Simplified Exhaustive Search Approach for Estimating the Nonhomogeneous Transition Probabilities for Infrastructure Asset Management

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Abstract: A simplified exhaustive search approach is proposed to estimate the nonhomogeneous transition probabilities for a particular infrastructure element. The yearly nonhomogeneous transition probabilities associated with discrete-time Markovian chains can be estimated for a given analysis period mainly using observed performance ratings. The proposed approach is applicable to Markov chains comprised of only two state transitions, namely remaining in the same current state or transiting to the next worse one. The exhaustive search aims at finding two optimal deterioration exponents that would yield the optimal initial and terminal transition probabilities subject to a minimal difference between the predicted and observed performance ratings for each transition. Therefore, the exhaustive optimization is mainly carried out with respect to two parameters only. A limited number of annual infrastructure performance ratings spanned over an analysis period is required to estimate the corresponding initial and terminal transition probabilities. In contrast, the intermediate transition probabilities for each transition can be estimated using either linear or quadratic approximation. The sample results presented for both hypothetical and actual performance data indicated the simplicity and efficiency of the proposed approach in yielding reliable optimal solutions. In particular, the results indicated that there is more than one compatible solution, and that a Markov chain with a smaller size is required when the deterioration rates are higher considering only two state transitions. **DOI: 10.1061/(ASCE)IS.1943-555X.0000660.** *© 2021 American Society of Civil Engineers.*

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Introduction

Infrastructure asset management has gained wide publicity in the last 3 decades, forcing public agencies to exercise special efforts to improve the effectiveness of their decision-making procedures. The main objective of infrastructure asset management is finding reliable and cost-effective solutions for maintaining and rehabilitating the nation's infrastructure systems that are subject to deterioration over time. The main researched systems have included road networks, bridge networks, airport systems, and sewer networks (Li et al. 2006; Marzouk and Omar 2013; O'Connor et al. 2013; Altarabsheh et al. 2016; Li et al. 2016; Thomas and Sobanjo 2016; Pérez-Acebo et al. 2019; Ansarilari and Golroo 2020). As a tool to solve the infrastructure management problem, specialized management systems have been developed such as the pavement management system (PMS), bridge management system (BMS), and airport management system (AMS) (Li et al. 2006; Thomas and Sobanjo 2016; Ansarilari and Golroo 2020). The main component of any management system is a performance prediction model that can accurately predict the future conditions of a particular system element. A system is typically broken into heterogeneous elements with different deterioration mechanisms. Examples of system elements include roadway pavement, airport pavements, concrete bridge decks, and sewer pipelines.

The prediction of future infrastructure conditions is essential for specifying appropriate remedy actions for a timely scheduled program. Deterioration prediction of a particular system element has been extensively investigated using stochastic models such as the Markovian-based models (Wang et al. 1994; Amin 2015; Abed et al. 2019). Different forms of the Markov model have been used to investigate infrastructure deterioration over time (Thomas and Sobanjo 2016; Fuentes et al. 2021; Yamany et al. 2021). In particular, the discrete-time Markov model with homogeneous, semihomogenous, and nonhomogeneous chains has widely been used to forecast the deterioration of different infrastructure elements as related to roads, airports, bridges, and sewer pipelines (Li et al. 2006; Marzouk and Omar 2013; O'Connor et al. 2013; Altarabsheh et al. 2016; Li et al. 2016; Ansarilari and Golroo 2020). The two main elements of the discrete-time Markov model are the discrete states representing specified infrastructure conditions, and the transition probabilities denoting the infrastructure deterioration rates from one condition state to another in a discrete-time interval called transition (Abaza 2021; Yamany et al. 2021). Much emphasis has been placed on the estimation of the transition probabilities as described next.

Generally, the transition probabilities are estimated from historical records of infrastructure performance typically collected on an annual basis. Several researchers have investigated different approaches to estimate the transition probabilities associated with infrastructure deterioration. For example, Ortiz-Garcia et al. (2006) deployed the minimization of sum of squared errors to propose three different approaches to estimate the transition probabilities associated with pavement deterioration. The three approaches involved original pavement records, a regression curve derived from original pavement records, and yearly distributions of pavement records. Kobayashi et al. (2010) estimated the deterioration transition probabilities from exponential hazard models defined using condition

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states with nonuniform intervals among the inspection time points. Costello et al. (2016) proposed an analytical tool that utilizes deterministic deterioration models coupled with an estimate of scatter to estimate the transition matrix using an optimization procedure that minimizes the difference between the condition distributions obtained from the deterministic model and the transition matrix itself.

Abaza (2017a) estimated the nonhomogeneous transition probabilities using an empirical model that took into consideration the impact of both increased traffic loading and decreased pavement structural capacity over time. Abu Dabous (2017) applied the Dempster-Shafer theory of evidence to estimate the transition probabilities associated with bridge deterioration. The evidence theory was proposed as a scientific expert judgement elicitation technique in lieu of the traditional probability theory for bridge deterioration modeling. Lethanh et al. (2017) applied the restricted least-squares optimization approach to estimate the transition probabilities for bridge management using proportional data obtained from the mechanistic-empirical deterioration models considering mainly reinforced concrete bridge element exposed to chloride-induced corrosion. Abaza (2021) applied the minimization of sum of squared errors to obtain both the homogeneous and nonhomogeneous transition probabilities using a transition probability matrix with three state transitions.

Research Objectives

This paper proposes a simplified exhaustive search approach that can effectively estimate the nonhomogeneous transition probabilities for a given infrastructure element mainly as a function of the annual performance ratings associated with an analysis period of n years. The exhaustive search approach deploys only two simple parameters in the search for the optimal transition probabilities, namely a deterioration rate factor that relates the terminal transition probabilities to the initial ones, and a deterioration rate exponent used to estimate the nonhomogeneous transition probabilities for a given year based on the corresponding values associated with the previous year. The main advantages of the proposed approach compared with other outlined approaches are its simplicity, efficacy, and minimal need for performance records. However, the use of the proposed approach is restricted to discrete-time Markov chains with only two state transitions, namely remaining in the same current state or transiting to the next worse one. The Markov chain size is to be selected depending on the infrastructure deterioration trend.

Compared with the approach presented by Abaza (2021), the new approach proposed here provides a much simpler, but yet effective, procedure to estimate the nonhomogeneous transition probabilities. The approach proposed in the former publication is based on the minimization of sum of squared errors (SSE) over a specified analysis period while considering a minimum of three state transitions. It had investigated both homogeneous and nonhomogeneous Markov chains. The mathematical modeling and computation efforts associated with the former publication are much more extensive. The simplicity of the new proposed approach stems from the fact that it is applicable to nonhomogeneous Markov chains with only two state transitions. This has led to the derivation of a closed-form formula [Eq. (7)], which can estimate the initial transition probability associated with the first transition, and the corresponding terminal transition probability is estimated using an appropriate deterioration rate factor.

Then, a new effective technique is proposed to estimate the transition probabilities for the subsequent duty cycles (i.e., transitions) as a function of the transitional deterioration rates defined using the observed condition ratings. The explicit use of deterioration rates and deterioration rate factors presents a new contribution in Markovian prediction modeling. Of course, Markovian modeling with only two state transitions had been extensively used by several researchers considering both pavements and bridges (Butt et al. 1987; Hatami and Morcous 2012; Wellalage et al. 2015; Abaza 2016; Abed et al. 2019; Galvis Arce and Zhang 2021).

Literature Review

Yamany et al. (2021) indicated that there are five methods reported in the literature to estimate the pavement transition probabilities, namely the expected-value, percentage transition, simulation-based, econometric models, and duration models. Yamany et al. (2021) also reported that the most popular one is the percentage transition method as defined in Eq. (1). Eq. (1) simply represents the basic logic associated with the Markov chain transitioning process. The transition probability $P_{ij,t}$ represents the probability of pavement condition transiting from state i to state j wherein $N_{j,t}$ represents the number of pavement sections that had been in state i and moved to state j during one duty cycle (i.e., transition interval), and $N_{i,t-\mathrm{DC}}$ is the number of pavement sections that exited in state i at time t minus one duty cycle

$$P_{ij,t} = \frac{N_{j,t}}{N_{i,t-\mathrm{DC}}} \quad (1)$$

Typically, pavement condition surveys require dividing the pavement network into small sections, which are then individually assessed. It was reported that better estimates of the transition probabilities can be obtained when using pavement sections with smaller length (Abaza 2016). Pavement assessment is normally conducted based on annual or biennial basis, which essentially implies that application of Eq. (1) requires a minimum of two consecutive cycles of pavement distress assessment separated by one duty cycle in order to estimate one set of transition probabilities. Application of the Markov model requires that the transition probabilities be estimated using the same transition interval (i.e., duty cycle). Unfortunately, many highway authorities are often unable to conduct regular pavement condition assessments. However, adequate numbers of pavement sections must transit to the various deployed condition states during one duty cycle in order to obtain reliable estimates of the corresponding transition probabilities using Eq. (1). Consequently, some of the transition probabilities may not be estimated because either none or inadequate pavement sections have transited to the various deployed condition states during one duty cycle. In addition, this essentially requires surveying a very large number of small pavement sections so that reliable estimates of the transition probabilities can be obtained.

