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THE CREATION OF MATHEMATICS IN SCHOOL TEXTBOOKS: PALESTINE AND ENGLAND AS EXAMPLE

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The language of mathematics textbooks, including symbols and diagrams, constructs particular views of the nature of mathematics and expectations about students' participation in mathematical activity. In a collaborative project between the Institute of Education and Birzeit University, we developed an analytic framework for examining the nature of mathematics and mathematical activity in textbooks. This framework, based on those developed by Tang, Morgan, & Sfard (2012) for the verbal mode and by Alshwaikh (2011) for the visual, enabled us to take account of the multimodal nature of mathematical texts. We applied the framework to analyse a sample of topics from the textbooks used in Palestinian schools and to a smaller sample of topics from textbooks commonly used in England. The research showed that, for younger students in both countries, mathematics is construed as involving practical activities. For students in Palestine, however, abstract mathematical reasoning is also prioritised from a much earlier age. This raises questions about how textbooks in the two countries may support students to move towards abstract mathematical reasoning.
Multimodal social semiotics; discourse analysis; nature of mathematics; learner agency

INTRODUCTION

In this paper we present and illustrate an approach to analysis of textbooks that addresses the ways that mathematics and learners are construed. Our study was motivated originally by the many complaints regarding Palestinian mathematics textbooks. In particular, studies have identified that they are densely packed with abstract concepts which makes mathematics a hard topic to learn and teach (Al-Ramahi, 2006; Alshwaikh, 2005). Some have suggested that the nature of textbooks is linked to the very modest achievement of Palestinian students in international and local studies (Rewadi, 2005).

In a project funded by The British Academy "Analysing the Palestinian school mathematics textbooks" (Alshwaikh & Morgan, 2013) we examined mathematics textbooks used in grades 4 to 10 (9 to 15 years old) in Palestine using a multimodal analysis that considers language and diagrams in mathematical meaning making. A comparison was made with textbooks used in the UK. We take the theoretical view that language, diagrams and other systems of communication function to construe the nature of our experience of the world and of the identities and relationships of participants. This view draws on multimodal social semiotics (Halliday, 1985; Kress & Van Leeuwen, 2006; Morgan, 2006). We also draw on the work of Sfard (2008) to consider the characteristics of mathematical discourse.

In order to analyse the textbooks we posed two research questions: What image of mathematics is presented in Palestinian and English textbooks? How is the learner of mathematics presented in these two contexts? We will give a short description of the contexts of the study, followed by the methodology and the analytic framework used in this study. Then we present an illustrative example of our analysis and, finally, introduce some concluding remarks.

THE CONTEXTS: PALESTINE AND ENGLAND

In both Palestine and England, the curriculum is mandated by the state. However, the degree of centralisation and control varies. In Palestine, the Ministry of Education is responsible for producing textbooks and distributes them to governmental schools. The other two types of schools (UNRWA and private) are also obliged to use the same textbooks from grade 1 (age 6 years) to grade 12 (age 18).[1] In contrast, schools in England are free to choose textbooks from a wide range produced by commercial publishers. In this study we chose to analyse textbooks from a series very widely used in schools in London at Key Stage 3 (Years 7-9, age 11-14 years) and a textbook for Key Stage 4 (Years 10-11, age 14-16) published by one of the commercial organisations responsible for setting the national examinations at age 16.

ANALYTIC FRAMEWORK AND METHOD

The first stage of the project was to construct a framework for analysis that would enable us to consider the multimodal nature of mathematical texts. This framework was constructed drawing on two main sources: the analytic framework developed by Tang, Morgan and Sfard (2012) for application to examination papers and that developed by Alshwaikh (2011) for application to geometric diagrams. The major components of the framework addressed in this paper are derived from Halliday's ideational and interpersonal metafunctions of language. An initial version of the framework was produced and iteratively refined through application to sample chapters from both Palestinian and English textbooks. The framework was applied to analyse texts on different topics and at different grade levels in order to check the general applicability of the framework across the school mathematics curriculum. An extract of the developed version of the analytical framework is in Table 1: each component is elaborated by questions that guide our analysis and indicators that allow us to identify relevant characteristics of the verbal and visual text. The structure of this framework is based on that proposed by Tang et al. (2012).

