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A Fault Detection Scheme for Uncertain Switched Systems Under Asynchronous Switching

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Abstract: The main focus of this paper is on analysis and design issues concerning with fault detection in asynchronous switching systems with uncertainties. To this end, the existence conditions for the application of observer-based fault detection systems are first derived. Based on these conditions, an observer-based fault detection scheme is proposed both for the matched and unmatched switching operations. The core of this scheme is a dynamic threshold. The theoretical results are finally illustrated by a numerical example.

Keywords: Switched system, $\mathcal{L}_2$ observer, fault detection, integrated design, linear matrix inequality, dynamic threshold.

1. INTRODUCTION

Over the past few decades, due to the increasing demands in facility safety and reliability, the researches on fault detection (FD) for complex dynamical systems have also been rapidly increased. As a comprehensive method of FD, observer-based fault detection techniques for different categories systems have received considerable attention. In addition, advancement have also been achieved in both model-based (Ding, 2013) and data-driven frameworks (Ding, 2014). Until now, significant efforts have been made in the integrated design of the fault detection system including an observer-based residual generator, a residual evaluator and the decision-making unit with a threshold, such as (Hwang et al., 2010).

The hybrid system is proposed as one of the settlement to analyze and study the complex systems. Gain scheduling methods have also been investigated for the analysis and integrated design of different types of complex systems, such as Markov jumping systems (Shi et al., 2013), T-S fuzzy systems (Li et al., 2016), linear parameter varying (LPV) systems (Stilwell & Rugh, 1997) and Switched Systems (Hai & Antsaklis, 2009). An estimation of the residual evaluation function variance in the fault-free case is recursively computed and consequently applied in a threshold setting (Saijai et al., 2012).These mathematical models attempt to fit the whole complex system by different subsystems and parameters or switching signals.

As an important branch of hybrid systems, switched systems have stimulated the great interest of researchers in both theoretical study and real applications. Switched systems consist of a class of subsystems and a logical switching rule, which can be applied to describe a large class of complex systems. Recent advancements in switched systems and other complex systems have been studied in many fields, such as filter design, robust control and fault detection. In (Hai & Antsaklis, 2009), stability and stabilizability analysis based on Lyapunov method are introduced, such as common Lyapunov method, multiple Lyapunov method and piecewise Lyapunov method. Another attempt of the stability issue that has been studied intensively is the average dwell time (ADT) method (Wu et al., 2010). Based on the stability research of switched systems, filtering or state estimation (Su et al., 2016) and fault detection (FD) (Han et al., 2017) have emerged as significant research areas. Observer-based fault detection has been studied for switched systems and considerable achievements have been made. FD issue for discrete-time and continuous-time switched systems has been discussed in (Li & Zhong, 2013) and (Wang et al., 2010) respectively. However these researches are proposed for the case that models and observers operating in synchronous manner. In practical applications, because of the time lag between the sub-models and the observers, the phenomenon of asynchronous switching between an observer and a subsystem generally exists. In (Wang et al., 2016) asynchronous switching problem in fault diagnosis has been explored, and a maximum dwell-time approach is employed to deal

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with the asynchronous switching control problem. Besides these achievements, non-synchronized FD issue and decision rule for the asynchronous switched system are necessary to be further studied. Motivated by these observations, switched systems with asynchronous manner between sub-models and observers are considered in this paper.

The main objective of this paper is to address the analysis and integrated design of the $\mathcal{L}_2$ observer-based fault detection for switched systems with uncertainties. To this end, the existence condition for $\mathcal{L}_2$ observer-based fault detection, the analysis and integrated design of the fault detection system is studied first. An observer-based residual generator is developed in both the synchronized and non-synchronized case respectively. Then a dynamic threshold is proposed and a procedure for estimating the switching mode is developed.

This paper is organized as below. Preliminaries for $\mathcal{L}_2$ observer-based FD for asynchronous switched systems and problem formulation are given in Section 2. Section 3 is dedicated to the design scheme of the $\mathcal{L}_2$ observer-based FD. Further on the decision rules are proposed in Section 4. Finally, a numerical example is given in Section 5.