Conducting regular and reliable condition assessment surveys has been a major challenge for developing and implementing an effective infrastructure management system. The simplified approach proposed in this paper requires much less effort in conducting the assessment surveys while ensuring the estimation of reliable transition probabilities for all involved condition states. Essentially, the main data requirement for applying the proposed approach is a few data points that represent the values of an appropriate condition indicator spanned over an analysis period. In essence, the data points need not be collected on a regular time basis. A best-fit performance curve is typically generated from the data points and used as the main requirement for applying the proposed approach. A single data point represents the average condition rating associated with a random sample of pavement sections surveyed at a specified service time, thus making it less susceptible to variability in section condition ratings. Therefore, the proposed approach can effectively be applied at the project level because the data requirement is minimal and affordable, and can lead to conclusive and reliable estimates of project-based transition probabilities. This would definitely make a positive impact in infrastructure management applications using Markovian-based performance prediction models.

Other recent research has applied a backward approach to estimate the transition probabilities (Abaza 2016; Yamany and Abraham 2021). This backward approach requires estimating the state probabilities at two consecutive transitions (i and $i+1$), which are then used to estimate the corresponding transition probabilities using the discrete-time Markov model in what is known as a backward solution. Abaza (2016) used this approach to derive closed-form formulas to compute the transition probabilities for a Markov chain with only two state transitions. Yamany and Abraham (2021) also used this approach wherein the state probability vector for the $i+1$ year is divided by the state probability vector for ith year to yield the corresponding transition probability matrix as per definition of the discrete-time Markov model. However, the involved computations are not simple depending on the number of elements incorporated in the transition matrix.

The new approach proposed in this paper is expected to be much simpler because the optimization is mainly carried out with respect to two parameters only. Moreover, extensive historical performance records are required to estimate the relevant state probabilities using the backward approach with state probabilities representing worst conditions cannot generally be estimated for infrastructures with low to moderate service lives, which results in some of the transition probabilities not being estimated. The proposed approach guarantees to estimate all transition probabilities for a particular transition just using two consecutive annual performance ratings.

Also, Yamany and Abraham (2021) incorporated the improvement rates into the transition matrix; however, the improvement rates can alternatively be applied to the state probabilities so that the transition matrix can only represent the pavement deterioration mechanism (Abaza and Murad 2007). Therefore, there are two viable options available for incorporating pavement improvement rates into Markovian modeling.

Overview of Markovian-Based Prediction Models

Several forms of the Markovian-based models have been used to study the long-term infrastructure deterioration mechanisms; however, the most popularly used ones are the discrete-time homogeneous and nonhomogeneous Markov models (Abed et al. 2019; Abaza 2021; Yamany et al. 2021). The main difference between the two types is that a varied transition probability matrix is used in the discrete-time nonhomogeneous Markov model, an indication of variable transition probabilities (i.e., deterioration rates) for each transition. Eq. (2) defines the discrete-time nonhomogeneous Markov model, which can incorporate a different transition probability matrix, $\mathbf{P(k)}$, for every transition (i.e., time interval).

The main outcome of Eq. (2) is the estimation of the state probabilities after n transitions, $\mathbf{Q(n)}$, as a row vector obtained from multiplying the initial state probability vector, $\mathbf{Q(0)}$, by the multiplication product of n transition probability matrices. The state probabilities represent the infrastructure proportions that are expected to exist in the various deployed condition states after n transitions. The discrete-time Markov model requires that both the transition length and number of condition states m to be integers. The transition length is the equal time interval between successive transitions, which is typically considered to be 1 year. The sum of state probabilities after n transitions must add up to one. It is typically assumed that all infrastructure elements are assigned to the best condition state when considering a new infrastructure, thus the initial state probabilities are equal to $(1, 0, 0, \ldots, 0)$

$$\mathbf{Q(n)} = \mathbf{Q(0)} \prod_{\mathbf{k}=1}^{\mathbf{n}} \mathbf{P(k)} \tag{2}$$

Eq. (3) provides a typical transition probability matrix with only two state transition outcomes. The two transition outcomes are either remaining in the same current state ($\mathbf{i}$) with probability of $P(k)_{i,i}$ or transiting to the next worse state ($\mathbf{i}+1$) with probability of $P(k)_{i,i+1}$. Therefore, all other matrix entries above the main diagonal are assigned a zero value. This matrix form can only predict the infrastructure deterioration in the absence of any maintenance and rehabilitation works because all entries below the main diagonal are also assigned zero value.

The transition matrix form indicated by Eq. (3) has been used by several researchers to model infrastructure deterioration mainly because of its minimal requirement for historical performance records (Butt et al. 1987; Hatami and Morcous 2012; Wellalage et al. 2015; Abaza 2016; Abed et al. 2019; Galvis Arce and Zhang 2021). However, there are certain requirements to be met for this model to be effective in predicting infrastructure deterioration, among which are the deterioration rate magnitudes, transition length, and number of deployed condition states m. One set of deterioration transition probabilities, $P(k)_{i,i+1}$, is typically determined from historical performance records collected over two consecutive cycles of in situ assessment. The sum of any row in the transition matrix must be equal to one

$$P(k) = \begin{bmatrix} P(k)_{1,1} & P(k)_{1,2} & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & P(k)_{2,2} & P(k)_{2,3} & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & P(k)_{3,3} & P(k)_{3,4} & 0 & 0 & \cdots & 0 \\ & & \vdots & & & & & \\ & & \vdots & & & & & \\ 0 & 0 & 0 & \cdots & & & P(k)_{m-1,m-1} & P(k)_{m-1,m} \\ 0 & 0 & 0 & 0 & 0 & 0 \cdots & 0 & 1.0 \end{bmatrix} \tag{3}$$

The nonhomogeneous deterioration transition probabilities, $P(k)_{i,i+1}$, are expected to increase over time due to the progressive increase in traffic loading and progressive decrease in pavement structural capacity (Abaza 2017a). Therefore, the deterioration transition probabilities for the $k+1$ transition are expected to be higher than the corresponding values associated with the k transition. Two key deterioration transition probabilities were found to have a major impact on the pavement deterioration mechanism, namely the initial transition probability, $P(k)_{1,2}$, and the terminal transition probability, $P(k)_{m-1,m}$ (Abaza 2017a, 2021). The other intermediate transition probabilities for a given transition can be estimated from the corresponding initial and terminal transition probabilities using either linear or quadratic approximation as outlined subsequently.

Generally, the infrastructure performance rating at any given transition can be estimated from the corresponding state probabilities. This estimation can either be based on the observed state probabilities or the predicted values as derived from the nonhomogeneous Markov model presented in Eq. (2). Eq. (4) can be used to estimate the predicted performance rating $R_p(k)$ for the k transition as a function of the state mean performance ratings $(\bar{R}_i)$ and corresponding state probabilities $Q_i(k)$. The $R_p(k)$ is essentially the expected value of m uniform probability density functions

$$R_p(k) = \sum_{i=1}^{m} \bar{R}_i \times Q_i(k) \quad (k = 1, 2, \ldots, n;\ i = 1, 2, \ldots, m) \tag{4}$$

where $\bar{R}_i = (\mathrm{LR}_i + \mathrm{UR}_i)/2$.

Typically, the condition states are defined using a continuous performance indicator with equal ranges so that LR_i and UR_i indicate the lower and upper range limits, respectively. The state mean performance rating $(\bar{R}_i)$ is the average of the lower and upper range ratings. For example, if the performance indicator is defined using a scale of 100 points, the $\bar{R}_i$ values for a Markov chain of 10 condition states are (95, 85, 75, …, 5) when equal ranges of (100–90, 90–80, 80–70, …, 10–0) are used. Similarly, the $\bar{R}_i$ values become (90, 70, 50, 30, 10) for a Markov chain with five condition states using equal ranges of (100–80, 80–60, …, 20–0).

Although Markovian-based models have gained popular usage in predicting infrastructure future conditions, there has been a main critic of these models, which is the underlying assumption that the infrastructure condition state at a given time is independent of its past improvement history. Therefore, several researchers have proposed different probabilistic approaches to overcome this limitation while considering infrastructure intervention history (Abaza and Murad 2007; Saeed et al. 2017; Yamany and Abraham 2021). In particular, Saeed et al. (2017) proposed a novel probabilistic approach that can enhance the infrastructure condition prediction while accounting for improvement effectiveness. This approach requires defining and quantifying the intervention types while incorporating newly introduced explanatory variables to account for future deterioration of bridge components. The deployed dependent variable defines the probability of a bridge component being in a given condition state at a specified age. The new approach proposed in this paper can estimate the nonhomogeneous transition probabilities for an original infrastructure without any improvement history, and also an infrastructure with intervention history. In both cases, the main data requirement is a number of average annual condition ratings spanned over its past service life.

