Comparison between Elastomeric Passive Isolators and LQR Active Control in Stone Cutting process: Modelling and Simulation

Ahmed Abu Hanieh\textsuperscript{a},*, Ahmad Albalasie\textsuperscript{a}

\textsuperscript{*Birzeit University, PO Box 14, Birzeit, Palestine}

Abstract

This paper focuses on the problem of mechanical vibrations induced in the cutting process of stone and marble in stone cutting machines. The study leads to sustainable stone cutting operations by saving energy, further finishing and providing noise clean environment. This periodic force induced in the cutting area is reduced passively by using an elastomeric pad that isolates the base of the machine from the head and the table. Adding less than 2.5% of passive damping ratio to the isolator could reduce the level of vibration on the saw blade and on the table. To reduce the propagation of these vibrations through the base into the floor, another elastomeric pad is mounted under the machine. This pad reduces the propagation of oscillations through the floor to the other places in the factory. Another solution is presented to reduce the vibrations in the whole structure significantly by regulating the input force this one is controlled by the LQR method. The controller has the capability to regulate the vibrations in the structure by solving the Riccati equation to compute the optimal response for the input force to perform the required tasks.

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1. Introduction

Stone and marble industry is one of the main income resources in the Middle East and North Africa (MENA) countries. For example, this sector employs more than 30% of the workers in Palestine and conforms 25% of the general exports. This fact refers to the rocky nature of the landscape and the good type of stones in these countries.

* Corresponding author. Tel.: +970-2-298-2115; Fax: +970-2-298-2984.
E-mail address: ahanieh@birzeit.edu

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Although this is a very important sector, it suffers from many social, economic and environmental problems [1]. The problem tackled in this paper is focused on seeking solutions for the mechanical vibrations and noise in the stone and marble cutting machines. Two approaches can solve the previous problem. The first solution is mainly passive one using elastomeric isolators (pads). This solution is affordable and can be integrated into existing machines easily. In the early seventies, NASA used the passive vibration isolation using elastomeric isolators for engineering applications as shown in [2]. In that report, NASA authors test the stiffness characteristics and damping properties of the elastomer mounts under constant temperature using an experimental test rig. While the second solution is to use the Linear Quadratic Regulator (LQR) controller to control the input force actuator. Consequently, the actuator regulates the vibrations in the whole structure of the stone cutting machine. This type of controller is used usually to regulate the vibrations in the structure e.g. LQR algorithm used to regulate the vibrations in two-link planar elastic manipulator as shown in [3]. Furthermore, this algorithm has the capability to apply the damping for the vibrations in a beam as discussed in reference [4].

Noise and vibrations in marble cutting machines are induced by the direct impact between the stone and the circular cutting saw teeth. The periodic frequency of this cutting process is determined by the angular speed of the cutting saw and the number of teeth on its circumference. The first stone cutting circular saw in its recent shape was invented by Milton Sprague in his US patent published in 1964 [5]. This patent explains the design of the circular saw with peripheral notches and abrasive material on the edges of the teeth. Nowadays, this circular saw in widely used in both hand tools and cutting machines for stone and marble. Energy consumed during cutting is discussed in [6], this paper investigates different parameters during cutting of marble in terms of energy consumption with some experimental results. Abhijit Ganguli focused in his Ph.D. thesis at the Free University of Brussels on the chatter and vibrations of metal cutting processes in turning and milling machines in [7]. Although metal cutting depends on rotating tools, but it differs from marble cutting by the existence of chatter that does not significantly influence the marble cutting process. The influence of mechanical vibration on granite is discussed in [8]. In this paper, the authors study the influence of mechanical vibrations on the granite-epoxy beams. This study was performed using an impact hammer and force sensor. The influence of vibrations on the health of workers was discussed by [9] tackling the problem of hand-arm vibration syndrome of hand-held machine tools and by [10] discussing Hand-arm vibration syndrome and dose-response relation for vibration-induced white finger among quarry drillers and stone carvers. In 2012 Sumida et al. invented a new type of low vibration saw blade in [11]. This new cutting saw blade replaces the notches and teeth with a layer of abrasive diamond grains on the peripheral edge of the disk. Another focus in this invention to reduce vibration is the use of a plastic pair of male and female including the central hole of the disk like a sandwich. Feng Lu et al. discuss the numerical control of stone column turning process using modal analysis as shown in [12]. Stress analysis of the saw blade is discussed in [13]. Finite element analysis is used in this paper to study the influence of stresses on the side of the saw blade resulting from cutting processes and vibration effects.