While some textual characteristics were counted during the analytic process, we have only used relative numbers of occurrence of specific indicators to support the construction of qualitative descriptions of the texts. Independent analyses of sample chapters were conducted by each of the two authors as part of the process of refining the framework. These give us confidence in the reliability of the analytic method.

AN EXAMPLE ANALYSIS

To illustrate the application of the framework in Table 1we will use a geometry unit from the Palestinian textbook for Grade 7 (an extract from this unit, translated from Arabic, is shown

[1] There are however, some private schools that follow international curricula, such as International Baccalaureate –IB, IGCSE and SAT, with different textbooks.

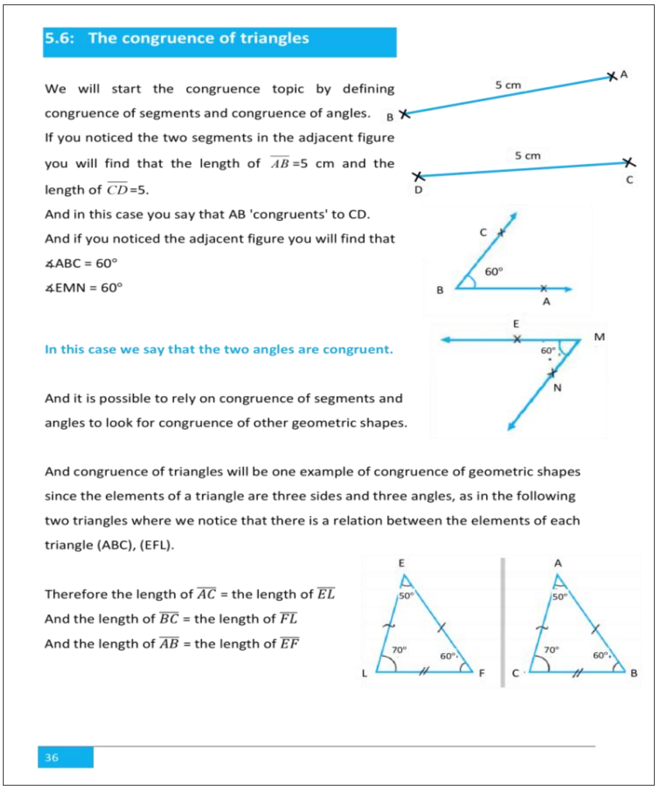
in Figure 1) and a section from an English textbook for the same age group (Year 8) (an extract is shown in Figure 2). Both deal with the topic of congruenceas we wish to compare the treatment of the same topic in both contexts. Given the limited space available in this paper, we will give details of the analysis using only that part of our framework shown in Table 1.

Table 1: An extract of the analytic framework, showing its structure

<table>
<tr><th></th><th>property of the discourse</th><th>specific questions guiding analysis</th><th>indicators in verbal text</th><th>indicators in visual text</th></tr>
<tr><td rowspan="2">How is the nature of mathematics and mathematical activity construed?</td><td>specialisation</td><td>To what extent is specialised mathematical language used?</td><td>- vocabulary used in accordance with mathematical definitions
- ‘conventional’ expressions
- mathematical symbols</td><td>- ‘conventional’ mathematical diagrams, charts, tables, graphs and labelling systems
- Conceptual diagrams (showing properties and relationships, not actions)</td></tr>
<tr><td colspan="4">further properties include:
objectification, alienation, logical structure, status of mathematical knowledge</td></tr>
<tr><td rowspan="2">How are the learners and their relationship to mathematics construed?</td><td>agency</td><td>What kind of activity is the learner expected to engage in?
What possibilities are there for learners to make decisions?</td><td>- ‘thinker’ or ‘scribbler’ processes ascribed to the learner
- imperative instructions or open questions?</td><td>- presence or absence of labelling (suggesting form of engagement with diagrams)</td></tr>
<tr><td colspan="4">further properties include:
authority; formality</td></tr>
</table>

