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On the Concept of Material Consequence

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Everyday reasoning is replete with arguments which, though not logically valid, nonetheless harbor a measure of credibility in their own right. Here the claim that such arguments force us to acknowledge *material* validity, in addition to logical validity, is advanced, and criteria that attempt to unpack this concept are examined in detail. Of special concern is the effort to model these criteria on explications of logical validity that rely on notions of substitutivity and logical form. It is argued, however, that such a parallel is not easily located and that it is uncertain that a construal of material validity can be fashioned after traditional accounts of logical validity. Attention is also given to the topics of enthymemes and to the proper domain of logic.

1. Introduction

The analysis of argument validity, implication and logical truth in terms of logical structure or form is familiar to every student of logic. Some writers virtually characterize the science of logic by appeal to this notion, for instance, Alonzo Church, who states that 'logic is concerned with the analyses of sentences or of propositions and of proof with attention to the *form* in abstraction from the *matter*.[1] By utilizing various techniques of paraphrase logicians and mathematicians have been able to scrutinize a wide array of inferences through concentration solely upon the abstract structures exhibited and to develop decisive criteria for adjudging validity. But while this has proved fruitful for many arguments, there are others which, though harboring some degree of credibility, are not, on the surface, sanctionable under acknowledged logical forms. This is so not only as regards mundane arguments like

(A1) This is a collie
therefore, this is a dog,

or

(A2) Albert is a partner of Henry
therefore, Henry is a partner of Albert,

but also with respect to the more sophisticated inference (mathematical, causal or otherwise) that abound in scientific texts and journals. If such arguments are *valid*,

1 Church *1956*, 1; compare Henkin *1967*, 61–62.

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then, assuming that it is correct to regard logic as the study of validity and implication, how might they be analyzed within established provinces of logic?

There are a variety of responses, or types of responses, to this query. Some claim that appropriate transformations into canonical form will reveal underlying logical structures that legitimate the arguments in question. Others argue that the inferences are valid only in some extended sense of 'logic'. Yet another view is that the arguments are *enthymemes* which, in the final analysis, require additional premises in order to become truly valid. Contrary to these approaches, however, is that which maintains that logical validity is only one species of validity *simpliciter*, that there are credible arguments which, though not *logically* valid, are nonetheless *materially* valid. This latter view is sometimes coupled with a further claim, namely that material validity, unlike logical or formal validity, is determined by the *content* of the argument instead of the *form*. Unfortunately, neither of these positions has been developed, and it is worth the effort to investigate them more carefully. In particular, if material validity is to be acknowledged, then something should be said about the competing approaches previously mentioned; and criteria should be advanced that are not only adequate for the cases at hand but which approximate the clarity of accepted construals of logical validity. These matters are the concern of this paper.

2. Substitution and logical form

According to the classical account of 'extra-systematic' validity (Haack *1978*) a conclusion follows from its premises just in case it is *impossible* that the premises be true and the conclusion false. There are, of course, those who repudiate this construal (e.g., Anderson and Belnap *1975*), but let us set their misgivings aside for the present and examine what can be accomplished within classical confines. A traditional manner of deepening the classical account appeals to truth-preservation, as evidenced in the writings of Bernard Bolzano where deducibility is characterized in terms of truth-preservation throughout uniform substitutions upon certain 'variable ideas' occurring in the propositions so related.[2] More recently, it has been customary to express this analysis linguistically by speaking of sentences and distinguishing between logical and extra-logical constants, or, to use Quine's terminology, between lexical items and logical particles. With this, the focus is apparently shifted from talk about validity *per se* to talk about *logical consequence*. One version of the analysis, then, is as follows:

(LSV) A sentence B is a logical consequence of sentences $A_1, \ldots A_n$ *if and only if* where $A_1', \ldots A_n', B'$ are the results of any uniform substitutions upon all the extra-logical constants occurring in $A_1, \ldots, A_n, B$, then B' is true if $A_1', \ldots A_n'$ are all true.[3]

2 Bolzano *1973*, 204–126; and see Kneale and Kneale *1962*, who write that Bolzano interpreted deducibility in terms of propositional structure and went on to apply this idea to the analytic-synthetic contrast as well.

3 See, for example, Tarski *1956*, 415; Quine *1970*, 47–51; and Harman *1973*, 75. It is common to characterize logical truth in terms of substitution and to then define other logical notions, specifically, logical consequence, in terms of logical truth, as in, for instance, Carnap *1937 passim*. With a criterion like (LSV) it must be allowed that each sentence "occurs in" itself despite the oddity of this mode of speech.

Naturally, a criterion of this sort relies on a suitable characterization of 'uniform substitution'. In particular, the substituends must be *appropriate* in the sense that extra-logical constants of a given category can only be replaced by terms of the same category; that is, singular expressions for singular expressions, predicates for predicates, and so forth.[4] The main thrust of (LSV) can be put somewhat differently. In explicating a relationship of logical consequence it is only the logical constants, or the operations expressed by such constants, that *occur essentially* or that carry implicational burden in the sentences so related. The occurrence of any particular extra-logical constant is inessential in that it, or the content it expresses, may be freely substituted for *salva consequentia*. Its occurrence is, in other words, *idle* so far as logical consequence is concerned.