The first motivation for this research is the effect of vibrations on the surface finish of the cut stones. Unstable saw blade causes bad corrugations and cracks on the stone surface as shown in Fig. 1. These corrugations force manufacturers to pass the stone to another finishing process to obtain the required final smooth surface finish. The design shown in Fig. 2 is used to study the vibration phenomena and its influence on stones. The second motivation is the noise level induced by this cutting process. This noise is caused by the propagation of sound through floors and walls affecting people in the neighbourhood. This is also a serious problem knowing that most of the small and medium stone cutting workshops in developing countries like Palestine are in the populated areas.
The vibration isolation tackled in this paper leads to a sustainable stone and marble industry in two ways. The first way is that through getting a non-corrugated clean stone cut will save the need for further finishing of the stone surface which saves time, money, operation, and energy. The second way is that vibration reduction leads directly to noise reduction obtaining a noise clean environment. This noise propagates into the surrounding environment annoying the neighbourhood people living around the stone factories.

Following the introduction and literature review, section 2 discusses a numerical model of the stone cutting machine in which a CAD model and block diagram of the machine are explained. Section 3 tackles the mathematical model of the marble cutting machine using second order linear differential equations and state space model. The following section discusses the LQR algorithm to control the system. This section is followed by numerical simulation, using MATLAB and based on the discussed mathematical model. All simulation results and conclusions are discussed and summarized each in its corresponding section. Finally, the conclusions section is discussed at the end of the paper.

2. Stone cutting machine model

The stone cutting machine has been designed and modelled using CATIA software. It consists of three main parts; the head, the table and the base. The head and the table are mounted on top of the base and the base is mounted on the ground of the workshop. Isolation elastomeric pads are suggested to be inserted between the different components and under the base as shown in Fig. 2.

Fig. 3 shows a block diagram that represents the cutting machine taking into account the vertical degree of freedom only where the input variable to the system is the periodic cutting force $F(t)$ as a function of time and the outputs of the diagram are the base force $F_B$ and the three displacements; $x_H$ (Head displacement), $x_T$ (Table displacement) and $x_B$ (Base displacement).

The head displacement ($x_H$) is derived from the input force by the transfer function $(1/m_Hs^2)$ where $m_H$ is the head mass and $s$ is the Laplace variable. The stiffness ($k_H$) and damping ($c_H$) cause forces of the isolation elastomeric pad, these forces are fed back to be subtracted from the input signal as depicted in Figure 4. The table displacement ($x_T$) is derived the same way from the input force. The base displacement ($x_B$) is derived also from the force transmitted to the base via the table and the head while the stiffness and damping forces of the base ($k_B$ and $c_B$ respectively) are fed back to the head and table translating the cross-talk between these three parts.

3. Mathematical analysis

The machine shown in Fig. 2 is represented in the lumped-mass symbolic form shown in Fig. 4 where each of the elastomers is represented by a spring and damper strut supporting the corresponding mass. The head and the table are simplified for the study and considered to be guided in the vertical direction to avoid lateral (horizontal) motion causing only vertical translational motion. This can be achieved practically by adding vertically installed guiding pins and collars between the table and base from one side and between the head and the base from the other side.
The governing dynamic equations of the system can be expressed in the following three equations:

\[ m_H \ddot{x}_H + c_H (\dot{x}_H - \dot{x}_B) + k_H (x_H - x_B) = F \]  
\[ m_T \ddot{x}_T + c_T (\dot{x}_T - \dot{x}_B) + k_T (x_T - x_B) = F \]  
\[-m_B \ddot{x}_B - c_B \ddot{x}_B - k_B x_B + c_T (\dot{x}_T - \dot{x}_B) - c_H (\dot{x}_H - \dot{x}_B) + k_H (x_H - x_B) + k_T (x_T - x_B) = 0 \]  

Where:
- \( F \) is the input vertical force on the cutting blade (function of time),
- \( m_H, c_H \) and \( k_H \) are respectively, the mass, displacement, damping, and stiffness of the head.
- \( m_T, c_T \) and \( k_T \) are respectively, the mass, displacement, damping, and stiffness of the table.
- \( m_B, c_B \) and \( k_B \) are respectively, the mass, displacement, damping, and stiffness of the Base.
- \( \dot{x}, \ddot{x} \) are the first and second time derivatives of the displacement.