Research Methodology

A sequential exhaustive search approach for estimating the nonhomogeneous transition probabilities is presented in this section. The approach mainly requires the average annual performance ratings associated with a particular infrastructure element for a specified analysis period. The deterioration of an infrastructure element is defined using key parameters, namely a deterioration rate factor that relates the terminal nonhomogeneous transition probabilities to the corresponding initial transition probabilities, and two deterioration exponents that depict the impact of deterioration rates on the estimation of the nonhomogeneous transition probabilities.

First-Transition Initial Transition Probability

The proposed sequential exhaustive search approach seeks to estimate the nonhomogeneous deterioration transition probabilities associated with n transitions. However, the initial transition probability, $P(1)_{1,2}$, associated with the first transition $(k = 1)$ can directly be derived in a closed form when considering a new infrastructure. The predicted performance rating for the first transition is computed using Eq. (4) and is set equal to the observed performance rating as indicated by Eq. (5)

$$R_p(1) = R_o(1) = \sum_{i=1}^{m} \bar{R}_i \times Q_i(1) \tag{5}$$

The state probabilities $Q_i(1)$ associated with the first transition $(k = 1)$ are obtained from Eq. (2) by multiplying the transition matrix defined in Eq. (3) by the initial state probability row vector for a new infrastructure, namely $(1, 0, 0, \ldots, 0)$. The outcome of this multiplication is a row vector with only the first and second state probabilities being assigned nonzero values, as indicated by Eq. (6). These state probabilities are mainly the two entries of the first row in the transition matrix

$$\mathbf{Q(1)} = [P(1)_{1,1}, P(1)_{1,2}, 0,\ 0,\ \ldots,\ 0] \tag{6}$$

where $P(1)_{1,1} + P(1)_{1,2} = 1.0$.

Now, substituting the state probabilities associated with the first transition into Eq. (5) and solving for the initial transition probability $P(1)_{1,2}$ of the first transition $(k = 1)$ yields Eq. (7). Therefore, the estimation of the first-transition initial transition probability mainly depends on the mean performance ratings associated with Condition states 1 and 2, namely $\bar{R}_1$ and $\bar{R}_2$. As outlined previously, the values of these two-state mean performance ratings depend on the number of deployed condition states m. For example, their values are 95 and 85 for a Markov chain with 10 condition states, and 90 and 70 for a Markov chain with five states if equal ranges of performance ratings are used to define condition states

$$P(1)_{1,2} = \frac{\bar{R}_1 - R_o(1)}{\bar{R}_1 - \bar{R}_2} \leq 1.0 \tag{7}$$

where $\bar{R}_1 > R_o(1)$, $\bar{R}_1 > \bar{R}_2$, and $R_o(1) > \bar{R}_2$.

Therefore, the selection of the appropriate Markov chain size m depends on the magnitude of change in the observed first-transition performance rating $R_o(1)$. This is achieved by requiring the value of first-transition initial transition probability $P(1)_{1,2}$, to be less than or equal to one according to Eq. (7). Otherwise, a smaller Markov chain size is required. For example, the observed first-transition rating $R_o(1)$ has to be greater than or equal to $\bar{R}_2 = 85$ to be able to use a Markov chain of size 10. However, if the drop in the observed first-transition performance rating is below 85 but above or equal to 70, then a Markov chain of size five is required. Generally, a 10×10 transition matrix is used when the drop in the performance rating is less than 10 points per transition, whereas a 5×5 matrix is required if the drop reaches 20 points per transition. This is under the assumption of using a 100-point scale indicator, with lower ratings representing inferior conditions.

The estimation of the other initial transition probabilities will be based on the corresponding value associated with the first transition and the deterioration rates associated with subsequent transitions as outlined in Eq. (8). The transitional deterioration rates are defined in terms of the observed performance ratings associated with two consecutive transitions, namely $R_o(k)$ and $R_o(k+1)$ with their ratio is being raised to the power $A(k)$ named as the deterioration rate exponent. These deterioration exponents are to be estimated from the sequential exhaustive search approach outlined in the next section

$$P(k+1)_{1,2} = P(k)_{1,2}\left(\frac{R_o(k)}{R_o(k+1)}\right)^{A(k)} \quad (k = 1, 2, \ldots, n) \quad (8)$$

The terminal transition probabilities, $P(k)_{m-1,m}$, can be estimated, as an option, from the multiplication of the initial transition probabilities and deterioration rate factors $F_d(k)$ as indicated by Eq. (9). The deterioration factor can be assumed constant over an analysis period of n transitions or it can be variable to be estimated from the exhaustive search approach outlined next

$$P(k)_{m-1,m} = F_d(k) \times P(k)_{1,2} \leq 1.0 \quad (k = 1, 2, \cdots, n) \quad (9)$$

The deterioration rate factor $F_d(k)$ is typically associated with a value in the range of about 2–4 for a performance trend with progressively increasing deterioration rates, and in the range of about 0.2–0.4 for a performance trend with progressively decreasing deterioration rates. These factor ranges are generally estimated based on research results obtained from the new approach proposed in this paper. Project A shown in Fig. 1 is an example of deterioration trend with progressively increasing deterioration rates, whereas Projects B and C represent examples of deterioration trend with progressively decreasing deterioration rates.

Sequential Exhaustive Search Approach

Estimation of the nonhomogeneous initial and terminal transition probabilities $P(k)_{i,i+1}$ and $P(k)_{m-1,m}$ for a specified number of transitions n is performed using a sequential exhaustive search approach. The proposed approach aims at minimizing the transitional difference $D(k)$ between the predicted and observed performance ratings. Therefore, it seeks to find a set of initial and terminal transition probabilities that would yield a predicted performance rating that differs from the corresponding observed value by less than or equal to a specified tolerable value D_a as indicated by Eq. (10)

$$\text{Minimize: } D(k) = R_p(k) - R_o(k) \leq D_a \quad (10)$$

Two optimization options are proposed to accomplish the sequential exhaustive approach in the search of the optimal nonhomogeneous initial and terminal transition probabilities $P'(k)_{i,i+1}$ and $P'(k)_{m-1,m}$, for an analysis period comprised of n transitions, as outlined next.

Optimization Option I: One Deterioration Exponent

The first proposed optimization option only involves one deterioration exponent $A(k)$ applied to the nonhomogeneous initial transition probabilities as indicated by Eq. (8). The nonhomogeneous terminal transition probabilities are computed using Eq. (9) as a multiplication of the corresponding initial transition probabilities and constant deterioration factor F_d. The corresponding optimization model is summarized by Eq. (11)

$$\text{Minimize: } D(k) = R_p(k) - R_o(k) \leq D_a \quad (k = 1, 2, \ldots, n) \quad (11)$$

Subject to

1. $P(k+1)_{1,2} = P(k)_{1,2}\left(\frac{R_o(k)}{R_o(k+1)}\right)^{A(k)}$
2. $P(k)_{m-1,m} = F_d \times P(k)_{1,2} \leq 1.0$

A sequential trial-and-error approach will be used to find the optimal deterioration rate exponent $A'(k)$, that will yield the minimum performance difference $D'(k)$ that satisfies Eq. (11). For each trial solution, the intermediate nonhomogeneous transition probabilities will be estimated using linear/nonlinear approximation as outlined subsequently. The transition matrix as indicated by Eq. (3) is now completely defined and can be used to estimate the corresponding predicted performance rating $R_p(k)$ using Eqs. (2) and (4). The observed performance ratings should be available from historical records.

Optimization Option II: Two Deterioration Exponents

In the second optimization approach outlined in Eq. (12), two deterioration rate exponents are proposed. The first exponent, $A(k)$, is applied to the deterioration rates associated with the initial transition probabilities, and the second one, $B(k)$, is similarly applied to the terminal transition probabilities. A simultaneous trial-and-error approach involving the two deterioration exponents will be executed in the search for the optimal exponent values $A'(k)$ and $B'(k)$. The optimal solution will yield the optimal initial and terminal transition probabilities associated with the minimal difference, $D'(k)$, between the predicted and observed performance ratings for each transition within the analysis period

$$\text{Minimize: } D(k) = R_p(k) - R_o(k) \leq D_a \quad (k = 1, 2, \ldots, n) \quad (12)$$

Subject to

1. $P(k+1)_{1,2} = P(k)_{1,2}\left(\frac{R_o(k)}{R_o(k+1)}\right)^{A(k)}$
2. $P(k+1)_{m-1,m} = P(k)_{m-1,m}\left(\frac{R_o(k)}{R_o(k+1)}\right)^{B(k)}$

Therefore, the dependency between the initial and terminal transition probabilities is removed, and thus a different deterioration factor, $F_d(k)$, can result in each transition, as indicated by Eq. (13)

$$F_d(k) = \frac{P(k)_{m-1,m}}{P(k)_{1,2}} \quad (13)$$

Generally, the initial and terminal transition probabilities associated with a particular transition are estimated from the initial and terminal transition probabilities associated with the preceding transition, and two deterioration rates defined based on the ratio of the observed distress ratings associated with the two involved transitions (k and $k+1$) as outlined by Constraints 1 and 2.