Notations: In this paper the notations adopted are fairly standard. $R^+ = [0, \infty)$. A function $\gamma : R^+ \rightarrow R^-$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and satisfies $\gamma(0) = 0$; it is of class $\mathcal{K}_\infty$ if $\lim_{k\rightarrow\infty} \gamma(k) = \infty$. $\|\cdot\|$ denotes the Euclidean norm of a vector. $L_{2,[0,\tau]} - norm$ of $u(k)$ is defined by $\|u_\tau\|_2 = (\sum_{k=0}^{\tau} \|u(k)\|^2)^{1/2}$. The set of real matrices with dimension $m \times n$ is denoted by $R^{m\times n}$. For a matrix $A \in R^{n\times n}$, the transpose is represented by A^T. Denote that $\texttt{Sym}\{A\} = A^T + A$. In a symmetric matrix, $\star$ represents the symmetric elements. In quadratic form $A^T PA$, $[\star]$ represents the transfer of aforementioned matrix, for instance $A^T PA$ denotes $A^T P[\star]$.

2. PRELIMINARIES AND PROBLEM FORMULATION

In this paper, an FD scheme for uncertain discrete-time switched systems is studied.

Consider the following class of uncertain discrete-time switched systems:

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + \Delta_A x(k) \\ \qquad\qquad +\Delta_B u(k) + M_d d(k) + E_{f,\sigma(k)} f(k) \\ y(k) = C_{\sigma(k)}x(k) + D_{\sigma(k)}u(k) + \Delta_C x(k) \\ \qquad\qquad +\Delta_D u(k) + N_d d(k) + F_{f,\sigma(k)} f(k) \end{cases} \tag{1}$$

where $x(k) \in \mathcal{R}^{n_x}$ is the system state with initial state $x(0) = x_0$, and $y(k) \in \mathcal{R}^{n_y}$ is the measurable output. $u(k) \in \mathcal{R}^{n_u}$ is the input signal, $d(k)$ is the unknown input vector with $\mathcal{L}_2$ bounded, $\|d(k)\|^2 \leq \delta_d^2$. M_d and N_d are known system matrices with appropriate dimensions. $\sigma(k) \in \Gamma, \Gamma = \{1, 2, 3, \ldots\}$ is the switching signal that specifies which subsystem is activated at the switching instant, the switching rules are random variables independent of all uncertainties. Δ_A, Δ_B, Δ_C and Δ_D are uncertain matrices which satisfy the following assumption.

Assumption 1. The uncertainties of the system (1) are assumed to be norm bounded, which satisfy

$$\begin{bmatrix} \Delta_A & \Delta_B \\ \Delta_C & \Delta_D \end{bmatrix} = \begin{bmatrix} E \\ F \end{bmatrix} \Delta(k) \, [\, G \;\; H \,], \tag{2}$$

where E, F, G, H are known matrices with appropriate dimensions and $\Delta^T(k)\Delta(k) \leq \delta_\Delta^2 I$.

The observer-based FD system for uncertain switched systems can be constructed by an observer-based residual generator, an evaluation function and a decision maker with a dynamic threshold. The dynamics of observer-based residual generator is given by

$$\begin{cases} \hat{x}(k+1) = A_{\sigma'(k)}\hat{x}(k) + B_{\sigma'(k)}u(k) \\ \qquad\qquad +L_{\sigma'(k)}(y(k) - \hat{y}(k)) \\ \hat{y}(k) = C_{\sigma'(k)}\hat{x} + D_{\sigma'(k)}u(k) \\ r(k) = y(k) - \hat{y}(k), \end{cases} \tag{3}$$

where the $\hat{x}(k) \in \mathcal{R}^{n_x}$ denotes the estimation of state $x(k)$, $\hat{y}(k) \in R^{n_y}$ is estimation of output, $r(k) \in R^{n_y}$ represents the residual signal. $\sigma'(k)$ is the switching rules of the observer. In this study the switching signal is unknown. As shown in switched systems (1) and residual generator (3), the switching rules are different.

