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Learning Geometry in Palestine: An Outlook at Students and Teachers

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ABSTRACT: *The current study is part of a combined general research that addressed the situation of learning and teaching geometry in Palestine. The Van Hiele levels of geometric reasoning attained by students and teachers were explored. For students, a sample of 1240 sixth, eighth, and tenth graders was studied. A random sample of 191 male and female mathematics teachers from the West Bank was selected. Results indicated low levels of geometric reasoning among students, as one third of them could not achieve the lowest Van Hiele level. About 18% of the Palestinian mathematics teachers are at the first level or below, of Van Hiele model, and 57% of the teachers did not achieve the formal deductive level.*

Key words: *Geometric thinking, Geometric reasoning, Van Hiele Levels, Palestine.*

INTRODUCTION

Despite the agreement among researchers that geometry is an important and a vital part of mathematics teaching and learning, many world countries suffer from poor students' performance in geometry (Fuys, Geddes, & Tischler, 1988; Senk, 1989; Battista & Clements, 1988). Additionally, teachers' lack of interest in geometry has negatively impacted teaching and learning of this subject (Backe-Neuwald, 1999).

One of the earliest Palestinian studies that aimed to identify the status of teaching and learning geometry in the Palestinian schools was conducted by Kamal and Masad (1991). This study investigated the level of learning mathematics among fourth and sixth graders in Jerusalem, Ramallah, and Bethlehem during the Palestinian uprising (Intifada) in 1987. Results revealed poor performance on basic mathematical skills and concepts; meanwhile the students' performance in geometry was the lowest.

Alshwaikh (2005) aimed to identify the geometric reasoning patterns among the Palestinian students. He also tried to interpret the reasons behind the students' weak performance in geometry. Later, another study had been conducted by Al-Ramahi (2006) in an attempt to reveal the reasons behind students' low performance in geometry. Al-Ramahi investigated the different levels of geometric thinking attained by Palestinian teachers of mathematics (in-service and pre-service).

The researchers became interested in trying to explain the reasons behind the weak performance of Palestinian students in geometry. As mentioned earlier, one possible reason was teachers' poor content knowledge in geometry. Thus the researchers decided to look at the teachers' levels of geometric thinking using the same theoretical framework used in studying students' geometric thinking, namely, the Van Hiele theory. The researchers thought that studying teachers' geometrical thinking might provide more insight into understanding reasons behind students' poor performance in geometry and into teachers' content knowledge of geometry. Both pre-sevice and in-service teacher education programs, might benefit from such information in designing their teacher education courses in the area of teaching geometry.

The current article brings the two studies together (Alshwaikh, 2005; Al-Ramahi, 2006) to investigate the geometry education in Palestine through responding to the following questions:

1. What are the patterns of geometric thinking among the Palestinian students?
2. What are the patterns of geometric thinking among the Palestinian teachers?

THEORETICAL FRAMEWORK

The evolution of geometry in the Soviet curriculum (at that time) had been mainly associated with two educators and psychologists: Piaget and Van Hiele. Van Hiele's ideas, though, had formed the basis of the new Soviet curriculum in teaching geometry (Fuys, Geddes, & Tischler, 1988). Pierre and Dina Van Hiele (1958), as cited in Wirzup (1976), had stated that the learning process of geometry is not connected and has leaps, which might suggest the presence of levels of geometric thinking. The levels suggested by the Van Hiele start from the holistic and analytical thinking, and then to non-formal and formal thinking, and finally to rigorous mathematical deduction. Different researchers investigated Hiele's levels; however, the description of the levels which are being adopted in the current study relied on Wirzup (1976) and Fuys, Geddes, and Tischler (1988) studies. These levels are:

- Level 0 - Visual: At this level, the child recognizes a geometric figure by its appearance, and not by thinking of its properties or parts. The objects of thought at this level are the individual figures as the child is not aware of their properties.
- Level 1 - Analysis: At this level, the child analyses a geometric shape by analysing its parts. The child understands the properties of a geometric figure and realizes the relationships between these parts through activities like measurement, paper folding, and congruence. For example, the child understands that the diagonals of a rectangle are congruent and that the rectangle has four right angles. The child understands, at this level, that the members of the same class have the same properties, but does not perceive the relationships between different shapes, implying that the child does not

link the properties of a square with the properties of a rhombus. The objects of thought at this level are the classes of geometric shapes such as the class of rectangles or the class of triangles.