Intermediate Nonhomogeneous Transition Probabilities

The presented sequential search approach has mainly focused on the estimation of the initial and terminal transition probabilities for a number of transitions. The other intermediate transition probabilities [i.e., $P(k)_{2,3}, P(k)_{3,4}, \ldots, P(k)_{m-2,m-1}$] shall be estimated so that the transition matrix indicated by Eq. (3) will be totally defined. This can be achieved by simply applying linear interpolation, which assumes all nonhomogeneous transition probabilities $P(k)_{i,i+1}$ fall on a straight line. Thus, the slope of the corresponding straight line, $S(k)$, is computed from the difference between the initial and terminal transition probabilities, as indicated by Eq. (14)

$$S(k) = \frac{P(k)_{m-1,m} - P(k)_{1,2}}{m-2} \quad (14)$$

Generally, there are two types of performance trend as depicted in Fig. 1. The first type is considered as a high-grade performance such as Project A shown in Fig. 1. It is typically associated with increasingly higher deterioration rates [i.e., $P(k)_{1,2} < \mathrm{P}(k)_{2,3} < P(k)_{3,4} \ldots < P(k)_{m-1,m}$]. The straight-line slope $S(k)$ associated with this performance type is positive according to Eq. (14). The second type is a low-grade performance such as Projects B and C

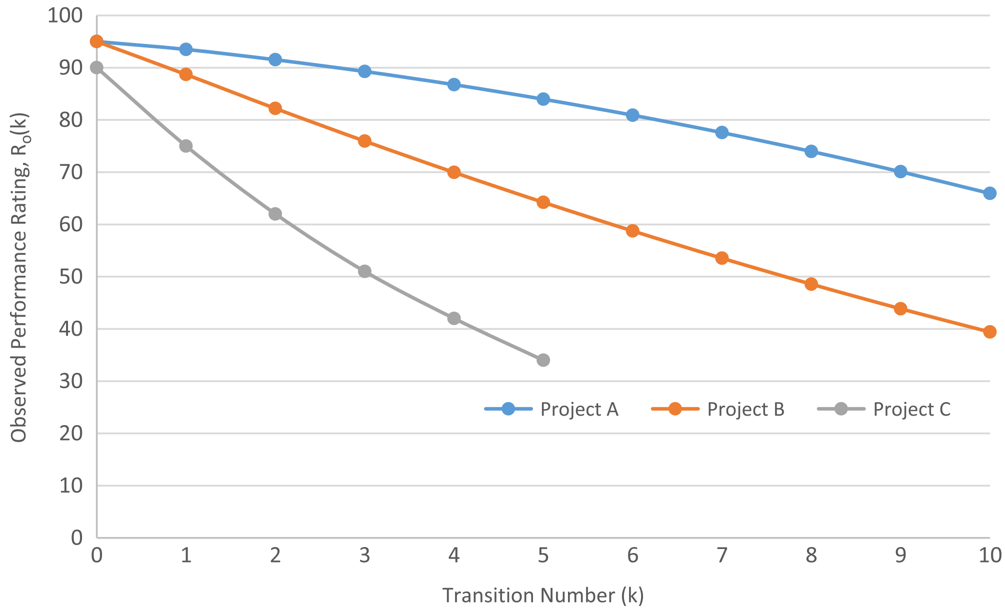

Fig. 1. Deterioration trends associated with sample Projects A, B, and C.

shown in the same figure. It is associated with decreasingly lower deterioration rates [i.e., $P(k)_{1,2} > P(k)_{2,3} > P(k)_{3,4} \ldots > P(k)_{m-1,m}$]. Therefore, the corresponding straight line slope $S(k)$ is negative. In both cases, the intermediate transition probabilities for a particular transition are estimated using Eq. (15). This linear estimation of intermediate transition probabilities had yielded satisfactory results in former Markovian-based studies (Abaza 2017b, 2021)

$$P(k)_{i,i+1} = P(k)_{i-1,i} + S(k) \quad (i = 2, 3, \ldots, m-2;\ k = 1, 2, \ldots, n) \tag{15}$$

It is also possible to estimate the intermediate transition probabilities using nonlinear approximation such as second-degree polynomial (i.e., quadratic) and third-degree polynomial (i.e., spiral) models. Different models can be proposed for both types of performance trend depicted in Fig. 1. For example, a best-fit quadratic model can be approximated using three data points, namely initial point (1, $P_{1,2}$), terminal point ($m-1$, $P_{m-1,m}$), and middle point [$m/2$, $F_q \times (P_{1,2} + P_{m-1,m})$]. The middle point approximation is based on a quadratic factor F_q. A linear model is associated with an F_q value equal to $1/2$, wherein F_q can be smaller or greater than $1/2$ for quadratic modeling depending on concavity shape.

Fig. 2 shows sample quadratic models for both high-grade performance with $P_{1,2} = 0.2$, $P_{m-1,m} = 0.6$, and $m = 10$, and low-grade performance with $P_{1,2} = 0.6$, $P_{m-1,m} = 0.2$, and $m = 10$ for a particular transition. The corresponding quadratic models are presented in Eqs. (16) and (17) for high- and low-grade performances, respectively. The two sample models are generated using a quadratic

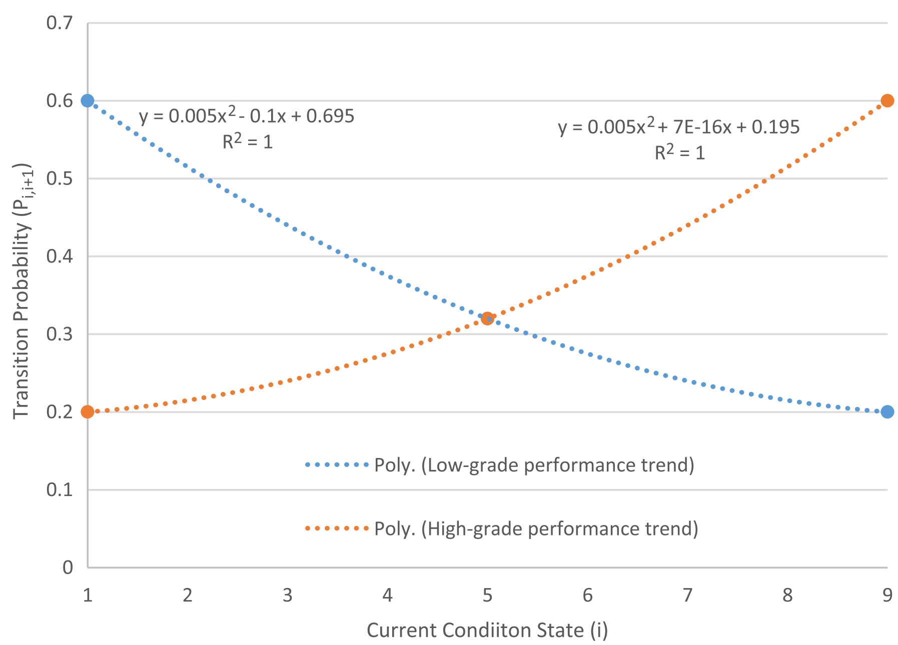

Fig. 2. Sample quadratic models for estimating the intermediate transition probabilities using a quadratic factor of $F_q = 0.4$.

factor of 0.4 so that both curves are concave upward to be consistent with the previously outlined deterioration trends. This results in the intermediate transition probabilities being progressively increasing in the case of high-grade performance, and progressively decreasing in the case of low-grade performance. Eqs. (16) and (17) are only applicable to a particular transition; thus, different models need to be developed for each transition using the corresponding initial and terminal transition probabilities

$$P_{i,i+1} = 0.005i^2 + 0.195 \quad (i = 1, 2, \ldots, m-1) \tag{16}$$

$$P_{i,i+1} = 0.005i^2 - 0.1i + 0.695 \quad (i = 1, 2, \ldots, m-1) \tag{17}$$

Generally, the initial transition probability associated with the first transition, $P(1)_{1,2}$, as defined in Eq. (7) is only applicable to the form of transition matrix presented in Eq. (3) considering a new infrastructure wherein all elements are assigned to State 1. For all other cases, an initial trial value of $P(1)_{1,2}$ can be selected to initiate the proposed sequential exhaustive search. The typical $P(1)_{1,2}$ value range is about 0.1–0.4 for a high-grade performance trend, and about 0.5–0.8 for a low-grade performance trend. Therefore, it is recommended to select the midrange value as an initial trial value in each case. These probability ranges are suggested based on prior experience (Abaza 2017a, 2021).