Correlated to the substitutional approach is a charaterization that appeals directly to the notion of logical form. Let us say that a *valid* logical form is one such that every statement (sentence, proposition) having it is true. Intuitively, a valid form is depictable by a matrix or schema containing only logical constants and having only true substitution instances, e.g., '$p \supset p$'. Then, exploiting the equivalence between logical consequence and the logical truth of a conditional, we have

(LFV) A statement B is a logical consequence of statements $A_1, \ldots A_n$ *if and only if* the statement '$(A_1 \ \& \ \ldots \ \& \ A_n) \supset B$' has a valid logical form.[5]

This criterion has one advantage over (LSV); if a ''statement'' is a non-linguistic item possessing logical structure, then we have a construal of logical consequence that does not seem dependent upon language in the way that (LSV) is. In this light one might suppose that (LFV) affords a deeper analysis of logical consequence that confers legitimacy upon the substitutional account (LSV).[6]

The substitutional approach is often contrasted with an alternative explication in terms of models and satisfaction. Tarski, in favoring the latter, felt that the substitutional analysis supplied only a necessary condition for logical consequence if the language in question fails to possess a 'sufficient stock of extra-logical constants'. (Tarski *1956*, 415–416). Quine, however, has argued that the two approaches are

4 Compare Tarski *1956*, 415, whose statement of the substitutional criterion embodies a reference to *like* signs.

5 See, for example, Wittgenstein *1961*, 5.131; Tarski *1956* 417; Strawson *1952*, 50–52; Lewis and Langford *1959*, 340; and Castañeda *1975*, 71. Quine *1966*, 37 also defines a 'valid form' in this way, and it is important to realize that the occurrence of 'every', in the definition given, must be understood as expressing a nomological or lawlike generalization—however, the notions of ''nomological'' and ''lawlike'' are themselves to be interpreted. It should be noted, finally, that the practice of representing logical forms through open sentences in which the only constants are logical constants is adopted throughout.

6 There are several philosophers who speak of the logical forms of non-linguistic items, *viz.*, propositions; for instance, Bolzano *1973*, who wrote of the forms of *Satz* (propositions); Strawson *1952*, ch.2; Castañeda *1975*, ch.3 and *1977*, 298–299 and 327–329; and, most recently, McCawley *1981*, 2–5. One who takes seriously the idea that logical forms belong to propositions might hold that sentences have their forms only derivatively and, similarly, that logical relationships among sentences are parasitic upon logical relationships among the propositions expressed by sentences.

equivalent if the language is 'reasonably rich: rich enough for elementary number theory' (Quine *1970*, 53). If the latter is correct, then (LSV) would require slight modification, though the same need not be so as concerns (LFV) for the reasons given. It is interesting to observe that Tarski, while rejecting the substitutional analysis, retained the idea that formal consequence is a relation 'uniquely determined by the form of the sentences between which it holds'.[7]

3. Paraphrase and the boundaries of logic

The foregoing analyses of logical consequence are unlikely to stir controversy—barring the misgivings of relevance logicians. The more controversial claim is that logical consequence, so construed, exhausts our intuitive notion of validity. It is necessary to evaluate the claim by means of examples provided by natural and scientific languages, examples that are not prejudged through reformulation in preferred linguistic structures. An antecedent investigation of natural language arguments is required, and if these cannot be brought into conformity with the criteria by means of grammatical, though otherwise unfettered, paraphrase then it is unlikely that we can do better with an artificial language.

At first blush, natural languages appear to contain an overwhelming host of counterexamples; for instance, the argument from 'he is a bachelor' to 'he is unmarried', an argument as solidly entrenched as any *modus ponens* inference, is not, at first glance, logically valid. It is commonly believed, however, that terms like 'bachelor' conceal logical complexity, that is, such lexical items are *decomposible* into linguistic units that reveal underlying logical structure. Thus, 'is a bachelor', we learn, means 'is a male and is unmarried' so that the argument actually has the valid logical form

$$\text{`}fx \,\&\, gx, \textit{ therefore, } gx\text{'}.$$

Similarly, 'collie' or 'is a collie' in (A1) is decomposible into a conjunction of predicates one of which is 'is a dog'; hence this argument, too, can be viewed as logically valid. An analogous account could be given of the argument from, 'this is a collie' to 'this has a pointed snout', where having a pointed snout is a differential property of the species in question.

Other cases, while invoking paraphrase, call into question the precise domain of logic. A case in point is mathematics. Assume that some theory framed in set-theoretical terms is capable of generating a desired body of mathematical truths; are the constants of this theory logical constants, and are its principles logical laws? This is, as we all know, a matter of debate. Equally problematic are the modalities. Few would question the validity of arguments taking the form 'necessarily *p*, *therefore*,

7 Tarski *1956*, 414: and see p.417 where, after presenting his analysis of consequence in terms of models and satisfaction, he writes that 'the consequence relation which holds between given sentences is completely independent of the sense of the extralogical constants which occur in these sentences'. Compare Quine *1970*, 58.

possibly *p*', but are such arguments logically valid? Perhaps, if the modalities are themselves logical modalities, but suppose that they are physical or causal modalities? Again, consider the argument

(A3) John knows that he knows that he is handsome
therefore, John knows that he is handsome,

where 'he' is used to attribute first-person reference to John.[8] The argument is credible enough, yet the implicational burden is carried by 'knows'; is this term, too, a logical constant? Are the theses of epistemic or doxastic "logic" to be classified as logical truths? The same question can be raised concerning so-called 'mereological' predicate logic by whose means some (e.g., Massey *1976*) would bring arguments like (A2) within the scope of logic.

The ensuing controversy is widespread and, becoming one with the issue of distinguishing logical from extra-logical constants, difficult to resolve.[9] Where the line is drawn is not crucial for present purposes, but it is important that the matter not be taken lightly. Each adjustment of the boundaries of logic theatens the autonomy of the discipline; as symbolic techniques develop there is a growing threat that what is called 'logic' will usurp metaphysics, epistemology, etc., and encroach upon the overlapping disciplines. The point, of course, is not to disparage symbolism, but only to stress that its mere introduction into a given field of study does not imply that its users are suddenly engaged in logical investigations. Caution must be urged in the application of the terms 'logic', 'logical' and 'logically' if the domain of logic is not to become too diffuse.