- Level 2 - Informal reasoning: At this level, the child understands that some properties of a geometric figure imply the fulfilment of another property. For example, the child realizes that a right angle in a rhombus implies that the figure is a square. At this level, the child understands the meaning of a necessary and sufficient condition. For example, the child realizes that the necessary and sufficient condition for a rectangle to be a square is the congruence of two adjacent sides. The child can write a brief definition of a geometric shape, such that the definition does not include properties that could be inferred from other parts of the definition. At this stage the child does not understand the role of formal proof, but he/she can present an argument to justify his/her conclusion. At this level, the objects of thought are the properties of shapes that can be linked together in a deductive way.
- Level 3 - Formal deduction: At this level, mathematical facts including theorems are proved deductively (i.e. building on axioms, definitions, and previous theorems such as in Euclidean geometry). The objects of thought at this level are the relationships between properties of geometric shapes.
- Level 4 - Rigor: At this level new geometries are built from non-Euclidean axioms, and proof of theorems in these geometries could be established. In most cases, only college students specializing in mathematics can perform at this level. The objects of thought at this level are geometric deductive axiomatic systems.

LITERATURE REVIEW

We identified three main areas which are relevant to the Van Hiele model of geometric thinking, and to the learning of geometry by Palestinian students. These were: Learning geometry among students, teachers' geometric thinking and their content knowledge, and geometry school textbooks.

Learning geometry among students

Wirzup (1976) had a major role in drawing attention towards Van Hiele's theory. Thereafter, in the early eighties, a great concern was set off in the United States: studying Van Hiele's theory and its effectiveness in teaching geometry. Later, two major projects were launched in the United States: The Chicago University Project (Usiskin, 1982) and Oregon University Project (Burger & Shaughnessy, 1986). These two projects highlighted students' geometric thinking that was measured through pencil and paper tests and through interviews.

The Chicago University Project (1979-1982) study included a sample of 2699 high school students from grades seven to twelve. Those participants were exposed to several tests designed to measure students' levels of geometric thinking. The study found that

29% of students who performed the test could not achieve the lowest Van Hiele level (level 0). It also showed that most of the students finished their study of school geometry without acquiring the basic geometric ideas and terminology.

On the other hand, the Oregon University Project (1979-1982) came in line with the Chicago University Project relating to students' learning of geometry. The main difference was that individual interviews were used instead of paper-pencil tests. The interviews included certain tasks such as drawing, identifying, defining, sorting, inference making about an unknown shape, and knowledge about axioms and theorems which was given only to secondary and college students (Burger & Shaughnessy, 1986).

The results of the project showed that students had more ideas and beliefs about geometry than expected; some students considered some non-triangles as triangles, whereas other students excluded some triangles from the set of triangles (level 0 of Van Hiele, visual recognition). In addition, some students considered the properties of a geometric shape as necessary but not sufficient to determine the concept of that shape. This implied that the role of the definition was not clear enough to students, and that they did not appreciate the importance, the benefits, and the need for necessary and sufficient conditions in introducing definitions (level 2 of Van Hiele, Informal reasoning). The results agreed with the Chicago University Project, that students in high school lacked the ability of shape reasoning. The project also came in parallel with the Brooklyn Project which highlighted the fact that language was a vital factor in students' transition from one level to another.

Furthermore, much concern was raised about the situation of learning and teaching geometry and a great amount of research was conducted in this field. For example, Senk (1989) investigated 241 secondary students' achievement in writing geometric proofs based on Van Hiele's levels and found that there was a strong correlation between the ability of writing a proof and the Van Hiele's levels attained by students. Similar results were found in Titi's (2001) study which was conducted with 264 secondary school students. It concluded that the students' ability in writing proofs increased as they acquired higher levels of geometric reasoning. Titi also showed that 60% of the Palestinian tenth graders in the sample were categorized in level one and two. Besides, Ayasra (2002) demonstrated a strong positive correlation between Jordanian students' geometric levels and their achievement in grades 6-10.