Proposed Sequential Approach Flowchart

The main steps required in the execution of the proposed exhaustive search approach are described in a flowchart. Fig. 3 shows the flowchart that outlines the basic steps and their logical sequence to be followed in the execution of the sequential exhaustive search to yield the optimal nonhomogeneous transition probabilities for Optimization option I. The same logical sequence is used in Optimization option II except that the second deterioration exponent, $B(k)$, will be required prior to the computation of the terminal transition probabilities $P(k)_{m-1,m}$ as defined in Eq. (12), Constraint 2. The flowchart shows that the initial transition probability associated with the first transition, $P(1)_{1,2}$, is computed only one time when solving a problem with n transitions. However, the initial, terminal, and intermediate transition probabilities for the remaining transitions are sequentially computed for every trial value of $A(k)$ until a satisfactory optimal solution is reached. The main steps outlined in the flowchart can easily be programmed using simple software packages such as Microsoft Excel.

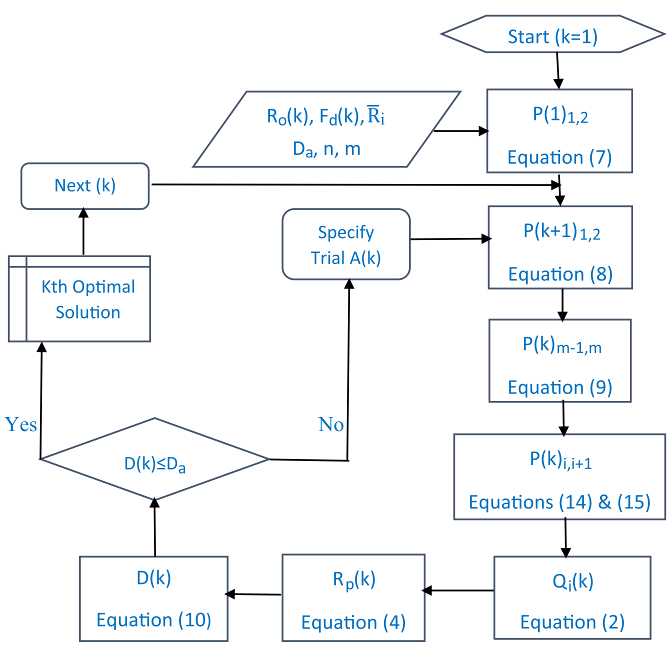

Fig. 3. Flowchart depicting the main steps used to estimate the nonhomogeneous transition probabilities according to Optimization option I.

Sample Presentation

In this section, sample nonhomogeneous transition probabilities are presented using the proposed sequential exhaustive search approach. In particular, two case studies are investigated. The first one consists of three pavement projects, namely A, B, and C, with deterioration trends represented by hypothetical second-degree polynomial models. Meanwhile, the second case study applies actual deterioration data using the international roughness index (IRI) as the pavement performance indicator.

Case Study I: Hypothetical Performance Data

The three pavement projects (A, B, and C) depicted in Fig. 1 are used to estimate the corresponding nonhomogeneous transition probabilities. The mathematical models that represent the deterioration trends of these three projects are provided in Eqs. (18)–(20) using the pavement condition index (PCI) as the performance indicator. These deterioration trends/models can be generated based on a limited number of data points using a best-fit technique. The ASTM (2007) manual has provided the procedure to estimate the PCI for a given pavement section considering 19 different distresses. The PCI is widely used in pavement management applications. It is estimated using a scale of 100 points, with higher values indicating superior pavement performance. The PCI values at different transition numbers (k) are computed and provided in Table 1 for the three projects. These PCI values represent the observed performance ratings $R_o(k)$ that form the main data input for estimating the corresponding nonhomogeneous transition probabilities.

It is clear from Fig. 1 that Project A has a high-grade performance trend, and Projects B and C have a low-grade performance trend, with Project C being associated with higher deterioration rates compared with Project B. The PCI values associated with Projects A and B are 65.93 and 39.40 after 10 years of service, respectively, and it is 34.18 for Project C after only 5 years of service. The length of one transition is considered to be equal to 1 year. The sample nonhomogeneous transition probabilities for the three projects have been estimated following the flowchart depicted in Fig. 3

$$\text{PCI}(k) = -0.136k^2 - 1.566k + 95.19 \text{ (Project A)} \tag{18}$$

$$\text{PCI}(k) = 0.128k^2 - 6.884k + 95.44 \text{ (Project B)} \tag{19}$$

$$\text{PCI}(k) = 0.911k^2 - 15.696k + 89.89 \text{ (Project C)} \tag{20}$$

Tables 2 and 3 provide sample nonhomogeneous transition probabilities derived to represent the deterioration trend of Project A using a Markov chain with 10 condition states. The tables mainly present the initial and terminal transition probabilities for 10 transitions (years). The first-year initial transition probability, $P(1)_{1,2} = 0.1510$, is computed from Eq. (7) based on $R_o(1) = 93.49$, $\bar{R}_1 = 95$, and $\bar{R}_2 = 85$.

Table 2 provides two sets of solution: the first set is derived using Optimization option I with a constant deterioration factor of $F_d = 2$, and the second set is obtained from Optimization option II with variable deterioration factor $F_d(k)$. The two sets of solution

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Table 1. Observed pavement condition index values

Project name	Transition number, k									
	1	2	3	4	5	6	7	8	9	10
A	93.49	91.52	89.27	86.75	83.96	80.90	77.57	73.96	70.08	65.93
B	88.68	82.18	75.94	69.95	64.22	58.74	53.52	48.56	43.85	39.40
C	75.11	62.14	51.00	41.68	34.18	—[a]	—	—	—	—

[a]Not applicable.

Table 2. Sample optimal nonhomogeneous transition probabilities obtained for Project A using a 10×10 Markov chain ($F_d = 2$)

Transition number, k	Optimization option I				Optimization option II					
	$P'(k)_{1,2}$	$P'(k)_{9,10}$	$A'(k)$	$D'(k)$	$P'(k)_{1,2}$	$P'(k)_{9,10}$	$F'_d(k)$	$A'(k)$	$B'(k)$	$D'(k)$
1	0.1510	0.3020	—[a]	0.0000	0.1510	0.3020	2.000	—	—	0.0000
2	0.1934	0.3867	11.61	6.46×10^{-5}	0.1937	0.3671	1.895	11.70	9.165	7.26×10^{-7}
3	0.2156	0.4312	4.378	5.47×10^{-5}	0.2168	0.4053	1.869	4.520	3.982	-1.86×10^{-6}
4	0.2352	0.4703	3.028	-2.21×10^{-5}	0.2370	0.4467	1.885	3.108	1.102	-2.44×10^{-5}
5	0.2529	0.5058	2.229	4.11×10^{-5}	0.2550	0.4879	1.913	2.240	2.701	6.30×10^{-6}
6	0.2689	0.5378	1.649	3.25×10^{-5}	0.2710	0.5247	1.936	1.640	1.957	5.10×10^{-6}
7	0.2831	0.5662	1.225	-3.18×10^{-6}	0.2854	0.5555	1.946	1.232	1.358	1.10×10^{-5}
8	0.2964	0.5928	0.965	6.16×10^{-5}	0.2988	0.5842	1.955	0.965	1.056	-2.75×10^{-5}
9	0.3072	0.6144	0.663	6.37×10^{-5}	0.3097	0.6075	1.962	0.662	0.726	-1.69×10^{-5}
10	0.3165	0.6330	0.487	9.42×10^{-5}	0.3190	0.6273	1.966	0.487	0.526	5.09×10^{-5}

[a]Not applicable.

are somewhat similar in terms of the values associated with the nonhomogeneous transition probabilities, with the optimal differences between the predicted and observed performance ratings, $D'(k)$, less than 1 in 10,000 (i.e., $D_a = 1.00 \times 10^{-4}$), the specified tolerable value. This means that the derived optimal solutions have estimated the annual performance ratings in terms of PCI with a high degree of accuracy.

The results also show that the optimal deterioration rate exponent $A'(k)$ has consistently decreased with the increase in service time, which resulted in increasing the value of the initial transition probability from 0.1510 to 0.3165 over 10 years of service life. The deterioration rate exponents $A'(k)$ and $B'(k)$ associated with the second optimal set exhibited similar trends as in the first optimal set, but they are not equal in values. The optimal deterioration factor $F'_d(k)$ is less than 2, and it is generally increasing with service time.