Though the boundaries of logic may be unsettled, it is illegitimate to include within them the subject-matter of the physical or social sciences. To be sure, much of the reasoning that occurs within the latter is rightly classified as purely logical or mathematical, but occasionally a given formula or law seems to relate to a given inference as a *rule*, or as a *rule-legitimating principle*, than as a premise. The latter is a common practice in pure mathematics, but it also occurs in the physical sciences as when a theoretician concludes $E = T + m_0c^2$ from $mc^2 = T + m_0c^2$ by using the well-known '$E = mc^2$' as a rule-legitimating principle and *not* as a premise. The said "rule" may never be articulated, though if it were it might read something like

$$\text{'from } f(E) = y \text{ infer } f(mc^2) = y\text{'},$$

where 'f' is a schema for any functional expression. Lack of articulation, of course, does not mean that the rule is not operative in the inference. To be sure, we cannot be certain of what goes on in the minds of mathematicians and physicists when they draw conclusions, but if expressed inferences are any guide then it appears that we

8 The expression 'he' in this context is, in Castañeda's terms, a 'quasi-indicator', i.e., an expression used to attribute indexical reference to agents (see Castañeda *1967* and *1980*).

9 The distinction between "logical" and "extra-logical" has long been recognized as problematic. Recent discussions of the issue can be found in Peacocke *1976*, Haack *1978* and Hacking *1979*.

have unearthed a class of arguments that are not logically valid as they stand and, yet, are plausible in their own right.

Similar remarks pertain to arguments involving predicates which do not seem decomposible, that is, those that express apparently simple properties, as in,

(A4) this is scarlet
therefore, this is extended,

though, admittedly, it is no easy matter to determine which properties are simple and which are complex. Additionally, while the devices of decomposition and paraphrase might bring inferences that move from determinates of species to differentia or determinables, e.g., (A1), into accord with (LSV) and (LFV), there are also those that go from genus or determinable to a disjunction of species or determinates, for instance, from 'this is a human' to 'this is a male or this is a female'. The latter becomes logically valid if determinable properties simply *are* disjunctions of determinates, but there is no way, short of metaphysical perversity, that we can simultaneously analyze determinables into disjunctions of determinates and determinates into conjunctions of their differentia and determinables. So, if logical validity is conferred upon (A1) it must be withheld from the argument that moves from 'this is a dog' to the disjunction 'this is a collie or this is a terrier or this is a dachshund or . . . '—provided that such a disjunction can exist. While some technique of regimentation is suitable to each type of determinable-determinate-differentia argument, there is no one technique suitable for all.

Neither the appeal to paraphrase nor the practice of extending the boundaries of logic is sufficient to render all credible arguments logically valid by means of the criteria (LSV) or (LFV). A third approach insists that the aberrant cases are not valid at all, and to this we now turn.

4. Enthymemes

Labeling an argument an 'enthymeme' is often taken as tantamount to discrediting it; an enthymeme, so goes the story, is, by that very fact, *invalid* as it stands, even though a 'complete' valid argument simmers underneath. Perhaps a word or two about enthymemes will shake the confidence of those who take refuge in this strategy. It must be asked; what is an enthymeme? Traditionally, the notion has been relativized to syllogistic logic; an enthymeme is a syllogism with a missing or suppressed premise. As logic outstriped the syllogism, however, this definition became too restrictive. An alternative is to characterize an enthymeme as any argument with a suppressed premise (Haack *1978*, 245). But this definition is too broad for the approach in question. Consider someone who expresses the following inference:

(A5) I am hungry
therefore, either I am hungry or my brother is hungry,

and yet who believes that the conclusion follows only if the premise is supplemented by the unexpressed conditional 'if I am hungry then either I am hungry or my brother

is hungry'. The latter qualifies as a suppressed premise of the argument, thereby rendering the argument (A5) enthymematic, at least for that person, but those familiar with truth-functions would be unwilling to conclude that the argument is, thus, invalid as it stands (assuming that the disjunction is inclusive). To salvage the strategy the definition can again be relativized, not to syllogisms, but to any reliable system of logical deduction. Quine, for example, writes that an enthymeme is '...a logical inference in which one or more of the premises are omitted from mention on the ground that their truth is common knowledge and goes without saying' (Quine *1974*, 169).[10] Presumably, as a logical inference, that is, a logically valid argument (if this is the correct reading of Quine), is *not* an enthymeme if the expressed premises logically imply the conclusion, no matter what other premises lurk underground; with this characterization the strategy is resurrected.

The notion of a 'suppressed premise' is less innocent than might first appear. Speaking loosely, a suppressed premise would seem to be "in the mind", unexpressed, though in some way operative in the psychological process constituting the inference. It should not be assumed, however, that every inference is actually expressed or that each so-called 'enthymeme' is a truncated expression of some complete argument in the mind. Perhaps a given episode of reasoning is itself enthymematic, in which case the idea of a suppressed premise becomes more problematic. Every inference requires some rule or 'leading principle' (to use Peirce's terminology) which is not included among the premises of the involved argument. How does one know that a suppressed statement—for instance, in the case of (A1), 'every collie is a dog'—is functioning as a *premise* rather than as a *rule*, or, more exactly, as a rule-legitimating principle? Undoubtedly, the issue merits close psychological scrutiny, but it is significant that some investigators have concluded that most everyday inferences cannot be modeled on extant logical calculi, and not all inferential rules are appropriately characterized as "logical" in any standard sense, and that content, in addition to logical structure, is crucial in accounting for *de facto* human reasoning.[11] Their findings suggest that the principles relating material contents enter into inferences as rule-legitimating principles and *not* as premises. As a consequence, it is presumptuous to hold that a person who sets forth what seems to be a plausible argument, but not logically valid as expressed, has some completed *logical* argument in mind.

It can be claimed that such psychological considerations are irrelevant to the objective matter of argument validity. To some extent this is correct, but it must be remembered that an appeal to *suppressed* premises, *ipso facto*, lifts the discussion

10 This definition of 'enthymeme' raises interesting problems if an inference or argument is construed as an ordering of sentences. If a given argument lacks a premise then it is not obvious that the (completed) "logical" inference even exists, unless we are willing to say that the latter is a possible ordering of sentences or an ordering of possible sentences *pace* Quine. The notion of a "suppressed" premise, in fact, suggests a construal of arguments in terms of orderings of *propositions*.