On the other hand, Kamal and Masad (1991) attempted to study students' achievement in mathematics. Their study was implemented on the fourth and sixth graders in the central region schools in the West Bank, Palestine. The results showed very low performance of students in six areas of school mathematics, namely: Computational skills, elementary geometry, estimation and rounding, measurement, number theory, and verbal problems. The percentages of correct answers in the six areas were 21.6% for fourth grade, and 16.3 % for sixth grade. The results also revealed that only 20% of the fourth graders were able to recognize the parallelogram (level 1 of Van Hiele levels, analysis).

In regard to young learners' knowledge about geometric shapes and their properties, Clements and Samara (2000) argued that children form their concepts about shapes during a long period of time before joining schools. They found that children could recognize most shapes through a holistic approach (level 0 of Van Hiele levels, visual recognition). For example, they distinguished and identified the rectangle because it resembles a door. They also focused only on one particular property of the shape such as recognizing the triangle because it is sharp. Furthermore, children recognized circles and squares more accurately than rectangles and triangles. Nevertheless, they thought that the slanted squares were not squares. Moreover, some researchers found that non-mathematical characteristics such as skewness and orientation of the figures would affect young children's classifications of these shapes (Hannibal & Clements, 2000). As stated in Clements (1998), many children identified shapes with four sides which are almost equal and with approximately right angles as squares. They also identified four-sided shapes with opposite "long" sides approximately parallel as rectangles. Some children, though, did not recognize triangles and rectangles because they were tiny or not wide enough.

Accordingly, it was appropriate to start questioning and investigating the reasons behind these difficulties in identifying geometric shapes. Some researchers considered teachers as responsible for these difficulties. Teachers may be responsible for the lack of this exposure, since they present prototypes of geometric shapes. Besides, it was found that teachers do not use a variety of manipulations nor do they facilitate their use by students, which could enable students to practice geometry rather than being observers and monitors (Prevost, 1985).

Teachers' geometric thinking and their content knowledge

One of the most important reasons behind students' weaknesses in writing proofs was attributed to the lack of teachers' content knowledge in geometry (Backe-Neuwald, 1999). Thus, teachers should have sufficient knowledge about geometry that might enable them to present geometric topics in different ways. Additionally, the geometric reasoning depends mainly on the mathematical background of teachers which was acquired when they were school students. Therefore, if this background was not solid enough, that could lead to a lack of ability to teach geometry to students (Ahuja, 1996).

In addition, some studies indicated that teachers' content knowledge was related to teaching methods and to teachers' beliefs, and had an impact on teachers' teaching practices inside classrooms and affected learning and teaching mathematics (Abu Sharekh et al., 2004; Ahuja, 1996; Backe-Neuwald, 1999). Besides, a study was conducted by the Ministry of Higher Education in Palestine to investigate the misconceptions that teachers develop in mathematics topics. The results indicated that 39.1% of teachers did not recognize the trapezium, and about one third of them thought that the parallelogram was a trapezium (Abu Sharekh et al., 2004) (level 1 of Van Hiele levels, analysis).

Moreover, it was highlighted that teachers' beliefs and attitudes had a significant impact on teaching geometry, and that many teachers consider geometry as a worthless topic (Backe-Neuwald, 1999). In the Palestinian context, for example, teachers' attitudes toward geometry could be one of the obstacles and may cause great difficulties in teaching and learning geometry. For example, Alshwaikh (2005) observed, during conducting the paper-pencil test and the interviews, that the teachers, school principals, and students' attitudes towards teaching were not positive.

METHODOLOGY

We aimed to investigate students and mathematics teachers' geometric thinking. For that purpose, a sample of sixth, eighth, and tenth graders (N=1240), and a random sample of male and female West Bank mathematics teachers (N=191) were selected. Both samples were given the Arabic translated version of the original Geometry Test used by the University of Chicago Project (Usiskin, 1982). All test items were given to both students and teachers except for the items on the last level of Van Hiele (Level 4) which were not given to students in grades 6 and 8.