The nature of mathematics and mathematical activity

Our framework identifies a number of properties that contribute to the image of mathematics and mathematical activity. Here we focus only on the issues arising from analysis of the **specialisation** of the two texts. The Palestinian text has a high density of specialised mathematical words (*congruence*, *segments*). Mathematical symbols ($\overline{AB}$, $\measuredangle$ABC) are used both within the verbal parts of the text and in independent symbolic statements. The diagrams consist of representations of named mathematical objects such as triangles and segments. These objects are identified conventionally by letters labelling vertices and their properties are communicated by conventional marks on the sides and angles. Most of the diagrams are conceptual (Alshwaikh, 2011), displaying the properties of objects and relationships between them rather than representing a process; the dominance of conceptual visual elements is a common characteristic of specialised scientific and mathematical text (Kress & Van Leeuwen, 2006).



5.6: The congruence of triangles

We will start the congruence topic by defining congruence of segments and congruence of angles.

If you noticed the two segments in the adjacent figure you will find that the length of $\overline{AB}$=5 cm and the length of $\overline{CD}$=5.

And in this case you say that AB 'congruents' to CD.

And if you noticed the adjacent figure you will find that

∡ABC = 60°

∡EMN = 60°

In this case we say that the two angles are congruent.

And it is possible to rely on congruence of segments and angles to look for congruence of other geometric shapes.

And congruence of triangles will be one example of congruence of geometric shapes since the elements of a triangle are three sides and three angles, as in the following two triangles where we notice that there is a relation between the elements of each triangle (ABC), (EFL).

Therefore the length of $\overline{AC}$ = the length of $\overline{EL}$

And the length of $\overline{BC}$ = the length of $\overline{FL}$

And the length of $\overline{AB}$ = the length of $\overline{EF}$

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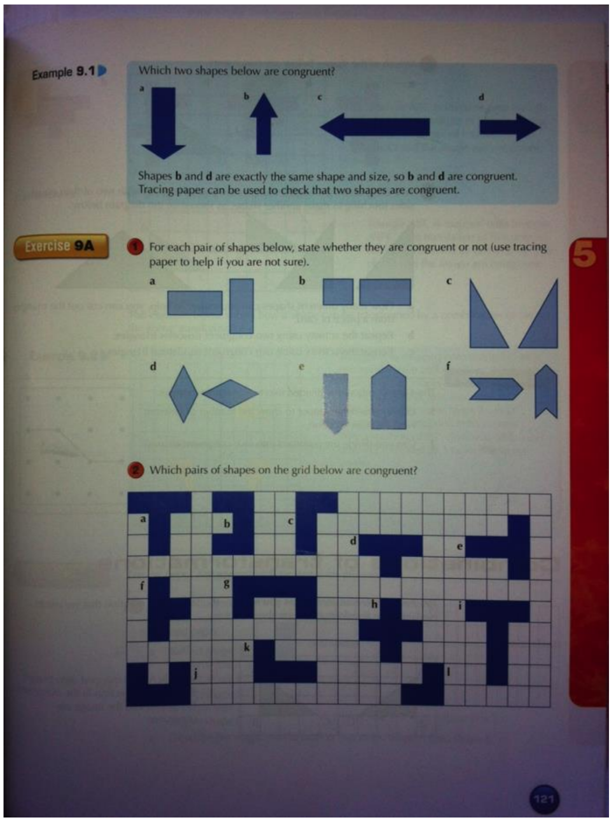

Example 9.1 Which two shapes below are congruent?

Shapes **b** and **d** are exactly the same shape and size, so **b** and **d** are congruent.
Tracing paper can be used to check that two shapes are congruent.

Exercise 9A

1 For each pair of shapes below, state whether they are congruent or not (use tracing paper to help if you are not sure).

2 Which pairs of shapes on the grid below are congruent?

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Figure 1: extract of (translated) Palestinian text

Figure 2: extract of English text

In the English text, the density of specialised vocabulary is lower. There is more "everyday" language, referring, for example, to *shapes* rather than naming specific mathematical objects. There is also reference to the use of practical equipment such as *tracing paper*, *piece of card*, etc., making it clear that the text is about concrete school activity rather than abstract mathematical reasoning. There is almost no use of mathematical symbolism. The diagrams include a wide variety of irregular objects, contrasting strongly with the overwhelming focus on triangles in the Palestinian text. Although, as in the Palestinian text, most of the diagrams are conceptual, the diagrams in the English text lack conventional features; in most cases the vertices of the polygons are not labelled. This lack of labelling is related to the lack of mathematical symbolism elsewhere in the text.