11 See Wason and Johnson-Laird *1972*, 93–95 and 245, who write that no existing formal calculus correctly models ordinary inferences and that 'it is clear that no conventional notion of logical form is viable for the analysis of ordinary deduction' (p.93). More strongly, they conclude that no purely formal calculus could succeed in modelling ordinary inference.

onto the psychological plane. It is best, perhaps, to drop the term 'suppressed' and define an enthymeme as an incomplete argument which when supplemented by an additional premise(s) becomes logically valid. Of course, such talk is elliptical; the original argument does not "become" valid; instead, one locates a further, though correlated, argument and recognizes it to be valid. The definition is peculiar, however, for now *any* invalid argument is an enthymeme no matter how outrageous. Inferences like (A1) and (A2) are no more and no less enthymematic than the reasoning of the hillbilly who concludes that his neighbor Ivan is a communist upon learning that Ivan hails from Russia. Insisting that the added premise be true improves matters little since truths which when combined with premises logically imply a conclusion are plentiful. Any bad argument with a false premise, for instance, 'London is in France, *therefore*, Paris is in Mexico', can be transformed into a valid argument upon supplementation with a material conditional whose antecedent is a conjunction of premises and whose consequent is the conclusion. To leave matters at this point is to fail to do justice to an important aspect of many so-called enthymemes, namely, their apparent credibility *as they stand*. Arguments like (A1) and (A2) will not be objected to as readily as those offered by the hillbilly or by the student who flunked both geography and logic, at least not by any reasonably informed and intelligent person. The weakness of the strategy of resorting to enthymemes is obvious; it simply fails to account for the initial credibility that arguments like (A1) and (A2) have and that flagrant *non sequiturs* lack.

To acknowledge that some inferences are not logical is to admit that reasoning occasionally proceeds by way of *material* inference rules, that is, by what Carnap once called 'P-rules' (Carnap *1937*). One attempt to discredit extra-logical or material validity reasons that since inferences can be carried out by means of logical rules alone, together with an increased stock of premises, material rules are dispensible and, consequently, there is no *need* to countenance any brand of argument credibility beyond that of logical validity. Though such reform of the ways we actually do reason is possible it should be observed that logical rules can similarly be dispensed with by restricting all inferences to a system characterized by material rules alone. Logically valid arguments will be enthymematic with respect to such a system, illustrating that being enthymematic is a relational property of virtually every argument vis-a-vis some inferential system or other. It would be extreme, naturally, to conclude that we have thereby disposed of or reduced logical validity. In sum, dispensability is no more a guarantee of reducibility than being enthymematic is of invalidity.

Related to these positions is a strategy of Carnap's that appeals to 'meaning postulates', but as this approach has been effectively criticized elsewhere discussion of it will be omitted here.[12] Without doubt, the understanding of any expressed inference depends upon knowing the meanings of certain terms used in its formulation. But this should not be taken to imply that material inferences, as expressed, presuppose 'definitions' of extra-logical constants *any more than* logically valid inferences, as expressed, presuppose definitions of logical constants. In this regard, material and logical inferences are on equal footing.

12 See Carnap's 'Meaning postulates', in *1958*, Supplement B, and criticisms of Carnap's views in Pap *1958*, 150–156, 407–416; Quine *1976*, 107–132; and Chisholm *1966*, 89.

There is a long-standing correlation between validity and implication, and also between implication and the necessity of a conditional. The attempts to dispense with material validity and to withhold legitimacy from the arguments mentioned above effectively deny that there is such a thing as extra-logical implication or necessity. This is a controversial position, to be sure, but it has the tenebrous result of coralling many of the common inferences of scientific and everyday thinking into the bin of invalidity. The important distinctions that would be lost through such a move are considerable, and this provides some rationale for examining approaches that take seriously an irreducible relation of material consequence.

5. Material validity

It is often said that while some arguments depend upon *form* others rest their credibility upon *content*. Unfortunately, the second conjunct of this claim is rarely developed, and to leave things at this stage is inadequate for the concerns of logical theory; principles characterizing material consequence are highly desired, if not demanded, as is an explanation why there should be this curious hiatus in the first place. If validity, whether logical or material, just *is* the impossibility of a false conclusion given true premises, as the classical account would have us believe, and if the appeal to substitution and form is a way of explicating this conception for logical validity, then why should not an analogous device work for material validity?

It is significant that the substitutional approach admits of a natural extension; while certain extra-logical constants are idle, so far as implication is concerned, and may be freely replaced, others bear implicational burden and must be regarded as occurring essentially. One criterion that nicely parallels (LSV) is as follows:

(MSV.1) A sentence B is a material consequence of sentences $A_1, \ldots A_n$ *if and only if* (1) B is not a logical consequence of $A_1, \ldots A_n$ and (2) where $A_1', \ldots A_n', B'$ are the results of any uniform substitution upon *some* of the extra-logical constants occurring in $A_1, \ldots A_n, B$, then B' is true if $A_1', \ldots A_n'$ are all true.

The occurrence of 'some' in the second clause must be interpreted as having largest scope otherwise we fail to obtain a sufficient condition for material consequence, that is, (2) is to be read as saying that some of the extra-logical constants occurring in $A_1, \ldots A_n, B$ are such that any uniform substitution upon *those* constants is truth-preserving. It is clear how this criterion can then handle cases like (A1)–(A4); in both (A1) and (A4) the indicator 'this' is idle and can be freely substituted for *salva consequentia*, whle in (A2) and (A3) the occurrences of proper names are inessential. In (A3), however, the first occurrence of 'he' must also be replaced by pronouns appropriate to whatever replaces 'John', though not necessarily the second, since 'he is handsome' is itself, as a whole, replaceable.