Validity

The geometry test used by the Chicago University Project (Usiskin, 1982) was translated into Arabic (Alshwaikh, 2005). The translated version was judged by a mathematics education specialist, a language specialist, and two other experienced mathematics teachers, who checked it for mathematical accuracy and appropriateness to the Palestinian context, and recommended some modifications. The items from 21 to 25 were deleted from the students' test for grades 6 and 8, whereas they were kept in the tests for grade 10 students and for teachers. This came in line with previous studies which confirm the difficulty in achieving level 4 by school students (Senk, 1989; Usiskin, 1982; Wirzup, 1976).

Reliability

Some criticism was raised to Van Hiele's test regarding its low reliability coefficient (Crowley, 1990; Wilson, 1990; Teppo, 1991, Usiskin, 1982). In the mentioned Chicago University Project, the reliability coefficients using Kuder - Richardson (KR20) method for each level was very low because of the few number of items for each level (Usiskin, 1982). The reliability could be increased if each level contained 25 items instead of 5 items. The Cronbach' Alpha coefficients for the students' test in the current study were 0.40, 0.09, 0.31, 0.23.

For the teacher's test, the reliability coefficient was calculated using test-retest procedure, where the test was administered on a random sample of 20 mathematics teachers, who belonged to the population of this study but not to the sample. Then the test was re-conducted again after two weeks, and the test-re-test Pearson correlation coefficient was found to be (0.82). On the other hand, Cronbach's Alpha coefficients were used to calculate the reliability of each Van Hiele level (the sub-tests) by adopting Usiskin's correction procedure. The five coefficients were found to be 0.73, 0.64, 0.71, 0.70 and 0.62.

Data collection

The students' test consisted of 25 multiple-choice items in the duration of 35 minutes. The test items were categorized into ascending order of five questions for each level of geometric reasoning. For example, the first five questions in the test measured the visual level (0), while the next five (6-10) measured the analytical level and so on.

Below, some test items examples are presented: one question from Level 2-Informal deduction (Question 14 from students' test and Question 23 from teachers' test) and one question from Level 3- Formal deduction (Question 18 from students' test and Question 35 from teachers' test):

14 (23). Which is true?

A. All properties of rectangles are properties of all squares.

B. All properties of squares are properties of rectangles.

C. All properties of rectangles are properties of all parallelograms.

D. All properties of squares are properties of all parallelograms.

E. None of (A)-(D) is true.

18(35). Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.

II: If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

A. To prove I is true, it is enough to prove that II is true.

B. To prove II is true, it is enough to prove that I is true.

C. To prove II is true, it is enough to find one rectangle whose diagonal bisect each other. D. To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.

E. None of (A)-(D) is correct.

After conducting the students' test, the students' responses were coded and analysed using SPSS. The students' incorrect responses were coded 0, and the correct responses were coded 1. Achieving a Van Hiele level was based on the following criteria (Usiskin, 1982):

- Achieving level 0 as a prerequisite, otherwise the students would not be classified.
- Achieving the lower level is a prerequisite to achieve the following level. For example, a student must achieve Level 0-Visual and Level 1-Analysis to qualify for consideration in Level 2-Informal deduction. Passing one level without passing the previous levels was considered as a result of guessing, and consequently was dropped out.
- Achieving at least three correct answers out of five.

On the other hand, the teachers' test consisted of 40 multiple choice items in the duration of one hour. The test items were categorized into ascending order as well, and each 10 items represented one level of geometric reasoning.

The teachers' responses were coded and analyzed in the same way as students' responses using SPSS, and achieving a Van Hiele level was based on the same criteria explained above for students, except that achieving a Van Hiele level required correctly answering at least six out of ten items.

RESULTS AND DISCUSSION

Here we report the results with discussion of both students' and teachers' responses to the test.

1. Students' results

The first question we asked was: What are the patterns of geometric thinking among the Palestinian students? The students' geometric thinking was investigated and analysed according to the five Van Hiele levels: recognition of the basic shapes, identifying the characteristics of the basic shapes, the recognition of the relations between shapes, the formal deduction, and rigor or the use of axiomatic systems. Table 1 shows percentages of correct responses on items (1-25) by grade level.