Since the switching signals of the switched systems and the observer-based residual generator are asynchronous, the dynamics of the process and the residual generator can be described by

$$\begin{cases} x(k+1) = (A_i + \Delta_A)x(k) + (B_i + \Delta_B)u(k) \\ \qquad\qquad +M_d d(k) + E_{f,i} f(k), \\ y(k) = (C_i + \Delta_C)x(k) + (D_i + \Delta_D)u(k) \\ \qquad\qquad +N_d d(k) + F_{f,i} f(k), \end{cases} \tag{4}$$

and

$$\begin{cases} \hat{x}(k+1) = A_j\hat{x}(k) + B_j u(k) + L_j(y(k) - \hat{y}(k)) \\ \hat{y}(k) = C_j\hat{x} + D_j u(k) \\ r(k) = y(k) - \hat{y}(k), \end{cases} \tag{5}$$

where $i \in \Gamma$ represents the sequence number of the sub-model, $j \in \Gamma$ is the sequence number of the corresponding residual generator.

As a result of the synchronous switching mechanism, there are two switching modes (SM), as described below.

- SM1: both the system under consideration and the observer are running with the “dynamics” corresponding to the same sub-model before the switching.
- SM2: the subsystem and the observer are running the different dynamics corresponding to the different sub-model before the switching.

In this paper, the main focus is on the analysis and integrated design of observer-based fault detection systems (4), the following definition (Li et al., 2016) is introduced.

Definition 1. (Li et al., 2016) System (1) is called $\mathcal{L}_2$ reconstructible if there exists a system (3), such that $\forall x, \hat{x}$

$$\sum_{k=0}^{\tau} \varphi_1(\|r(k)\|) \leq \sum_{k=0}^{\tau} \varphi_2(\|\tilde{u}(k)\|) + \gamma_0(x(0), \hat{x}(0)), \tag{6}$$

where $\tilde{u}(k) = [u^T(k) \quad d^T(k)]^T$. $\varphi_1(\cdot) \in \mathcal{K}$, $\varphi_2(\cdot) \in \mathcal{K}_\infty$, and $\gamma_0(\cdot) \geq 0$ is a finite constant for given $x(0), \hat{x}(0)$.

Remark 1. It is worth mentioning that in the definition of $\mathcal{L}_2$ reconstructible in (Li et al., 2016), $u(k)$ represents the deterministic input. In this paper, $\tilde{u}(k)$ expands to both the known and unknown input signals.

Theorem 1. Given system (1) and observer-based residual generator (4), if there exist (i) a set of switched Lyapunov functions, $V_{ij}(k, x(k), \hat{x}(k)), i, j \in \Gamma$, (ii) functions $\varphi_1(\cdot) \in \mathcal{K}, \varphi_2(\cdot) \in \mathcal{K}_\infty$, such that, $\forall i, j, l, q \in \Gamma$,

$$\begin{aligned} V_{lq}(k+1, x(k+1), \hat{x}(k+1)) - V_{ij}(k, x(k), \hat{x}(k)) \\ \leq -\varphi_1(\|r(k)\|) + \varphi_2(\|\tilde{u}(k)\|), \end{aligned} \tag{7}$$

then the system (1) is $\mathcal{L}_2$ reconstructible.

Proof. Let $V(k+1, x(k+1), \hat{x}(k+1)) = V_{lq}(k+1, x(k+1), \hat{x}(k+1))$ and $V(k, x(k), \hat{x}(k)) = V_{ij}(k, x(k), \hat{x}(k))$ be the switched Lyapunov functions, where $i, j, l, q \in \Gamma$ satisfy

$$\begin{aligned} \{l,q\} &= \{i,j\} \quad , \text{ if there exists no switching at } k; \\ &= \{l,j\} \quad , \text{ if the system switching occurs at } k; \\ &= \{i,q\} \quad , \text{ if the observer switching occurs at } k; \\ \{l,q\} &\neq \{i,j\} \quad , \text{ if the switching occurs both to the model and the observer at } k. \end{aligned}$$

It is evident from (7) that

$$\begin{aligned} V(k+1, x(k+1), \hat{x}(k+1)) - V(0, x(0), \hat{x}(0)) \leq \\ -\sum_{k=0}^{\infty} \varphi_1(\|r(k)\|) + \sum_{k=0}^{\infty} \varphi_2(\|\tilde{u}(k)\|). \end{aligned}$$

Then, we have

$$\sum_{k=0}^{\infty} \varphi_1(\|r(k)\|) \leq \sum_{k=0}^{\infty} \varphi_2(\|\tilde{u}(k)\|) + V(0, x(0), \hat{x}(0)). \tag{8}$$

Thus, the proof is completed. ■

The $\mathcal{L}_2$ reconstructibility serves as a sufficient condition for the existence of an $\mathcal{L}_2$ observer-based FD system and threshold setting.