Unfortunately, (MSV.1) proves to be too liberal; if we choose our subset of extra-logical constants in such a way that none occurs in the conclusion then any argument will be valid so long as the conclusion is true. Some adjustment is needed, and it is helpful to look at a definition offered in George *1972* of what he calls 'enthymematic

consequence', where some sharing of constants by premises and conclusion is required. Paraphrasing George, the criterion is:

(MSV.2) A sentence B is a material consequence of sentences $A_1, \ldots A_n$ *if and only if* (1) B is not a logical consequence of $A_1, \ldots A_n$ and (2) there is a set S of extra-logical constants occurring in $A_1, \ldots A_n, B$ some members of which occur both in B and in one or more of $A_1, \ldots A_n$ such that if $A_1', \ldots A_n', B'$ are the results of any uniform substitution upon the members of S then B' is true if $A_1', \ldots A_n'$ are all true.[13]

In each of (A1)–(A4) some sharing of constants by premises and conclusion occurs, and (MSV.2) is able to account for the validity of each while circumventing the problem affecting (MSV.1). Since many plausible arguments from natural language do not exhibit such constant-sharing, however, there is still a need for regimentation. There are no shared extra-logical constants in the arguments 'the tallest man is a bachelor, *therefore*, some mammal is unmarried', and no constants appear idle, but the alleged decomposibility of 'bachelor' suffices to bring the argument into line with the new criterion. George cites the example of 'it rains, *therefore*, the streets are wet', and suggests that it is to be paraphrased as 'it rains at t, *therefore*, the streets are wet at t', where 't' is a schema for a replaceable temporal (and spatial) constant (George *1972*, 115). Any substitution upon instances of this schema is truth-preserving. A similar procedure may be followed in accommodating an inference taken from relativistic mechanics where the mathematical formalism conceals implicit constants (or schemata):

(A6) $E = mc^2$
Therefore, $T + E_0 = \int_0^S Fds + m_0 \times$ ½600,000 k/sec.

This inference is materially valid given $E = T + E_0$ and

$$mc^2 = \int_0^S Fds + m_0 \times \text{½600,000 k/sec},$$

assumed within the theory, as principles legitimating material rules of inference. On the face of it, this argument exhibits no shared constants, but this is remedied when it becomes evident that expressions like 'E' and 'm_0' are elliptical for 'the energy of x'

13 This criterion bears some similarity to W.T. Parry's notion of 'analytic implication' (see Parry *1933*) in that a statement A implies a statement B only if there is some sharing of content between A and B. More strongly, Parry's system requires that the conclusion of a valid argument contain no more than what is contained in the premises, i.e., that the conclusion simply "unpacks" what is already present in the premises. A metatheorem of this system is that if B is analytically implied by A then all the variables occurring in B also occur in A; as a consequence, the system does not contain the law of addition $p \rightarrow (p \vee q)$, where '$\rightarrow$' symbolizes analytic implication. See Anderson and Belnap *1975*, 430–432, for a more precise exposition of Parry's system. It should be noted that (MSV.2) does not place such stringent demands upon material validity since it does not imply that whatever is present in the conclusion is already contained in the premises.

and 'the initial mass of x', where 'x' is a schema for constants designating physical objects. Paraphrase frequently brings such implicit constants into the open, illustrating that a wide variety of arguments can, with some plausibility, be brought into conformity with (MSV.2).[14]

An interpretation of material validity in terms of form might seem unlikely; after all, does not the essential occurrence of extra-logical constants indicate that material consequence is determined by content and not form? Does the parallel between substitutivity and form break down at this point? The answer to these questions depends upon what exactly a *form* of a statement is, and this is an issue that has received comparatively little philosophical attention. Traditionally, the primary linguistic access to logical forms has been by way of open sentences or schemata in which some variables occur freely and the only constants are logical constants. One extension of this representational device is to allow the occurrence of extra-logical constants in form-specifying matrices, retaining free occurrences of certain variables, so that 'x is extended', say, specifies a *material* form just as 'x is f' represents a logical form. Accordingly, it would be the presence of some free variables or other rather than any particular constants that indicates form, and it is worth observing that some writers appear to be operating with just some such assumption.[15] Stipulations can be introduced that would forbid the employment of 'form' in this fashion, but the real question is whether the very grounds upon which logical forms are posited, namely, to deepen the classical account of validity or to provide it with additional content, might not also lend credence to the view that the class of forms is broader than is usually thought. Any mystery surrounding this idea is dispelled if forms, in general, are construed as *statement types* relevant to the analysis of implicational relationships and to the classification of statements into various categories. The advantage is a uniform treatment of validity and the sustention of the correlation between form and substitutivity.

This in mind, an analogue of (LFV) can be proposed:

(MFV) A statement B is a material consequence of statements $A_1, \ldots A_n$ *if and only if* (1) B is not a logical consequence of $A_1, \ldots A_n$ and (2) the statement '$(A_1 \,\&\, \ldots \,\&\, A_n) \supset B$' has a valid material form.

With the broadened notion of "form" it is easy enough to locate validating forms for arguments like (A1)–(A4); that of (A4), for instance, is represented by the conditional matrix

'x is scarlet $\supset$ x is extended',

14 George *1972* reports, with respect to his version of (MSV.2), that he has '. . . not been able to find any intuitive enthymemes which are not warranted under the definition, though certain cases yielded only after some analysis' (p.115). His treatment of time involves taking times as individuals open to quantification, a feature of his view that will be exploited in section 6 below.

15 Certain philosophers have used the terms 'form' and 'propositional form' in just this broader fashion, for example, Russell *1964*, 85–88 and *1919*, 158; Lewis and Langford *1959*, 264; and Castañeda *1975*, ch.3. On such views one could hold that constants suffice to differentiate among types of propositional forms.

where 'x' is the sole free variable. Mathematical arguments, if these are outside the boundaries of logic, also submit to a ready analysis; trivially, the argument

'7>5, 5>3, *therefore*, 7>3'

is validated by the form specified by

'$(x>y \;\&\; y>z) \supset x>z$'.