Table 1

Percentages of Correct Responses on Items (1-25) in students' test by Grade Level

Item	Item objective	Sixth	Eighth	Tenth
1	Identifying the square	90.2	91.6	96.6
2	Identifying the triangle	34.0	61.8	56.2
3	Identifying the rectangle	54.5	68.6	75.8
4	Identifying leaning square	75.8	75.8	75.8
5	Identifying Parallelogram	26.4	45.0	50.2
6	The Characteristics of Square	23.2	33.7	40.4
7	The Characteristics of Rectangle	64.8	66.7	80.0
8	The Characteristics of the rhombus	18.4	24.6	30.2
9	The Characteristics of Isosceles Triangle	32.4	44.6	44.2
10	The Characteristics of a kite shape	18.6	22.8	37.7
11	Informal deduction about the triangle and the rectangle	21.5	27.1	38.1
12	Triangle and isosceles triangle	22.7	36.6	40.8
13	The relations between the square and the rectangle	9.6	16.2	19.6
14	Relations between squares, rectangles and parallelograms	9.6	13.8	14.0
15	Rectangles and Parallelograms	16.8	20.9	28.7
16	Deductions about right triangles	16.0	27.7	21.1
17	Logical statements about the characteristics of the square, the rectangle and the two diameters	21.1	18.1	18.5
18	A proof about the rectangle and its two diagonals	17.6	24.2	30.2
19	Basics about the structure of Geometry	16.0	16.0	14.3
20	Proof's interpretation (the reason behind the parallelism of two lines)	14.1	9.0	6.8
21	Non-Euclidean Geometry: Intersection and parallelism	*	*	10.2
22	The impossibility of trisecting an angle	*	*	26.4
23	Non- Euclidean geometry: Total Triangle angles	*	*	22.3
24	Non- Euclidean geometry: rectangle	*	*	26.8

	characteristics			
25	Non- Euclidean proof	*	*	20.8

From the table above, the following results can be highlighted:

1.1 Recognition of basic shapes

- Students' recognition of the five geometric shapes, ordered from easiest to hardest were: the square, the leaning square, the rectangle, the triangle, and the parallelogram.
- Only 34% of sixth graders could identify the triangle, 66% of them could not recognize that the shape in Figure (1a) as a triangle. 46% of grade six students and 31% of grade 8 students and 24% of tenth grade students could not recognize the shape in Figure 1(b) as a rectangle.

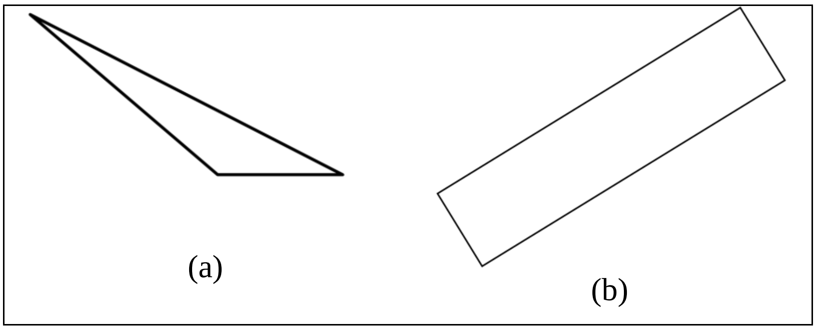

Figure 1. A triangle and a rectangle

In general, the Palestinian students' geometric reasoning was found to be too deficient, as most students could only achieve the lowest level (level 0). Except for being able to identify the square, they had difficulties in identifying the other shapes. In identifying the tilted square only 75.8% of students in grades 6, 8, and 10 answered the question correctly.

1.2 Identifying the characteristics of basic shapes

Items 6 to10 in the test aimed at identifying students' knowledge about properties of some shapes: the square, the rectangle, the rhombus, the isosceles triangles, and the kite shape.

From the table above, the following results can be highlighted:

- 75% of grade six students, 66% of grade eight students, and 60% of grade ten students could not recognize that the diameters of the square are orthogonal.
- 80% of grade six students, 75% of grade eight students, and 70% of grade ten students thought that the two diameters of the rhombus are equal.

- 67% of grade six students, about 50% of each of grades eight and ten students could not recognize that the isosceles triangle has at least two equal angles in measurement.