If the conditions given in Theorem 1 are feasible, the $\mathcal{L}_2$ observer-based FD system can be applied. In this paper, $\mathcal{L}_2$ restructibility serves as a sufficient condition for the existence of an $\mathcal{L}_2$ observer-based FD system and threshold setting.

Notice that (7) can be formulated as

$$\begin{aligned} V_{lq}(k+1, x(k+1), \hat{x}(k+1)) - V_{ij}(k, x(k), \hat{x}(k)) \\ \leq -r^T(k)r(k) + \gamma^2 \tilde{u}^T(k)\tilde{u}(k), \end{aligned} \tag{9}$$

with

$$\varphi_1(\|r(k)\|) = r^T(k)r(k),$$
$$\varphi_2(\|\tilde{u}(k)\|) = \gamma^2 \tilde{u}^T(k)\tilde{u}(k),$$

where γ^2 is a positive constant. In real applications, a moving window $[t_1, t_2]$ is implemented for the real-time fault detection by defining the evaluation function as

$$J = \sum_{k=t_1}^{t_2} r^T(k)r(k), \tag{10}$$

and the threshold as

$$J_{th} = \sum_{k=t_1}^{t_2} \gamma^2 \tilde{u}^T(k)\tilde{u}(k) + V(0, x(0), \hat{x}(0)). \tag{11}$$

3. ASYNCHRONOUS FD APPROACH FOR SWITCHED SYSTEMS

In this section, the integrated design of FD systems for switched systems is proposed.

Let the error signal between the state and the estimation as

$$e(k) = x(k) - \hat{x}(k),$$

and the vector $\tilde{x}(k)$ is set to contain both information of the state and the estimation

$$\tilde{x}(k) = [e^T(k) \quad x^T(k)]^T.$$

For our purpose, we first rewrite the observer-based residual generator as

$$\begin{cases} \tilde{x}(k+1) = (\bar{A}_{lq} + \bar{\Delta}_{A_lq})\tilde{x}(k) + (\bar{B}_{lq} + \bar{\Delta}_{B_lq})\tilde{u}(k), \\ r(k) = (\bar{C}_{lq} + \bar{\Delta}_{C_{lq}})\tilde{x}(k) + (\bar{D}_{lq} + \bar{\Delta}_{D_{lq}})\tilde{u}(k), \end{cases} \tag{12}$$

where

$$\begin{aligned} \bar{A}_{lq} &= \begin{bmatrix} A_q - L_q C_q & (A_l - A_q) - L_q(C_l - C_q) \\ 0 & A_l \end{bmatrix}, \\ \bar{B}_{lq} &= \begin{bmatrix} B_l - B_q - L_q(D_l - D_q) & M_d \\ B_l & 0 \end{bmatrix}, \\ \bar{C}_{lq} &= \begin{bmatrix} -C_q & C_l - C_q \end{bmatrix}, \\ \bar{D}_{lq} &= [D_l - D_q \quad N_d], \end{aligned}$$

and

$$\begin{aligned} \bar{\Delta}_{A_{lq}} &= \begin{bmatrix} 0 & -L_q \Delta_C + \Delta_A \\ 0 & \Delta_A \end{bmatrix}, \bar{\Delta}_{C_{lq}} = \begin{bmatrix} 0 & \Delta_C \end{bmatrix}, \\ \bar{\Delta}_{B_{lq}} &= \begin{bmatrix} -L_q \Delta_D + \Delta_B & 0 \\ \Delta_B & 0 \end{bmatrix}, \bar{\Delta}_{D_{lq}} = \begin{bmatrix} \Delta_D & 0 \end{bmatrix}. \end{aligned}$$