A validating form can even be found for physical inferences like (A6), specifiable by a matrix in which a free variable replaces the implicit constant.[16]

Many questions arise at this point. What sorts of constants can occur in matrices or open sentences that are allowed to specify material forms? Does every open sentence determine some such form? What becomes of the traditional form-content distinction once a criterion like (MFV) is permitted?[17] What *is* a form anyway? Does the posit of material form prove to be of any value in the treatment of material consequence? This latter question, in particular, must be answered affirmatively if the notion of material form can be taken seriously by logical theory.

6. Some difficult cases

There are two types of cases that pose apparent counterexamples to (MSV.2) and for which no intuitive paraphrase is of any help. The first of these are arguments in which every extra-logical constant appears to occur essentially so far as the validity of the arguments is concerned, while the second involves cases in which, though not every constant occurs essentially, truth is not preserved throughout all uniform substitutions. Let us examine each type of case in turn.

From 'Nixon is angry' it follows logically that some individual is angry, and it follows materially that Nixon is an emotive state, assuming that 'Nixon' is a singular expression designating some individual person. It would seem that 'some individual is in an emotive state' is a material consequence of 'Nixon is angry'. Perhaps by introducing a shared temporal parameter, *à la* George, this argument can be adequately handled by (MSV.2). Consider, however, the following:

16 It is difficult, of course, to know *which* scientific inferences are materially valid—this is a matter for specific investigations. Every example will presuppose the correctness of some measure of current scientific theory, and nothing should be thought to depend upon the particular cases cited since each is representative of a given *type* of argument.

17 Some might argue that the posit of material forms defaces the time-honored distinction between form and content since the so-called 'material forms' effectively smuggle in content and, so, illicitly parade under the banner of 'form'. Perhaps such dissenters are correct, but they face the challenge of explaining why it is that only logical constants can occur in form-specifying matrices. Given the usage of 'form' in ordinary English, this stricture is arbitrary; and if there are materially valid arguments, then such constants are not alone in expressing implicationally relevant content. How else is their restriction to be justified? In the absence of such justification another alternative is to insist that all claims about 'form' and 'content' be *relativized* to given propositional or statement types and to acknowledge with F.H. Bradley that there is '...no absolute divorce of matter from form, but there remains after all a relative distinction' (*1883*, 524).

(A7) Nixon is angry
therefore, some individual is, at some time or other, in an emotive state,

The conclusion of this argument is a complete generalization upon the premise in the sense that each extra-logical constant in the premise expresses an element that is generalized with respect to in the conclusion and no such constant is shared. The generalization with respect to time suggests that the introduction of a temporal parameter is of little aid. On the other hand, it could be argued that a proper name like 'Nixon' conveys content, e.g., being an individual, that must be brought out in canonical paraphrase. In a given context, for example, the description 'the individual who was President of the U.S.A. in 1970' might denote exactly what 'Nixon' does, and in this case 'individual' is itself a replaceable constant.

There is a sizeable controversy over this treatment of proper names, however, and its introduction brings with it a theoretical price. It is even less obvious that the first-person indicator 'I' can be replaced by descriptions in such a fashion. Substituting 'I' for 'Nixon' and generalizing to 'person' rather than to 'individual' yields an argument that constitutes an even stronger challenge to (MSV.2):

(A8) I am angry
therefore, some person is, at some time or other, in an emotive state.

But nothing should be thought to turn on the presence of singular expressions, whether proper names, descriptions or indexicals, as evidenced by the following argument:

(A9) some collies are brown
therefore, some dogs are colored,

where both premise and conclusion may be understood as implicitly involving a generalization with respect to time as in the conclusions of (A7) and (A8).[18] Calls for lexical decomposition will, no doubt, arise immediately as regards (A9), but the discussion of determinable-determinate-differentia arguments in section II above should be recalled to forestall hasty conclusions. In addition, attention should also be directed towards cases in which decomposition is of dubious value, as with,

(A10) some blue object is a cube
therefore, something extended is not a sphere,

where 'object' is used in a wider sense than 'physical object'.

18 In cases (A7)–(A9) times are construed as individuals open to quanitification after the manner of George *1972* (see footnote 14 above). This is, however, an accidental feature of these arguments since the same point can be made even if time is handled differently, e.g., through sentential operators or through tensed copulae or predicates. If, on the other hand, predications are timeless then George's strategy of introducing temporal constants to save (MSV.2) is to no avail.

Arguments like (A7)–(A10) can be used to show that material consequence, *à la* (MSV.2), fails to be transitive. The premises of (A9), for instance, implies a proposition expressed by 'some collies are colored', and the latter implies the conclusion of (A9). Both implications are sanctioned by (MSV.2) and the shared constants are replaceable (within limits). Since the premise does not imply the conclusion by (MSV.2), however, then this criterion fails to support transitivity of material consequence and this seems sufficient to render it a failure as an explication of any relation of consequence.

An interesting class of arguments involving implicit constants provide counter-examples of the second sort. These arguments contain principles that have different but overlapping ranges of applicability or domains of discourse. For instance, given current physical theory, certain laws which hold of photons, say, do not hold of all physical entities, e.g., $E = h\nu$ and, hence, $h\nu = mc^2$, where h is Planck's constant. Let us use $E = mc^2$ as well as

$$mc^2 = \int_0^s Fds + m_0 \times \tfrac{1}{2}600{,}000 \text{ k/sec}.$$

as rule-legitimating principles, and suppose that the description 'the first photon emitted in our next experiment with black body radiation' is an implicit constant in the equational expressions. Consider the following argument:

(A11) $E = mc^2$
Therefore, $h\nu = \int_0^s Fds + m_0 \times \tfrac{1}{2}600{,}000$ k/sec.