The previous results indicated that the Palestinian students did not know the characteristics of the basic shapes they have learned. They also lacked the knowledge of the fundamental properties of the basic geometric shapes.

1.2 The recognition of the relations between shapes

Items 11 to 15 explored children's knowledge about the relations between geometric shapes and their ability to use informal deduction. The items either required explicit identification of the relationship between geometric shapes or test children's ability to use informal deduction.

The following is a summary of the findings:

- Only 23% of sixth graders, 37% of eight graders, 41% of tenth graders concluded that an isosceles triangle has two equal angles.
- More than 80% of students in the three grade levels did not recognize that the square is a special case of a rectangle.
- 86% of students (in grade eight and ten) did not recognize the inclusion relations between squares, rectangles and parallelograms

1.3 The formal deduction

Items 16 to 20 of the test aimed at measuring students' abilities to make formal deductions or deductions supported by full proofs. For example, students were asked to prove that two straight lines are parallel if both are orthogonal to a third line.

Students' performance on these items was extremely low and ranged from 6.8% to 30%. This result was close to the level expected through sheer guessing, which is 20% for each item.

1.4 Rigor or Axiomatic deduction

Items 21 to 25 in the test elicit responses to check students' abilities to deal with abstract geometric system, which is different from what students used to learn in Euclidean geometry. This section of the test, however, was provided for tenth graders only.

The percentages of correct responses were extremely low and ranged from 10.2% to 26.8% which are at sheer guessing level or below. The researchers inferred that this result was achieved through guessing.

The previous results allowed us to conclude that the Palestinian students mainly relied on the general appearance of the geometric shapes, and that their geometric thinking was restricted to the visual recognition of these shapes. They also could not recognize these basic shapes when they were presented in a different orientation. Moreover, they considered geometric shapes independent of each other with no relations between them. These conclusions came in line with several previous studies that had been reviewed before (see Wirszup, 1976; Fuys, Geddes, & Tischler, 1988; Clements, 1998).

It seemed that the Palestinian students were not exposed to sufficient and appropriate experiences in learning geometry, and that teaching geometry is probably only restricted to observing or memorizing some facts and rules, which has a negative impact on students' attitudes towards geometry. Moreover, it seemed that geometry was usually presented in one pattern, for example the shape of a rectangle was always viewed as a window. Thus, students become unable to transfer their learning to other contexts (Bransforrd, Brown & Cocking, 1999). Teachers' poor knowledge in geometry affected students' understanding of geometry, and consequently affected negatively their geometric knowledge and thinking. Moreover, the Palestinian National Tests for fourth, sixth and tenth graders investigated the Palestinian students' abilities through a mathematics general evaluation. These evaluations checked three main aspects: Conceptual understanding, Procedural knowledge, and Problem solving (Ministry of Education/Measurement and Evaluation Center, 1998, 2000a, 2000b). These tests came with a conclusion that students' general performance in mathematics was very poor. The initial report of a study, which investigated the achievement level of sixth graders, stated:

> The students' performance in all mathematical topics is very poor and weak. Geometry, graphic representations, and the proportionality are the most difficult to students, and they need much concern. The feedback that is provided by mathematics teachers is right; that teaching geometry in the beginning of the scholastic year rather than at the end of it, along with giving the time needed to teach the geometric concepts would make a big improvement in students' performance. Besides, it is essential to connect geometry concepts to practical experiences such as a paper folding and cutting cardboards, since that will be beneficial as long as the student is the performer of the task not just the observer (p. 30).

2. Teachers' results

Our question regard teachers was: What are the patterns of geometric thinking among the Palestinian teachers? Results of the teachers' study indicated poor performance of in-service mathematics teachers, while results of the pre-service teachers were very disappointing. Figure 2 summarizes the results of both pre-service and in-service teachers on the Van Hiele Test.