Theorem 2. Consider a switched system (1) and residual generator (3), if there exist constants $\gamma^2 > 0, \epsilon > 0, \delta_\Delta^2 > 0$, matrices L_q and

$$P_{lq} = \begin{bmatrix} P_{1,lq} & 0 \\ 0 & P_{2,lq} \end{bmatrix} > 0, P_{ij} > 0, \forall i, j, l, q \in \Gamma,$$

such that the following LMIs are feasible:

$$\begin{bmatrix} \Sigma & M & N^T \\ \star & -\frac{\epsilon}{\delta_\Delta^2} I & 0 \\ \star & \star & -\frac{1}{\epsilon} I \end{bmatrix} < 0, i, j, l, q \in \Gamma, \tag{13}$$

where

$$\begin{aligned} \Sigma &= \begin{bmatrix} -P_{ij} & 0 & \bar{A}_{lq}^T P_{lq} & \bar{C}_{lq}^T \\ \star & -\gamma^2 I & \bar{B}_{lq}^T P_{lq} & \bar{D}_{lq}^T \\ \star & \star & -P_{lq} & 0 \\ \star & \star & \star & -I \end{bmatrix}, \\ G_l &= \begin{bmatrix} -P_{1,lq}^T L_q & P_{1,lq}^T \\ 0 & P_{2,lq}^T \end{bmatrix}, \quad K = \begin{bmatrix} I \\ 0 \end{bmatrix}, \\ M &= [0 \quad G \quad H \quad 0], \quad N = [0 \quad G_l \quad K] \begin{bmatrix} E \\ F \end{bmatrix}, \end{aligned} \tag{14}$$

then it holds that

$$r^T(k)r(k) \leq \gamma^2 \tilde{u}^T(k)\tilde{u}(k) + \tilde{x}^T(0)P_0\tilde{x}(0),$$

where P_0 depends on the switching mode at initial time.

Proof. Consider the Lyapunov equations:

$$V_{lq}(k+1,\tilde{x}(k+1)) = \tilde{x}^T(k+1)P_{lq}\tilde{x}(k+1),$$
$$V_{ij}(k,\tilde{x}(k)) = \tilde{x}^T(k)P_{ij}\tilde{x}(k),$$

where i denotes the sequence number of sub-model at time k while l at time $k+1$, index j represents the sequence number of observer at time k and q is for time $k+1$.

It follows Theorem 1 if

$$\begin{array}{c}\tilde{x}^T(k+1)P_{lq}\tilde{x}(k+1) - \tilde{x}^T(k)P_{ij}\tilde{x}(k)\\ +r^T(k)r(k) - \gamma^2\tilde{u}^T(k)\tilde{u}(k) \leq 0\end{array} \quad (15)$$

then the system is $\mathcal{L}_2$ reconstructable. It is easy to see that the left-hand-side (LHS) of (15) is equivalent to

$$LHS(15) = \begin{bmatrix}\tilde{x}_k\\ \tilde{u}_k\end{bmatrix}^T \Psi \begin{bmatrix}\tilde{x}_k\\ \tilde{u}_k\end{bmatrix} \quad (16)$$

where

$$\Psi = \begin{bmatrix}\bar{A}_{lq}^T + \bar{\Delta}_{A_{lq}}^T\\ \bar{B}_{lq}^T + \bar{\Delta}_{B_{lq}}^T\end{bmatrix} P_{lq}\,[\star] + \begin{bmatrix}\bar{C}_{lq}^T + \bar{\Delta}_{C_{lq}}^T\\ \bar{D}_{lq}^T + \bar{\Delta}_{D_{lq}}^T\end{bmatrix}[\star] - \begin{bmatrix}P_{ij} & 0\\ 0 & \gamma^2 I\end{bmatrix}.$$

By applying Schur complement, we have,

$$\Psi = \begin{bmatrix}-P_{ij} & 0 & \bar{A}_{lq}^T + \bar{\Delta}_{A_{lq}}^T & \bar{C}_{lq}^T + \bar{\Delta}_{C_{lq}}^T\\ \star & -\gamma^2 I & \bar{B}_{lq}^T + \bar{\Delta}_{B_{lq}}^T & \bar{D}_{lq}^T + \bar{\Delta}_{D_{lq}}^T\\ \star & \star & -P_{lq}^{-1} & 0\\ \star & \star & \star & -I\end{bmatrix} < 0. \quad (17)$$

Note that the inverse of the Lyapunov function matrices P_{lq}^{-1} is involved in function (17) which leads to the coupling between the system matrices and Lyapunov function matrices. To deal with this issue, a congruence transformation to function (17) by $diag\{I, I, P_{lq}, I\}$ is performed.