This argument is on the same footing as (A6) with respect to validity, that is, if (A6) is valid by virtue of the fact that its rule-legitimating principles are true then so is (A11). However, there is an important difference between the two; any object that satisfies $E = mc^2$ will satisfy the conclusion of (A6), but the same does not hold for the conclusion of (A11) since $E = h\nu$, while a law governing the behaviour of all photons, fails for other sorts of objects. So, it is not the case that *every* substitution upon the implicit constant, i.e., 'the first photon emitted in our next experiment with black body radiation', is truth-preserving. Still, there is a sense in which the occurrence of *this* constant is idle and inessential; any other constant designating a *photon* can be substituted for it with truth being preserved. (MSV.2), however, cannot be rescued by this fact nor can it accommodate the credibility of (A11).

No doubt there are simpler cases to illustrate the point concerning different domains. But (A11) has the advantage of being set forth in the relatively clear language of mathematical physics while bearing continuity with the earlier example (A6). Together with (A7)–(A10), it indicates the limitations to the applicability of (MSV.2). More strongly, these five arguments are counter-examples to this principle, and *if* (MFV) is equivalent to (MSV.2) then to (MFV) as well. A further criterion which would drop the demand that any extra-logical constant be shared yet require that both premises and conclusion contain substitutable constants would fare no better with respect to (A7)–(A11). Attempts to defend (MSV.2) that appeal to the concepts of enthymemes and implicit definitions must be measured against the

arguments of section 4 above. The crucial fact that virtually any lawlike generalization can be related to an argument as a rule-legitimating principle cannot be overlooked. It is advisable, hence, to examine an amended account of material consequence that can handle cases like (A7)–(A11).

7. Further proposals and problems

There is an alternative account of material validity that fits within the framework of the substitutional approach and the allied analysis in terms of form. While retaining the idea that each valid argument contains factors that are idle and replaceable it introduces restrictions upon the class of available substitution instances; instead of holding that validity requires truth-preservation throughout all substitutions *per se* it insists merely that truth be preserved throughout all substitutions of a specified sort. To explain, let us recall that the notion of an "appropriate" substituend is central to the substitutional analysis of validity, logical or material, as pointed out in section 2. Previously, this notion was introduced in terms of linguistic types, but it can also be presented in relation to logical categories; an expression E_1 is an *appropriate substituend* of an expression E_2 just in case E_1 and E_2 designate or signify items falling under the same logical category, e.g., logical subjects, predicables, etc . . . It seems fair to construe logical categories as very general sortals or kinds—i.e., as sortal or kind properties or concepts expressible through common nouns or common noun phrases. This in mind, a further relativization is in order; given *any* sortal or kind K, expression E_1 is a *K-appropriate substituend* of expression E_2 just in case E_1 and E_2 designate or signify items or entities of kind K. For example, 'red' is an appropriate substituend of 'angry' with respect to the higher-order logical category of being a predicable. The singular term 'Reagan' is an appropriate substituend of 'Carter' with respect to the category of being a logical subject as well as with respect to the kind (property) of being a person. With this, (MSV.2) can be forfeited in favor of the following:

(MSV.3) A sentence B is a material consequence of sentences $A_1, \ldots A_n$ *if and only if* (1) B is not a logical consequence of $A_1, \ldots A_n$ and (2) there is a set S of extra-logical constants occurring in $A_1, \ldots A_n, B$, some of which occur in $A_1, \ldots A_n$ and some in B, such that for some kinds $K_1, \ldots K_j$, where each c_i in S signifies an item of kind K_i, $1 \leqslant i \leqslant j$, and if $A_1', \ldots A_n', B$, are the results of any uniform replacement of each c_i by a K_i—appropriate substituend then B′ is true if $A_1', \ldots A_n'$ are all true.

The differences between this characaterization and (MSV.2) are two; (i) (MSV.3), unlike (MSV.2), does not require that any constant be shared by premises and conclusion, and (ii) (MSV.3) allows for restrictions to be placed upon the admissable substituends of a constant with respect to *any* kind of entity signified by that constant, whereas (MSV.2) requires that only *logical* kinds, i.e., logical categories, can determine the class of admissible substituends. In connection with this last point it is important to note that (MSV.3) states that an argument is valid just in case there is *some* set of constants substitutions upon which, with respect to *some* kinds, are

truth-preserving. The occurrences of 'some' are crucial, for not every substitution upon a given constant will be truth-preserving—even if the constant and the substituend both designate entities of a given kind K.

(MSV.3) is obviously a complex and, perhaps, unintuitive account of material consequence. Its merit must be shown through a careful examination of particular examples. First of all, given that logical categories are included among the kinds quantified over in the second clause of (MSV.3), it is clear that the criterion is capable of accommodating all the arguments that (MSV.2) can. But how does (MSV.3) help in explaining arguments (A7)–(A11)? Concerning (A11), it is sufficient to restrict the admissible substituends to constants designating *photons* so that the transition from the premise to the conclusion will never involve a shift from truth to falsity given replacements for the implicit constant. In this case, the kind property in question is the sortal "being a photon" while the replaceable constant is, as before, 'the first photon emitted in our next experiment with black body radiation'. Cases (A7)–(A10) require a slightly different move, namely, an acknowledgment that predicate and sortal expressions may themselves signify attributes (concepts) falling under given kinds. For instance, 'angry' expresses not only a specific emotional state, it also signifies a property that can be classified as a state, a mental state, an emotive state, and so forth, while 'individual' expresses a kind of property that is had by, among other things, persons. For (A7), then, the set S of replaceable constants might be the set

{'Nixon', 'individual', 'angry'}

where 'Nixon' can be replaced by any constant designating a person, 'individual' by any constant designating a property of each and every person (e.g., being a living thing), and 'angry' by any constant designating an emotive state; *any* substitution upon *these* constants with respect to *those* kinds of truth-preserving. By means of similar selections and substitutions (A8)–(A10) can also be handled. In addition, because (MSV.3) harbors no requirement of constant sharing it escapes the criticism of (MSV.2) concerning transitivity. Seemingly, (MSV.3) succeeds where (MSV.2) fails.