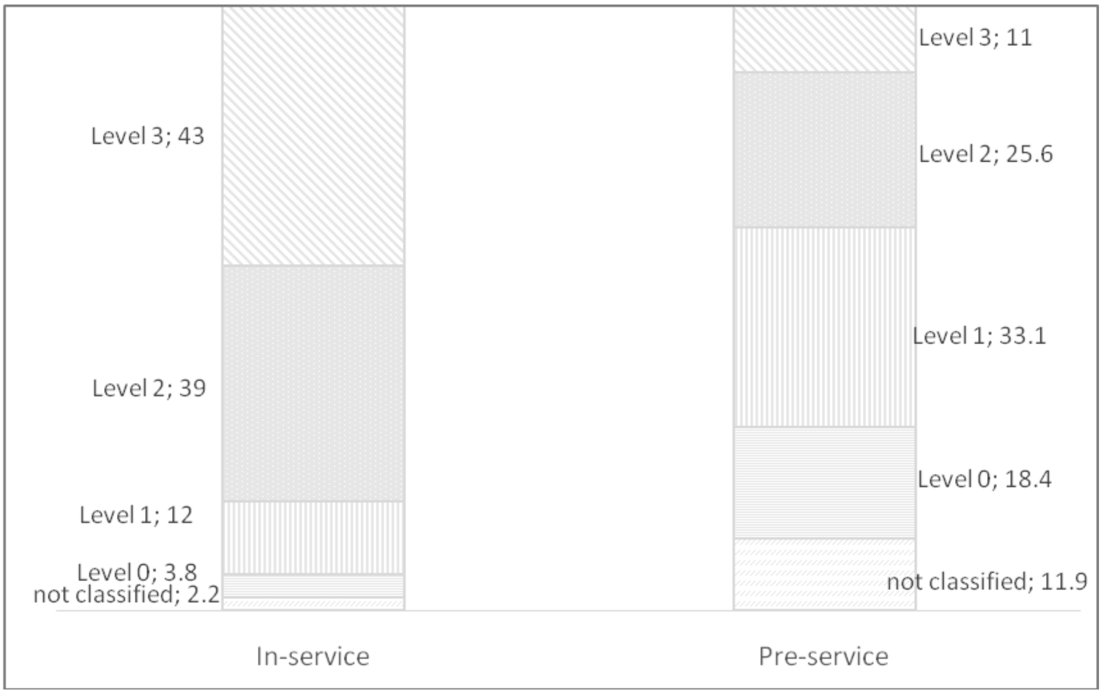

Figure 2. Percentage distribution of teachers (pre-service and in-service) by thevan hiele level attained (levels 0-3 in addition to those who were not classified)

Figure shows that 2.2% of the Palestinian in-service teachers had not been classified. The major difficulty of those teachers seemed to be in achieving the formal deduction level. In spite of the fact that formal deduction was an important constituent of geometry textbooks from grades 8 to 10, only 43% of the mathematics teachers achieved this level. Moreover, the results also show that 18% of in-service mathematics teachers had not reached the informal deductive level. This result was in line with some of the studies that had been previously reviewed (Fuys et al., 1988; Swafford et al., 1997; Surizal, 2003). On the other hand, the results of the current study were consistent with the results of (Abu Sharekh et al., 2004), a Palestinian study aimed to identify the misconceptions among mathematics teachers who teach eighth, ninth and tenth grade students. It also investigated teachers' knowledge about the characteristics of a circle, a rectangle and a trapezium. The results showed that 26.7% of the Palestinian teachers thought that the points inside the circle were a part of it. Whereas 32.4% of them thought that the two diameters in the rectangle divide its angles equally, and the characteristics of the rectangle and the square are the same. Additionally, 35.2% of the teachers thought that the parallelogram is a trapezium.

It is obvious that the teachers of eighth, ninth and tenth grades lacked a sound knowledge of geometry content, which probably caused these misconceptions. These difficulties with geometry may also be explained by the poor mathematics curriculum when these teachers were school students. The geometry syllabi did not help teachers or students to develop a good understanding of basic geometry, and probably developed a negative attitude towards this subject. Most mathematics teachers believed that geometry was not an essential topic. Both pre-service and in-service teachers seem to lack basic knowledge needed to advance to higher levels of geometric thinking.