Table 3 provides similar results using a constant deterioration factor of $F_d = 3$ for the case of Optimization option I, which generally resulted in a decrease in the initial transition probabilities but is compensated for by an increase in the terminal transition probabilities. The two deterioration exponents $A'(k)$ and $B'(k)$ are very similar and very close in values to the exponent associated with Optimization option I. Therefore, the two sets of solution are almost identical, with prediction accuracy as high as the one associated with the solutions provided in Table 2. This means there is more than one set of optimal solutions that can predict the deterioration trend of Project A.

Although it cannot be proven that any one of the obtained solutions is the absolute optimal solution, it is clear they are all reliable solutions that met the objective set in the optimization procedure, which is minimizing the performance rating difference $D(k)$ to be below a tolerable value of 1.00×10^{-4}. It is true that simplified exhaustive optimization procedures may not always yield the absolute optimal solution, but near-optimal solutions often serve the purpose. Therefore, even if there are relatively better solutions than the obtained ones, they would practically make no impact on the overall performance prediction. However, if higher accuracy is needed, it can be achieved by simply reducing the tolerance value.

Tables 4 and 5 provide sample optimal nonhomogeneous transition probabilities obtained to characterize the deterioration trend of Project B using a Markov chain with 10 condition states. The first-year initial transition probability $P(1)_{1,2} = 0.6320$ is obtained

Table 3. Sample optimal nonhomogeneous transition probabilities obtained for Project A using a 10×10 Markov chain ($F_d = 3$)

Transition number, k	Optimization option I				Optimization option II					
	$P'(k)_{1,2}$	$P'(k)_{9,10}$	$A'(k)$	$D'(k)$	$P'(k)_{1,2}$	$P'(k)_{9,10}$	$F'_d(k)$	$A'(k)$	$B'(k)$	$D'(k)$
1	0.1510	0.4530	—[a]	0.0000	0.1510	0.4530	3.000	—	—	0.0000
2	0.1898	0.5695	10.75	1.84×10^{-5}	0.1898	0.5694	2.999	10.75	10.74	1.05×10^{-7}
3	0.2070	0.6210	3.475	-2.74×10^{-5}	0.2070	0.6211	3.000	3.475	3.489	1.48×10^{-6}
4	0.2204	0.6613	2.198	8.17×10^{-5}	0.2204	0.6611	2.999	2.198	2.182	-2.05×10^{-5}
5	0.2313	0.6938	1.470	-3.44×10^{-5}	0.2313	0.6940	3.000	1.471	1.483	2.90×10^{-5}
6	0.2398	0.7194	0.974	-1.63×10^{-5}	0.2398	0.7194	3.000	0.974	0.971	4.74×10^{-6}
7	0.2462	0.7387	0.629	4.14×10^{-5}	0.2462	0.7385	2.999	0.629	0.623	1.49×10^{-6}
8	0.2514	0.7543	0.439	-1.81×10^{-5}	0.2514	0.7543	2.999	0.439	0.444	3.60×10^{-5}
9	0.2543	0.7628	0.208	8.35×10^{-5}	0.2543	0.7627	2.999	0.208	0.206	2.55×10^{-5}
10	0.2559	0.7676	0.103	-3.33×10^{-5}	0.2559	0.7676	3.000	0.103	0.105	4.44×10^{-5}

[a]Not applicable.

Table 4. Sample optimal nonhomogeneous transition probabilities obtained for Project B using a 10 × 10 Markov chain ($F_d = 0.2$)

Transition number, k	Optimization option I				Optimization option II					
	$P'(k)_{1,2}$	$P'(k)_{9,10}$	$A'(k)$	$D'(k)$	$P'(k)_{1,2}$	$P'(k)_{9,10}$	$F'_d(k)$	$A'(k)$	$B'(k)$	$D'(k)$
1	0.6320	0.1264	—[a]	0.0000	0.6320	0.1264	0.2000	—	—	0.0000
2	0.6938	0.1388	1.227	2.16×10^{-5}	0.6938	0.1387	0.1999	1.227	0.601	3.84×10^{-6}
3	0.7158	0.1432	0.394	4.56×10^{-6}	0.7158	0.1432	0.2000	0.394	0.396	-2.32×10^{-7}
4	0.7401	0.1480	0.406	1.77×10^{-5}	0.7401	0.1480	0.1999	0.406	0.405	-6.42×10^{-7}
5	0.7645	0.1529	0.380	-1.84×10^{-5}	0.7645	0.1529	0.2000	0.380	0.382	-9.80×10^{-7}
6	0.7917	0.1583	0.392	-1.01×10^{-5}	0.7917	0.1583	0.1999	0.392	0.391	8.02×10^{-7}
7	0.8190	0.1638	0.364	-1.99×10^{-5}	0.8190	0.1638	0.2000	0.364	0.364	-2.38×10^{-6}
8	0.8476	0.1695	0.353	1.25×10^{-5}	0.8476	0.1695	0.1999	0.353	0.353	3.17×10^{-6}
9	0.8794	0.1759	0.361	8.26×10^{-6}	0.8794	0.1759	0.2000	0.361	0.361	1.63×10^{-6}
10	0.9110	0.1822	0.330	-1.52×10^{-5}	0.9110	0.1822	0.2000	0.330	0.330	5.81×10^{-6}

[a]Not applicable.

Table 5. Sample optimal nonhomogeneous transition probabilities obtained for Project B using a 10 × 10 Markov chain ($F_d = 0.3$)

Transition number, k	Optimization option I				Optimization option II					
	$P'(k)_{1,2}$	$P'(k)_{9,10}$	$A'(k)$	$D'(k)$	$P'(k)_{1,2}$	$P'(k)_{9,10}$	$F'_d(k)$	$A'(k)$	$B'(k)$	$D'(k)$
1	0.6320	0.1896	—[a]	0.0000	0.6320	0.1896	0.3000	—	—	0.0000
2	0.6880	0.2064	1.116	-1.94×10^{-5}	0.6880	0.2064	0.3000	1.116	1.118	4.6×10^{-7}
3	0.7028	0.2108	0.269	-1.61×10^{-5}	0.7028	0.2108	0.3000	0.269	0.268	8.99×10^{-6}
4	0.7189	0.2157	0.275	6.25×10^{-5}	0.7189	0.2156	0.2999	0.275	0.274	1.81×10^{-7}
5	0.7338	0.2202	0.241	1.17×10^{-5}	0.7338	0.2202	0.2999	0.241	0.242	2.26×10^{-6}
6	0.7500	0.2250	0.244	6.85×10^{-6}	0.7500	0.2250	0.3000	0.244	0.244	3.06×10^{-6}
7	0.7646	0.2294	0.207	-2.02×10^{-5}	0.7646	0.2294	0.3000	0.207	0.207	1.65×10^{-6}
8	0.7786	0.2336	0.187	-1.83×10^{-5}	0.7786	0.2334	0.2999	0.187	0.186	-3.54×10^{-6}
9	0.7934	0.2380	0.185	1.03×10^{-6}	0.7934	0.2380	0.2999	0.185	0.185	-5.81×10^{-6}
10	0.8057	0.2417	0.144	5.46×10^{-6}	0.8057	0.2417	0.2999	0.144	0.144	7.31×10^{-6}

[a]Not applicable.

from Eq. (7) based on $R_o(1) = 88.68$, $\bar{R}_1 = 95$, and $\bar{R}_2 = 85$. This high initial probability value is an indication of low-grade performance trend, as depicted in Fig. 1. Table 4 provides the optimal nonhomogeneous transition probabilities for a constant deterioration factor of $F_d = 0.2$. The two sets of optimal solution as derived from the two optimization options are the same in most cases.

Similarly, Table 5 provides the optimal solutions for a constant deterioration factor of $F_d = 0.3$ with the two optimal sets are being very much identical. However, the optimal initial transition probabilities are lower in value compared with the corresponding ones provided in Table 4. This is because of the higher deterioration factor, which resulted in an increase in the optimal terminal transition probabilities, thus compensating for the decrease in the initial transition probabilities to still yield the same performance ratings. Again, Tables 4 and 5 practically provide two sets of optimal solution that are highly compatible in terms of yielding the minimum performance difference $D(k)$ between the predicted and observed performance ratings.

Table 6 provides sample optimal nonhomogeneous transition probabilities for Project C using a Markov chain with five condition states. The performance trend of this project could not be predicted using 10 condition states because the first-year initial transition probability would be greater than one when computed using Eq. (7). The first-year initial transition probability $P(1)_{1,2} = 0.7445$ is computed from Eq. (7) using $R_o(1) = 75.11$, $\bar{R}_1 = 90$, and $\bar{R}_2 = 70$.

Table 6 provides two sets of optimal solution based on Optimization option I for only five transitions (i.e., 5 years) using 0.2 and 0.3 as constant deterioration factors. The two solutions derived are compatible except for the last transition, wherein using the 0.2 constant deterioration factor failed to yield an error difference $D(k)$ within the specified tolerable value of 1.00×10^{-4}. This is because the corresponding initial transition probability has almost reached the limit value (0.9999). However, the optimal solution associated with the 0.3 constant deterioration factor has generally achieved the objective by yielding error differences within the specified tolerance.