How does the move to (MSV.3) affect the analysis of material validity in terms of material form? The objections to (MSV.2) apply equally to (MFV) if material forms can only be specified by matrices whose freely occurring variables range over everything, or, for that matter, over all and only elements of given logical categories. If, on the other hand, form-specifying matrices are permitted to contain freely occurring *restricted* variables, i.e., variables whose ranges, with respect to any domain, are determined by specific sortal concepts other than those determining the logical categories, then (MFV) can be retained. Allowing restricted variables in this way is the counterpart to paring down the class of appropriate substituends for constants. To illustrate, 'x_p is angry' expresses a material form just as 'x is angry' does, where 'x_p', unlike 'x', ranges over all and only *persons*. Letting 'f_P' be a predicate variable ranging over all and only kind properties of each and every person (with common noun expressions as substituends), and 'f_E' a predicate variable ranging over emotive states, then a validating form of (A7) is represented by the conditional matrix

'x_P is f_E $\supset$ some f_P is, at some time or other, in an emotive state'.

For (A11), let a perspicuous rendering of the premise by '$E(a) = m(a)c^2$' *where* 'a' abbreviates the implicit constant noted earlier now made explicit, and similarly for the conclusion. A validating form of this argument is specified by

$$'E(y) = m(y)c^2 \supset hv(y) = \int_0^s Fds(y) + \mathrm{m}_0(y) \times \tfrac{1}{2}600{,}000 \text{ k/sec}'$$

—where 'y' is a variable ranging over all and only photons. Arguments (A8)–(A10) receive a similar treatment.

Though they retain vestiges of traditional approaches, (MSV.3) and (MFV) constitute an unwieldy account of material validity. Whatever defense they can be given must be judged in terms of adequacy and comprehensiveness, not elegance. Regrettably, however, (MSV.3), like its predecessor (MSV.1), proves too much; unless some restrictions are placed upon the substitution kinds or upon the replaceable constants arguments will be valid which would not intuitively be so. For example, let A be any true statement and let B be any statement, true or false; it follows that the statement (sentence) $B \supset A$ is true. There *seem* to be cases, however, where A is not a *consequence* of B even if this holds, *viz.*, where A is not a necessary truth and B is not a necessary falsehood, e.g., where A is 'I am now reading this paper' and B is 'I am now 75 miles away from Sophia Loren'. However, given $B \supset A$, B is a member of the class of statements determined by the following kind property 'being a statement p such that $p \supset A$', and A is a member of a class of statements determined by the kind 'being a statement p such that $B \supset p$'. Any substitution upon B and A with respect to those kinds will be truth-preserving, so, by (MSV.3), it follows that A is a (material) consequence of B. This result runs contrary to a widely held belief that there are true statements which are implicationally independent of each other, regardless if the implication or consequence relation in question is logical or material.

The same phenomenon emerges with respect to substitutions upon individual and predicate constants. Consider a village in which the police chief is, albeit contingently, the mayor, and suppose that both 'the police chief is angry' and 'some mayor is, at some time or other, in an emotive state' are true. Does the former *imply* the latter? A negative response seems in order; surely the latter *could* be false even if the former is true. In fact, a valid argument appears only when the sentence 'the police chief is the mayor' is added as a premise, and it is important to notice that this sentence does not express a *principle* capable of legitimating an inference rule. As such, the argument from 'the police chief is angry' to 'some mayor is, at some time or other, in an emotive state' is not on a par with arguments (A1)–(A11). On (MSV.3), however, this argument turns out valid since all substitutions upon 'angry' with respect to the kind "being an emotive state" and upon 'mayor' with respect to the kind "being an attribute of the police chief" are truth-preserving. (MSV.3) is, therefore, an embarrassment. Analogous considerations apply to (MFV) if this criterion is joined by the liberal policy of representing material forms by means of matrices containing free variables of arbitrary restriciton.

There are ways of salvaging (MSV.3) and (MFV). One might allow that every true proposition is, in some sense, *necessary*, so that there is no oddity in saying that B

implies A whenever $B \supset A$ is true. But embracing this Spinozistic perspective is a drastic move, too drastic for many that are otherwise sympathetic to material validity. An alternative is to modify (MSV.3) by requiring that the substitution kinds be *essential* to the items designated by the replaceable constants. Similarly, concerning (MFV), it can be mandated that the kinds or concepts to which variables are restricted be essential to the values of those variables. With these strictures the counterexamples could be blocked; as to the first, B is not essentially a statement p such that $p \supset A$, while, regarding the second, 'mayor' does not express a kind property that is, essentially, an attribute of the police chief. But while moves of this sort might be welcome to some, it must be observed that they generate the monumental task of accommodating not only essentialism but also essential attributes (kinds) of higher order entities like statements and properties as well.

For those who spurn essentialism or abstract entities while remaining partial to the idea that some arguments are materially valid, there are other alternatives.[19] The failure of the three criteria (MSV.1)–(MSV.3) might indicate the futility of attempting to account for material consequence by appeal to form and substitutivity, and perhaps there is more than meets the eye to the contention that some implicational relationships are dependent upon *content* rather than *form*. Or, again, maybe the real culprit is the classical analysis of validity; examination of materially valid arguments merely reveals this fact in a novel manner. These hypotheses are, obviously, important contrasts to the traditional approaches that have been pursued here and whether the latter can be salvaged by further refinements remains to be seen. In the meantime, we face the bleak result that our understanding of material consequence and, hence, of consequence *simpliciter*, is still at a rudimentary and unarticulated stage.

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19 There is an alternative explication of material consequence that appeals to the notions of satisfaction and models, analogous to Tarski's preferred account of logical consequence. The actual definition of material consequence would be identical to that of logical consequence, *viz.*, truth-preservation in all models, save for the fact that the conditions placed upon the models would be far more stringent and numerous in order to anchor the acknowledged implications in the sets constituting the models. However, this development would be an uninteresting and tiresome extension of the model-theoretic analysis.

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