR value of 71.32 and 71.29, respectively, at a total M&R cost of \$3.0 million. The minor improvement obtained from the iterative linear model can mainly be attributed to the higher efficiency of the linear programming method compared to the deployed nonlinear optimization method. The incompatible single target nonlinear model has generated a 3-year average system PCR value of 76.94 with only seven M&R variables contributing to the optimal solution which requires a first year investment of \$2.25 million. The computer time required to solve the sample problem using the iterative linear model is very minimal compared to that required to solve the same problem as a nonlinear model using the deployed nonlinear optimization method.

Conclusions and Recommendations

The presented nonhomogenous SPMM model has been demonstrated to be effective in yielding optimum pavement conditions considering both deployed decision policy options. The nonhomogenous SPMM model has retained the feature of incorporating a distinct M&R plan for each time interval within the specified analysis period as evidenced from the presented sample applications. However, the optimization of the compatible complicated nonlinear model has tremendously been simplified when the model is iteratively solved using linear programming. The optimal solution for a particular iteration is used as the input for the subsequent iteration wherein the state probabilities associated with an optimal M&R solution plan become the initial state probabilities for generating the next optimal M&R plan. This approach makes the development of a long-term M&R program very feasible regardless of the length of the deployed analysis period.

It is recommended that both decision policy options be applied to a particular pavement system even though the presented sample results have indicated that the two corresponding maximization and minimization models are quite compatible. The maximization model can be used when transitional budgets are already allocated to produce optimum pavement conditions. The minimization model can be used to determine minimum M&R costs required for meeting desired transitional performance requirements, which can be valuable information in fund raising campaigns. Also, both iterative linear models can be useful in generating optimum M&R schedules as demonstrated in the sensitivity analysis. This can be done by initially conducting an exploratory search to determine a feasible search pattern to be followed by another exhaustive search with the hope of getting near optimal solutions. The entire search process can be programmed which mainly requires interactive access to a linear programming software package.

The main challenge for the effective use of the presented SPMM model is that the deployed M&R actions must carefully be defined so they can produce the intended improvement outcomes. Also, reliable estimates of the transition probabilities and M&R cost rates are required to obtain dependable M&R solution plans. Reliable estimation of the transition probabilities requires conducting a detailed assessment of pavement distress that can result in producing an accurate pavement condition rating for each surveyed pavement section. The M&R cost rates and annual budget can be different for each year, but the transition probabilities are expected to remain unchanged for a period up to 5 years (Butt et al. 1987; Abaza and Ashur 1999; Abaza et al. 2004). However, the transition probabilities must separately be estimated for pavement structures with different historical M&R records. Therefore it is recommended that pavements with similar historical M&R records be considered as an independent pavement system with its own Markov chain.

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