Table 6. Sample optimal nonhomogeneous transition probabilities obtained for Project C with a 5 × 5 Markov chain using Optimization option I

Transition number, k	Deterioration rate factor $F_d = 0.3$				Deterioration rate factor $F_d = 0.2$			
	$P'(k)_{1,2}$	$P'(k)_{4,5}$	$A'(k)$	$D'(k)$	$P'(k)_{1,2}$	$P'(k)_{4,5}$	$A'(k)$	$D'(k)$
1	0.7445	0.2234	—[a]	0.0000	0.7445	0.1489	—	0.0000
2	0.7848	0.2354	0.2784	0.99×10^{-5}	0.8091	0.1618	0.4392	-6.00×10^{-5}
3	0.8252	0.2476	0.2539	-9.30×10^{-6}	0.8862	0.1772	0.4604	-8.58×10^{-5}
4	0.8550	0.2565	0.1759	-8.92×10^{-5}	0.9708	0.1942	0.4521	1.79×10^{-5}
5	0.8663	0.2599	0.0658	3.23×10^{-5}	0.9999	0.1999	0.1491	-3.38×10^{-1}

[a]Not applicable.

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Case Study II: Actual Performance Data

In this section, actual IRI data are used to estimate the nonhomogeneous transition probabilities considering the urban arterial system in Ramallah, Palestine. The IRI is an internationally used performance indicator for pavement management applications. Birzeit University has recently acquired a new device called IRIMETER-2 (Englo LLC, Tallinn, Estonia) for measuring the roadway longitudinal roughness. The device estimates the IRI by measuring the vehicle vibration via two sensors installed on the vehicle front axle, one next to each wheel.

As an initial experimentation with the new device, several road segments were tested for IRI. The selected segments belong to the arterial system, which exhibits similar traffic loading, pavement structure, and drainage and subgrade conditions. The selected segments were rehabilitated within the last 5 years using cold milling and asphaltic overlay, which is a popular rehabilitation strategy in Palestine. However, the pavement conditions of selected segments are diverse due to differences in rehabilitation age. The segments were selected so that the rehabilitation age covered a period of 5 years. The average IRI value is obtained for all segments with approximately the same rehabilitation age as follows:

- Rehabilitation age (year): 1, 2, 3, 4, and 5, and
- IRI (m/km): 0.95, 1.35, 1.89, 2.48, and 3.17.

The device measures the IRI at intervals of 5 m for both the left and right wheels. The IRI provided above is estimated as an average value for both wheels considering all segments with the same age. Pavement deterioration requires that the IRI value increases over time, as evidenced from the preceding IRI data, wherein higher IRI values indicate inferior pavement performance. Although, these annual IRI values do not belong to the same roadway segments, they provide a means to estimate the corresponding nonhomogeneous transition probabilities for a period of 5 years. A Markov chain with five condition states and equal IRI ranges of (0–1, 1–2, 2–3, 3–4, and 4–5) are used in deterioration modeling. The corresponding state mean performance ratings $\bar{R}_i$ become equal to 0.5, 1.5, 2.5, 3.5, and 4.5.

Table 7 provides the corresponding optimal nonhomogeneous initial and terminal transition probabilities estimated for 5 years using Optimization option II. The first-year initial transition probability is estimated using Eq. (21), which is a modification of Eq. (7) accounting for a performance indicator that increases over time such as the IRI. The first-year initial transition probability $P(1)_{1,2} = 0.450$ is computed from Eq. (21) using $R_o(1) = 0.95$, $\bar{R}_1 = 0.5$, and $\bar{R}_2 = 1.5$. In this case, Optimization option II was successful in yielding optimal solutions with variable deterioration factor $F'(k)$ and variable deterioration rate exponents $A'(k)$ and $B'(k)$ as indicated by the results provided in Table 7. All optimal solutions have met the specified error tolerance of 1.00×10^{-4}, although a smaller value can be specified because the IRI scale is typically much smaller than the PCI scale

$$P(1)_{1,2} = \frac{R_o(1) - \bar{R}_1}{\bar{R}_2 - \bar{R}_1} \leq 1.0 \tag{21}$$

where $R_o(1) > \bar{R}_1, \bar{R}_2 > \bar{R}_1$, and $R_o(1) < \bar{R}_2$.

Additionally, the sequential computation of the initial and terminal transition probabilities have been computed using the two constraints provided in Eq. (12). However, the deterioration rate ratio has been reversed to become $R_o(k+1)/R_o(k)$, which keeps the ratio value greater than one. This is required in order to obtain positive exponent values instead of negative ones.

Sample optimal intermediate transition probabilities have been estimated using both linear and quadratic approximations for Project B considering Optimization option I. The corresponding optimal solutions are provided in Table 8 for a limited number of transitions. A quadratic model similar to the one presented in Fig. 2 has been used with an $F_q = 0.4$ quadratic factor. Generally, both linear and quadratic approximations have yielded compatible results in terms of meeting the specified tolerance limit of 1.00×10^{-4}, whereas the associated deterioration rate exponents $A'(k)$ are different in values. It can also be noted that the initial transition probability $P'(k)_{1,2}$ associated with quadratic modeling is consistently higher than the corresponding value associated with linear approximation. However, the intermediate transition probabilities are generally lower in the case of quadratic modeling. Hence, both linear and quadratic models are viable options for estimating the intermediate transition probabilities while meeting the specified optimization objective.

Table 7. Sample optimal nonhomogeneous transition probabilities obtained from actual IRI data with a 5×5 Markov chain using Optimization option II

Transition number, k	Optimal parameter					
	$P'(k)_{1,2}$	$P'(k)_{4,5}$	$F'_d(k)$	$A'(k)$	$B'(k)$	$D'(k)$
1	0.4500	0.1020	0.2267	—[a]	—	0.0000
2	0.4519	0.1061	0.2347	0.012	0.111	-2.49×10^{-5}
3	0.6790	0.1884	0.2774	1.210	1.707	7.20×10^{-6}
4	0.8427	0.2973	0.3528	0.795	1.680	1.80×10^{-5}
5	0.9434	0.5702	0.6044	0.460	2.653	-1.56×10^{-5}

[a]Not applicable.

Table 8. Sample optimal intermediate transition probabilities obtained for Project B using Optimization option I ($F_d = 0.2$)

Optimal parameter	Transition number, k							
	$k=2$		$k=3$		$k=4$		$k=5$	
	Linear	Quadratic	Linear	Quadratic	Linear	Quadratic	Linear	Quadratic
$P'(k)_{1,2}$	0.6938	0.7193	0.7158	0.7700	0.7401	0.8245	0.7645	0.8796
$P'(k)_{2,3}$	0.6245	0.6096	0.6442	0.6527	0.6660	0.6988	0.6881	0.7456
$P'(k)_{3,4}$	0.5551	0.5108	0.5726	0.5468	0.5920	0.5854	0.6116	0.6246
$P'(k)_{4,5}$	0.4857	0.4226	0.5010	0.4525	0.5180	0.4844	0.5352	0.5167
$P'(k)_{5,6}$	0.4163	0.3453	0.4294	0.3697	0.4440	0.3958	0.4587	0.4223
$P'(k)_{6,7}$	0.3469	0.2788	0.3579	0.2984	0.3700	0.3195	0.3823	0.3409
$P'(k)_{7,8}$	0.2775	0.2230	0.2863	0.2387	0.2960	0.2556	0.3058	0.2727
$P'(k)_{8,9}$	0.2082	0.1780	0.2147	0.1906	0.2220	0.2040	0.2294	0.2177
$P'(k)_{9,10}$	0.1388	0.1438	0.1432	0.1540	0.1480	0.1649	0.1529	0.1759
$A'(k)$	1.2266	1.6995	0.3936	0.8633	0.4063	0.8314	0.3803	0.7579
$D'(k)$	2.16×10^{-5}	1.11×10^{-6}	4.56×10^{-6}	-1.30×10^{-5}	1.77×10^{-5}	1.37×10^{-5}	-1.84×10^{-5}	7.35×10^{-6}

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Conclusions and Recommendations

In all presented sample results, it can be noticed that both the initial and terminal transition probabilities have consistently increased over time, as would be expected due to the progressive increase in traffic loading and progressive decrease in pavement structural capacity (Abaza 2017a). Furthermore, there is more than one reliable solution to achieve the objective of minimizing the differences between the predicted and observed performance ratings. They have all provided accurate prediction of the investigated deterioration trends, although none of them may be the absolute optimal solution. Additionally, the two proposed optimization options have provided solutions that are very compatible in terms of yielding accurate performance predictions. Therefore, either one of them can be used to obtain the nonhomogeneous initial and terminal transition probabilities for a given performance trend. The data requirement is very minimal, consisting mainly of annual performance ratings estimated over the analysis period, which can be estimated from historical distress records when dealing with pavement structures. The sample results have been obtained for a maximum 10-year analysis period because this is typically adequate in most pavement management applications. However, the proposed approach can be used for a longer period if so desired.