Based on the transformation, (17) is feasible if and only if the following inequality holds:

$$\begin{bmatrix}-P_{ij} & 0 & (\bar{A}_{lq}^T + \bar{\Delta}_{A_{lq}}^T)P_{lq} & \bar{C}_{lq}^T + \bar{\Delta}_{C_{lq}}^T\\ \star & -\gamma^2 I & (\bar{B}_{lq}^T + \bar{\Delta}_{B_{lq}}^T)P_{lq} & \bar{D}_{lq}^T + \bar{\Delta}_{D_{lq}}^T\\ \star & \star & -P_{lq} & 0\\ \star & \star & \star & -I\end{bmatrix} < 0. \quad (18)$$

Considering the LHS of (18), we have

$$LHS(18) = \Sigma + H_{0,l}^T\bar{\Delta} + \bar{\Delta}^T H_{0,l},$$

and

$$\bar{\Delta}H_{0,l}^T = \begin{bmatrix}\Delta_1\\ 0\end{bmatrix}[\,0\ G_l\ K\,],$$

where

$$\Delta_1 = \begin{bmatrix}0 & 0\\ \Delta_A^T & \Delta_C^T\\ \Delta_B^T & \Delta_D^T\end{bmatrix},$$
$$G_l = \begin{bmatrix}-P_{1,lq}^T L_q & P_{1,lq}^T\\ 0 & P_{2,lq}^T\end{bmatrix}, K = \begin{bmatrix}I\\ 0\end{bmatrix},$$

together with (2), we have

$$\Delta_1 = \begin{bmatrix}E\\ F\end{bmatrix}\Delta(k)\,[\,0\ G\ H\,].$$

It is known that

$$\begin{aligned}LHS(18) &\leq \Sigma + \mathtt{Sym}\{M^T\Delta^T(k)N^T\},\\ &\leq \Sigma + \frac{\delta_\Delta^2}{\epsilon}M^T M + \epsilon N N^T,\end{aligned}$$

here M, N and Σ are defined in (14).

By Schur complement, (18) can be written as

$$\begin{bmatrix}\Sigma & M & N^T\\ \star & -\frac{\epsilon}{\delta_\Delta^2}I & 0\\ \star & \star & -\frac{1}{\epsilon}I\end{bmatrix} < 0.$$

Thus, the proof is completed. ■

4. DECISION RULE

One of the main objectives of this paper is to judge whether the fault occurs or there is a switching case.

Because switched systems contain several subsystems the parameters of one subsystem differ form others, and the expression of thresholds can also be difficult to the ones of expressed into one unified form. In case to distinguish the faulty condition and switching condition, other observers are introduced. Moreover, by defining a norm of $r(k)$ as the evaluation function there are some functions about evaluation functions and thresholds.

(1) J represents the evaluation function defined in function (10).
(2) $J_{th,ij}$ represent the thresholds when the sequence number of system is ith subsystem and the sequence number of observer is jth, $i, j \in \Gamma$.
(3) $J_{th,max}$ represents the maximum threshold of the whole system.

For the purpose of threshold setting, we first minimize γ^2 subject to (13). Then, the evaluation function J and different thresholds are set as

$$\begin{aligned}J &= r^T(k)r(k)\\ J_{th,ij} &= \gamma^2(u^T(k)u(k) + \delta_d^2) + \tilde{x}^T(0)P_0\tilde{x}(0)\\ J_{th,max} &= \max(\sup_{i,j\in\Gamma} J_{th,i,j}).\end{aligned}$$

When the system is activated, there are two thresholds set to detect the fault, one is the threshold of the ith subsystem and jth observer denoted by $J_{th,ij}$, another is $J_{th,max}$, which is the upper bound of all $J_{th,ij}, i, j \in \Gamma$.