In order to study this system on MATLAB-SMULINK software, state space approach was used taking the following parameters:

Input: Cutting force \( \{ F \} \)

Outputs: Three displacements and force \( \{ x_H, x_T, x_B, F_B \} \)

State variables: The displacement and velocity of each part \( \{ x_H, \dot{x}_H, x_T, \dot{x}_T, x_B, \dot{x}_B \} \)

Taking the previous parameters into account, the system matrix of the state space model reads:
The input vector is given by:

\[
 B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{m_H} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_T} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{m_B} & \frac{-k_B}{m_B} & \frac{k_T}{m_B} & \frac{-k_T}{m_B} & \frac{-c_B}{m_B} & \frac{1}{m_B} \\ \end{bmatrix}^T
\]

Knowing that force transmitted to the base is given by:

\[
 F_B = k_B x_B + c_B x_B
\]

The output matrix is:

\[
 C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & k_B & c_B \end{bmatrix}
\]

And the feedthrough matrix is:

\[
 D = \begin{bmatrix} 0 & 0 & 0 \\ \end{bmatrix}^T
\]

4. Numerical Finite Element Simulation

The foregoing mathematical model has been translated numerically using MATLAB software using numerical values where the masses of the base, table, and head are 337 kg, 100 kg, and 19.2 kg respectively. The stiffness of the elastomeric pads of cutting head holding the saw, the moving table, the base block are equal to 120 kN/m, 100 KN/m, and 100 kN/m respectively. Three vibrational normal modes were found in the system; these modes are calculated by using the finite element method (FEM) on CATIA software. The normal mode of the head against the base is found at a frequency = 77.9 rad/s = 12.4 Hz as shown in Fig. 5. The FEM model is simplified to take only the vertical degree of freedom other degrees of freedom will be taken in further study.
5. Passive isolation using elastomeric passive pads

The elastomeric passive isolators were mounted as shown in Fig. 2. The damping coefficient of the elastomers is calculated to give a reasonable transmissibility where the transmissibility of the head reached 10%, this causes a damping ratio of the head equals to 2.6% and a damping ratio of the table equals to 1.2%. Fig. 6 (a) shows a Frequency Response Function (FRF) between $F$ as an input and the $x_H$ as an output. The dotted blue line expresses the vibrational dynamic behaviour in the frequency domain (Bode plot) of the head against the base assuming that there are no elastomers; while the solid red line expresses its behaviour with the elastomeric pad inserted in the system as shown in Figure 3. This FRF shows that using passive elastomers could reduce the overshoot about 15 dB in the first mode related to the head and about 20 dB in the table and base modes. It also depicts the response in the time domain of the dynamic displacement of the head; once without passive damping and the other curve is with 2.6% passive damping. The simulation has been extended to 100 seconds to show the influence of damping which has a slight effect on the oscillations. Fig. 6 (b) shows a FRF between $F$ as an input and $x_T$ as an output.

The dotted blue line expresses the vibrational dynamic behaviour in the frequency domain of the table against the base assuming that there are no elastomers; while the solid red line expresses its behaviour with the elastomeric pad inserted in the system as shown in Fig. 6 (b). This FRF shows that using passive elastomers could reduce the overshoot between 15 to 20 dB in the modes of the head and the table. This figure depicts the response in the time domain of the dynamic displacement of the table; once without passive damping and the other curve is with 1.2% passive damping.
6. Active damping using LQR controller

Linear Quadratic Regulator LQR is classified as a state feedback controller. This class of controller is focused on calculating the gain matrix \( K \) to stabilize the system and/or to enhance the performance of the system. According to [14], LQR is calculating the optimal response for the system by solving the reduced Riccati equation. Therefore, the algorithm is searching on the minimum optimal value in a quadratic cost function \( J \). This cost function is also known as the performance index as shown in Eq. 5. Consequently, the LQR has the capability to regulate the vibrations in the whole mechanical structure by forcing the states to go to zero i.e. it is killing the vibrations. On the other hand, it has the ability to minimize the energy consumption by selecting proper values for the design weighting matrices \( Q \) and \( R \). In our case, the cost function contains the input force and the states as shown in Eq. 5.