The selection of the appropriate size of Markov chain is an essential task. The two most popular ones are the Markov chains associated with 5 and 10 condition states. Generally, a 10×10 Markov chain is used when the average annual drop in performance rating is below 10 on a scale of 100 points, whereas a 5×5 Markov chain is employed when the average annual drop reaches 20 points. In either case, the first-year initial transition probability has to be less than one when computed from Eq. (6).

The proposed sequential search approach has mainly focused on dealing with a transition matrix with only two transitions per state, as indicated by Eq. (2). The sample results have indicated that this form of transition matrix is effective in modeling performance trends similar to the ones depicted in Fig. 1. However, there may be other performance trends that this form of transition matrix may not be able to predict even with using different Markov chain sizes. The solution in this case would require using additional state transitions such as three or even four transitions per state, which would require more extensive mathematical modeling (Abaza 2021).

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

References

Abaza, K. A. 2016. "Back-calculation of transition probabilities for Markovian-based pavement performance prediction models." *Int. J. Pavement Eng.* 17 (3): 253–264. https://doi.org/10.1080/10298436.2014.993185.

Abaza, K. A. 2017a. "Empirical approach for estimating the pavement transition probabilities used in non-homogenous Markov chains." *Int. J. Pavement Eng.* 18 (2): 128–137. https://doi.org/10.1080/10298436.2015.1039006.

Abaza, K. A. 2017b. "Empirical Markovian-based models for rehabilitated pavement performance used in a life-cycle analysis approach." *Struct. Infrastruct. Eng.* 13 (5): 625–636. https://doi.org/10.1080/15732479.2016.1187180.

Abaza, K. A. 2021. "Optimal novel approach for estimating the pavement transition probabilities used in Markovian prediction models." *Int. J. Pavement Eng.* 1–12. https://doi.org/10.1080/10298436.2021.1873326.

Abaza, K. A., and M. M. Murad. 2007. "Dynamic probabilistic approach for long-term pavement restoration program with added user cost." *Transp. Res. Rec.* 1990 (1): 48–56. https://doi.org/10.3141/1990-06.

Abed, A., N. Thom, and L. Neves. 2019. "Probabilistic prediction of asphalt pavement performance." Supplement, *Road Mater. Pavement Des.* 20 (S1): S247–S264. https://doi.org/10.1080/14680629.2019.1593229.

Abu Dabous, S. 2017. "Assessing transition probability of bridge deterioration using Dempster–Shafer theory of evidence." *Bridge Struct.* 12 (3–4): 97–106. https://doi.org/10.3233/BRS-170105.

Altarabsheh, A., A. Kandil, and M. Ventresca. 2016. "Multi-objective optimization algorithm for sewer network rehabilitation using life cycle cost analysis and semi-Markov deterioration models." In *Proc., Construction Research Congress 2016*, 2089–2099. Reston, VA: ASCE.

Amin, M. S. R. 2015. "The pavement performance modeling: Deterministic vs. stochastic approaches." In *Numerical methods for reliability and safety assessment*, edited by S. Kadry and A. El Hami, 179–196. Cham, Switzerland: Springer.

Ansarilari, Z., and A. Golroo. 2020. "Integrated airport pavement management using a hybrid approach of Markov chain and supervised multi-objective genetic algorithms." *Int. J. Pavement Eng.* 21 (14): 1864–1873. https://doi.org/10.1080/10298436.2019.1571208.

ASTM. 2007. *Standard practice for roads and parking lots, pavement condition index surveys*. ASTM D6433-07. West Conshohocken, PA: ASTM.

Butt, A. A., M. Y. Shahin, K. J. Feighan, and S. H. Carpenter. 1987. "Pavement performance prediction model using the Markov process." *Transp. Res. Rec.* 1123: 12–19.

Costello, S. B., J. J. Ortiz-Garcia, and M. S. Snaith. 2016. "Analytical tool for calculating transition probabilities for pavement performance prediction." *Road Transport Res.* 25 (2): 30–39.

Fuentes, L., R. Camargo, J. Arellana, C. Velosa, and G. Martinez. 2021. "Modelling pavement serviceability of urban roads using deterministic and probabilistic approaches." *Int. J. Pavement Eng.* 22 (1): 77–86. https://doi.org/10.1080/10298436.2019.1577422.

Galvis Arce, O. D., and Z. Zhang. 2021. "Skid resistance deterioration model at the network level using Markov chains." *Int. J. Pavement Eng.* 22 (1): 118–126. https://doi.org/10.1080/10298436.2019.1578882.

Hatami, A., and G. Morcous. 2012. "Deterioration models for life-cycle cost analysis of bridge decks in Nebraska." *Transp. Res. Rec.* 2313 (1): 3–11. https://doi.org/10.3141/2313-01.

Kobayashi, K., M. Do, and D. Han. 2010. "Estimation of Markovian transition probabilities for pavement deterioration forecasting." *KSCE J. Civ. Eng.* 14 (3): 342–351. https://doi.org/10.1007/s12205-010-0343-x.

Lethanh, N., J. Hackl, and B. T. Adey. 2017. "Determination of Markov transition probabilities to be used in bridge management from mechanistic-empirical models." *J. Bridge Eng.* 22 (10): 04017063. https://doi.org/10.1061/(ASCE)BE.1943-5592.0001101.

Li, L., F. Li, Z. Chen, and L. Sun. 2016. "Use of Markov chain model based on actual repair status to predict bridge deterioration in Shanghai, China." *Transp. Res. Rec.* 2550 (1): 106–114. https://doi.org/10.3141/2550-14.

Li, Y., A. Cheetham, S. Zaghloul, K. Helali, and W. Bekheet. 2006. "Enhancement of Arizona pavement management system for construction and maintenance activities." *Transp. Res. Rec.* 1974 (1): 26–36. https://doi.org/10.1177/0361198106197400104.

Marzouk, M., and M. Omar. 2013. "Multiobjective optimization algorithm for sewer network rehabilitation." *Struct. Infrastruct. Eng.* 9 (11): 1094–1102. https://doi.org/10.1080/15732479.2012.666254.

O'Connor, A. J., E. Sheils, D. Breysse, and F. Schoefs. 2013. "Markovian bridge maintenance planning incorporating corrosion initiation and nonlinear deterioration." *J. Bridge Eng.* 18 (3): 189–199. https://doi.org/10.1061/(ASCE)BE.1943-5592.0000342.

Ortiz-Garcia, J., S. Costello, and M. Snaith. 2006. "Derivation of transition probability matrices for pavement deterioration modeling." *J. Transp. Eng.* 132 (2): 141–161. https://doi.org/10.1061/(ASCE)0733-947X(2006)132:2(141).

Pérez-Acebo, H., N. Mindra, A. Railean, and E. Rojí. 2019. "Rigid pavement performance models by means of Markov chains with half-year step time." *Int. J. Pavement Eng.* 20 (7): 830–843. https://doi.org/10.1080/10298436.2017.1353390.

Saeed, T. U., Y. Qiao, S. Chen, K. Gkritza, and S. Labi. 2017. "Methodology for probabilistic modeling of highway bridge infrastructure condition: Accounting for improvement effectiveness and incorporating random effects." *J. Infrastruct. Syst.* 23 (4): 04017030. https://doi.org/10.1061/(ASCE)IS.1943-555X.0000389.

Thomas, O., and J. Sobanjo. 2016. "Semi-Markov models for the deterioration of bridge elements." *J. Infrastruct. Syst.* 22 (3): 04016010. https://doi.org/10.1061/(ASCE)IS.1943-555X.0000298.

Wang, K. C. P., J. Zaniewski, and G. Way. 1994. "Probabilistic behavior of pavements." *J. Transp. Eng.* 120 (3): 358–375. https://doi.org/10.1061/(ASCE)0733-947X(1994)120:3(358).

Wellalage, N. K. W., T. Zhang, and R. Dwight. 2015. "Calibrating Markov chain–based deterioration models for predicting future conditions of railway bridge elements." *J. Bridge Eng.* 20 (2): 04014060. https://doi.org/10.1061/(ASCE)BE.1943-5592.0000640.

Yamany, M. S., and D. M. Abraham. 2021. "Hybrid approach to incorporate preventive maintenance effectiveness into probabilistic pavement performance models." *J. Transp. Eng. Part B. Pavements* 147 (1): 04020077. https://doi.org/10.1061/JPEODX.0000227.

Yamany, M. S., D. M. Abraham, and S. Labi. 2021. "Comparative analysis of Markovian methodologies for modeling infrastructure system performance." *J. Infrastruct. Syst.* 27 (2): 04021003. https://doi.org/10.1061/(ASCE)IS.1943-555X.0000604.