Besides, mathematic teachers might receive teaching in one context, which makes them unable to transfer what they learned into new contexts (Brandsford, et al., 1999). Teachers' difficulties with geometry might be explained by the fact that mathematics teachers were not qualified enough during their college years. Pre-service teachers' performance on the Van Hiele test was extremely poor. Results show that about 63% of them are at the analysis level or below (i.e. level one or below). A child at age 9-10 is expected to achieve the descriptive level. This conclusion agrees with previous studies in this field (Ahuja, 1996; Mayberry, 1983). Also, it agrees with the results of a study conducted in Singapore which revealed that 8.3% of pre-service teachers did not achieve higher than the visual level (Ahuja, 1996).

CONCLUSIONS

The present study is an outcome of two studies aimed to investigate geometric thinking in the Palestinian context. This study adopted the Van Hiele Model and the data collected from the test, as the main instrument of the study, clarified the situation of geometric thinking among students and mathematics teachers.

The study confirmed that the majority of students were unable to achieve Level 0-Visual and Level 1-Analysis since they were unable to recognize the basic geometrical shapes (the square, rectangle, triangle and parallelogram) especially when these shapes were presented in an unfamiliar way. For example, tilting a square made it look like a diamond and hence was considered a rhombus. Besides, students' ability of geometric explanations did not improve much as students progressed through school grade levels. The majority of them, for instance, did not recognize that there were at least two equal angles in an isosceles triangle. Students' performance at Level 2-Informal reasoning for each of the three grade levels in recognizing that the square is a special case of a rectangle was extremely poor.

The low performance of Palestinian students in geometry was not surprising, when compared with other students' performance around the world. Wirzup (1976) found that the highest level attained by the majority of high school students is Level 1-Analysis. The results of this study were consistent with many relevant studies in several countries such as USA (Usiskin, 1982), Spain (Gutierezz & Jaime, 1988), Japan (Whitman et al., 1997), Jordan (Ayasra, 2002), India (Ali, Baghwati, & Sarmah, 2014), Nigeria (Adolphus, 2011), and South Africa (Mogari, 2003), to name but a few. Most of these studies reported poor performance of students on geometry tests or tasks. A common reason behind this poor performance is probably high school students' negative attitude towards geometry (Hoffer, 1981).

The results of the current study indicated that the majority of in-service teachers were still at Van Hiele Level 1 or 2 (Analysis and Informal reasoning). Similar results were used in other countries such as the United States (Knight, 2006), and Saudi Arabia (Khalid, 2015). Teachers who were incapable of making formal reasoning and construct simple geometric proofs are not qualified to teach high school geometry. A large proportion of teachers need training which must focus on developing their content and pedagogical knowledge in geometry. In parallel with teachers training, developments of the subject matter and activities presented in the school textbooks can help both teachers and students. A quick survey of the geometry units in the school textbooks revealed that these activities belong to the lower Van Hiele levels and are probably insufficient to promote the advancement of students' geometric thinking to higher reasoning levels (Al-Ramahi, 2014).

Also the results indicated very poor performance of pre-service teachers, since 11.9% of them were not classified to any level, and only 11% of them achieved the formal deductive level. These results agreed with the results of a study conducted in Singapore revealed that 8.3% of pre-service teachers did not achieve higher than the visual level and 38.6. % of them achieved the analytical level or lower level. Additionally, 42.8% of teachers achieved the informal deduction, or lower level, whereas 8.3% of them achieved the formal deduction (Ahuja, 1996).

The weak performance of the Palestinian students in geometric thinking could be linked to pre-service and in-service teachers' lack of sufficient content and pedagogical knowledge of geometry and teaching geometry, so more efficient teacher training systems are needed to build the capacity of teachers to understand geometry and to develop their methods of teaching geometry, but it seems that they resist such changes. Hashweh (1986) pointed out that one of the major factors which prevented teachers from acquiring new concepts is due to their prior background and knowledge. Based on that, it is recommended that teachers' prior concepts be acknowledged and targeted for change.

Accordingly, there are many procedures that can be applied in Palestine so as to enhance the status of geometry education. Additionally, there is a need to emphasize the definitions of the shapes and their characteristics to be a reference in recognizing the shapes and improving the geometric reasoning among teachers and students. Besides, teachers and students should be encouraged and engaged in problems that require discussions and exchanging opinions to develop the students' geometric language.

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