Suppose that at the beginning, the process is working in mode i (ith sub-system), and the residual generator is operating in parallel with the process, we have the following decision rule:

- If $J < J_{th,ii}$, no alarm is given.
- If $J \geq J_{th,ii}$, all the residual generators are activated with the initial value as the current state value of the residual generator. The detection logic is proposed as follows:

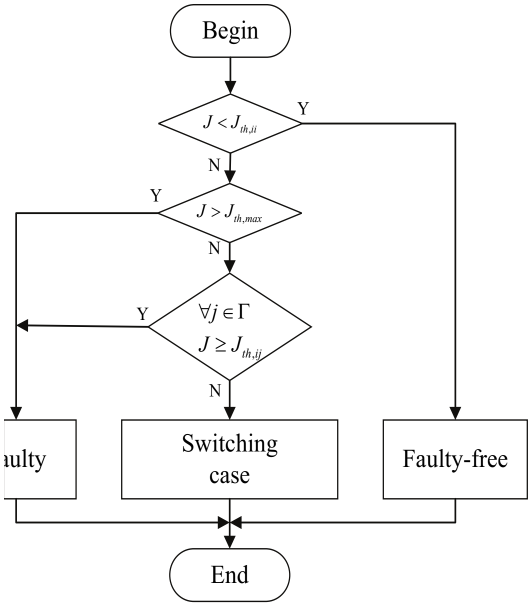

Fig. 1. Decision rule of fault detection scheme.

$$\begin{cases} If\ \exists\ j,\ J \geq J_{th,max} \implies \text{faulty} \\ If\ \forall\ j,\ J_{th,ij} \leq J < J_{th,max} \implies \text{faulty} \\ If\ \exists\ j,\ J < J_{th,ij} \implies \text{fault}-\text{free} \end{cases}$$

If no alarm is given, the process is working in different mode j from observer mode i. The switching mode will be estimated by the following procedure

- If there exists only one j, such that $J < J_{th,ij}$, the process is working in mode $m = j$.
- If there exists more modes $j = l, q, \ldots$ such that $J < J_{th,ij}$, the process is working in the mode

$$m = \arg_j \min_{J<J_{th,ij}} \frac{J}{J_{th,ij}}.$$

Then, the mth residual generator will be implemented for FD purpose.

For the calculation of evaluation function and the threshold, a time window $[t_1, t_2]$ is used which results in a truncated $\mathcal{L}_2$ norm over $[t_1, t_2]$.

5. AN ILLUSTRATIVE EXAMPLE

In this section, one example is performed to demonstrate the effectiveness of $\mathcal{L}_2$ observer-based FD scheme used in uncertain switched system.

Consider the switched system with the following matrices:

SUBSYSTEMS #1:

$$A_1 = \begin{bmatrix} 0.1 & 0.5 \\ -0.05 & 0.2 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix},$$
$$C_1 = [0.1\ 0.2], \quad D_1 = -0.9.$$

SUBSYSTEMS #2:

$$A_2 = \begin{bmatrix} 0.7 & 0.1 \\ 0.5 & -0.2 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix},$$
$$C_2 = [0.1\ 0.2], \quad D_1 = -0.9.$$

SUBSYSTEMS #3:

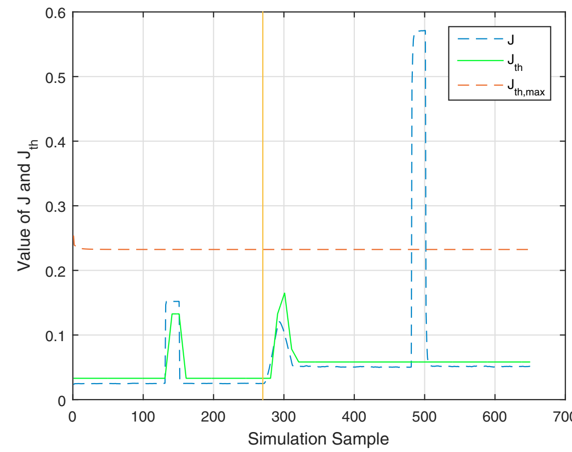

Fig. 2. Fault detection in switched system

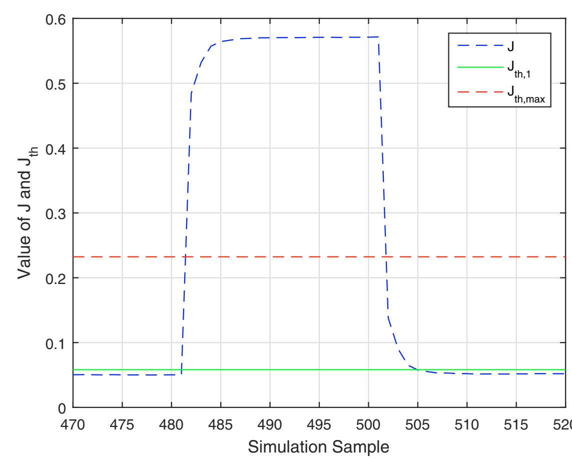

Fig. 3. Faulty case

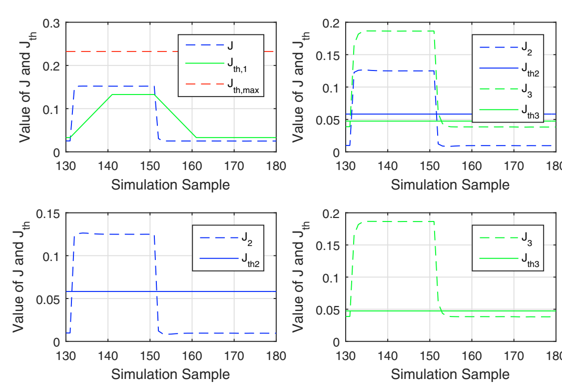

Fig. 4. Common fault case

$$A_3 = \begin{bmatrix} 0.4 & -0.4 \\ 0.5 & -0.2 \end{bmatrix}, B_3 = \begin{bmatrix} 0.1 \\ -0.5 \end{bmatrix},$$
$$C_3 = [0.1 \quad 0.2], \quad D_1 = -0.9.$$

The initial condition is $x(0) = [0 \quad 0]^T$.

$$M = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, N = 0.1. \tag{19}$$

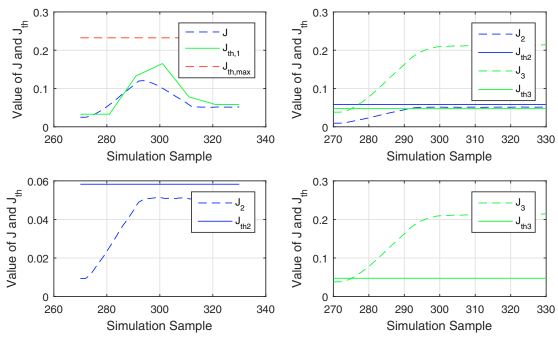

Fig. 5. Switching case

In this study, the input is chosen as $u(k) = 0.3(e^{-0.5k} - 1/k)$ and $d(k) = 0.1$. The uncertainties are described as

$$\begin{bmatrix} \Delta_A & \Delta_B \\ \Delta_C & \Delta_D \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Delta(k) \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \tag{20}$$

with $\delta_\Delta^2 = 10^{-3}$. In the sample time 130 and 480, faults occur, where $f(130) = 1$ and $f(270) = 2$; in sample time 270, the system switches from subsystem 1 into subsystem 2, in sample time 280, the observer switches form 1 into 2.

In Fig. 2., fault cases and the switching case are given. A fault can be observed that $J_1 > J_{th,max}$ (as shown in Fig. 3.). For the case $J_{th,i} < J_i < J_{th,max}$, all the residual generators are activated with the initial value as the current value. If for all residual generators $j \neq 1$ there exist $J_{th,j} < J_j < J_{th,max}$, the fault is observed (as shown in Fig. 4.). If at least one residual generator has that $J_2 < J_{th,2}$, the switching occurs (as shown in Fig. 5.), if no alarm is given, the process is judged switching from mode 1 into mode 2.

6. CONCLUSION

In this paper, we have addressed the design scheme of fault detection systems for a class of discrete-time switched systems with uncertainties. To be specific, the existence condition for an $\mathcal{L}_2$ observer-based FD system has been presented first. Then the design scheme for the FD systems has been addressed for matched period and unmatched period respectively, followed by a decision rule with dynamic threshold. A numerical example has been given in the end to show the effectiveness of the proposed approach.

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