In summary the Palestinian text maintains the mainstream conception about mathematics as timeless, impersonal and dealing with a specialised domain that is separate from everyday experience (Davis & Hersh, 1981; Morgan, 1996). While this specialised image of mathematics is also present in the English text, this text is more mixed, containing features that link to more everyday and concrete objects and activities and lacking use of conventional notations.

The nature and role of the learner

In order to address the question of learner **agency**, we distinguish between engagement in material processes (e.g. *measure*, *calculate*) that construe a role as a 'scribbler' and in mental processes (e.g. *consider, prove*), construing a 'thinker'. According to Rotman (1988), doing mathematics involves undertaking both of these roles: performing operations and reflecting on them. The Palestinian text engages learners (using an inclusive *we* or *you*) in mental processes (e.g. *define*, *notice*) as well as in material processes (e.g. *find*) that construe learners as 'scribblers'. In the section shown in Figure 1, these roles are combined (*if you noticed the adjacent figure you will find*). There is thus expectation that learners will be 'thinkers' (e.g. *show*, *prove, notice*, *consider*), engaged in observation, reflection and reasoning as well as operating on mathematical objects. In the visual component of the text, figures are labelled with specific measurements or marks indicating equality. Learners are thus construed as observing and reasoning about the properties of the shapes. The learners' activity is elicited not only through imperatives but also through use of questions allowing choice in the mode of response, (e.g. *If you try to measure AB and RP, what do you notice?).*

The English text construes learners as active mainly in 'scribbler' activity, including manipulation of concrete objects. Moreover, where there are mental processes, such as *notice*, these tend to refer to observation of facts rather than to engagement in reasoning. In the visual component of the text, the lack of labelling with measures or marks indicating properties suggests that the task of determining congruency is one of visualisation rather than analysis. In some cases, shapes are positioned on grids, suggesting that learners might be expected to check equality of lengths by counting. The use of questions such as: *Which pairs of shapes are congruent?* allows a choice of method. However, the absence of any formal definition, method or example of reasoning and the marking of use of concrete manipulation as less desirable (*use tracing paper to help if you are not sure*) mean that learners seem to be expected to rely on everyday notions of shape and size and informal methods of visualisation and measurement rather than focussing on specific properties.

In summary, the Palestinian learners appear to be expected to engage in both the scribbler and thinker activities that make up the work of a mathematician. They are construed as attending to specific mathematical properties of shapes and reasoning about these. In contrast, the English learners are construed as using their own informal methods to compare shapes and the thinker role is largely absent or implicit.

CONCLUDING REMARKS

The two texts deal with the same topic of congruency in different ways: While the Palestinian text uses a specialized discourse, emphasising formally defined objects and reasoning about properties, the English text uses a mixed discourse, emphasising processes of practical manipulation and visualisation. Both sets of learners are expected to engage in material activity; the Palestinian text, however, also construes the learner as a 'thinker'. The combination of scribbler and thinker activity suggests that the Palestinian text as a whole seeks to apprentice learners into specialised mathematical discourse (cf. Dowling, 1998), while the English text does not offer this opportunity for apprenticing.

However, while Palestinian textbooks appear to construe learners as engaged in reasoning, the actual teaching and learning of mathematics in Palestinian schools reflects a different reality. Observation of a number of geometry lessons in Palestine suggests that, in practice, teaching focuses mainly on the sections of the textbook identified as ‘Exercises and Problems’, which construe a predominantly scribbler role. Solving problems usually come after an example, and the student is expected to mimic what has been done in the example, using similar wording and a similar problem. Students are thus expected to act as scribblers: to follow what the authors have done or simply to repeat the activity already illustrated. Our analytical framework allows us to identify what opportunities are offered by the texts but the extent to which these are manifested in classrooms depends on how teachers make use of the texts provided. Knowledge of the context is thus essential to interpretation of the text.

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