\[
J(u) = \int_0^\infty (x^TQx + u^TRu + 2x^TMu)dt
\]  

(5)

Substitute to
\[
\dot{x} = Ax + Bu
\]
\[
y = Cx + Du
\]

Where: \( R \) is a weighting matrix, but it must be a positive definite matrix, \( Q \) is another weighting matrix, but it must be a symmetric positive semi-definite matrix, and \( M \) is the cross term that relates \( u \) to \( x \) in the performance index. Based on that, selecting suitable values for the weighting matrices lead to guarantee the stability of the system if the model is linear. This is achieved by choosing large values of matrix \( Q \) and this leads to force the eigenvalues to go to the left side on the s-plane i.e. the states go to zero faster than the original case [14]. But, this increases the input force therefore this leads to increase the energy consumption. On the other hand, increasing the values of matrix \( R \) leads to opposite effects. However, matrix \( P \) is computed by solving the reduced Riccati equation see Eq. 6 after selecting the weighting matrices \( Q \) and \( R \) [14]. Consequently, the gain matrix \( K \) is computed by using Eq. 7.

\[
A^TP + PA - PBR^{-1}B^TP + Q = 0
\]  

(6)

\[
u = -Kx = -R^{-1}B^Tx
\]  

(7)

In this subsection, the LQR algorithm has been used to control the system by applying the input force \( F \) as an input. The selected weighting matrices \( Q \) and \( R \) as follows \( Q = 3000*\text{eye} \) (6) and \( R = 0.1 \). In addition, a full state observer has been used to estimate the unmeasured states.

The FRF between \( F \) as an input and \( x_h \) as an output is shown in Fig. 7 (a). The solid red line expresses the dynamical behaviour of the head with using the LQR controller. On the other hand, the dotted blue line expresses the dynamical behaviour of the head against the base without using the controller. Fig. 7 (a) shows the effect of using the LQR controller on reducing the overshoot up to 55 dB. However, the impulse response of the head in the time domain with and without using the LQR controller is also shown in this figure. Based on that, the vibrations on the head is regulated in 5 seconds. The FRF between \( F \) as an input and \( x_T \) as an output is shown in Fig. 7 (b). The effect of using the LQR controller is summarized by increasing the virtual damping in the system which leads to kill the vibrations faster than the original case. It can be seen in Fig. 7 (b) that the peak values are reduced in some cases up to 55 dB. While the impulse response of the table in the time domain with and without using the LQR is shown in this figure. Based on that, the vibrations on the table is regulated also in 5 to 6 seconds.

7. Conclusions

The foregoing paper discusses the possibility of integrating passive elastomers in the stone cutting machines for the purpose of isolating the cutting head and table from vibrations induced in the cutting process. The main goal is to reduce the corrugations formed on the surface of the cut stone and to protect the machine parts from the influence of the propagating vibrations. Another elastomer is mounted under the base of the machine to avoid transmitting these vibrations into the ground and reducing noise pollution. Another technique has been introduced to kill the vibrations in the whole structure by using the LQR controller which leads to perform the required task besides reducing the energy consumption. Numerical simulation using MATLAB software showed a slight influence for these elastomers...
where it caused an amplitude reduction of 15 to 20 dB for the head and the table which are considered the most stringent parts of the machine. The base is also influenced by this technique. These reductions were obtained with a passive damping ratio between 1.2% and 2.6%. On the other hand, the LQR controller has better effects on the head, the table and the base respectively because it caused an amplitude reduction of 50 to 55 dB. Therefore, the numerical simulation shows a better result for the LQR technique with a comparison to passive elastomers technique. This vibration isolation has a great influence on energy reduction and noise reduction which helps in creating sustainable cutting process free of noise and with the lowest energy consumption due to saving further finishing processes.

![Image](image-url)  
(a) FRF of cutting force and the head displacement  
(b) FRF of cutting force and the table displacement

Fig. 7. Frequency response and time response functions due to applying LQR controller

References