

# Identification of Roadway Wavelengths Affecting the Dynamic Responses of Bridges due to Vehicular Loading

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**Abstract**—The bridge vibration due to traffic loading has been a subject of extensive research during the last decades. A number of these studies are concerned with the effects of the unevenness of roadways on the dynamic responses of highway bridges. The road unevenness is often described as a random process that constitutes of different wavelengths. Thus, the study focuses on examining the effects of the random description of roadways on the dynamic response and its variance. A new setting of variance based sensitivity analysis is proposed and used to identify and quantify the contributions of the roadway's wavelengths to the variance of the dynamic response. Furthermore, the effect of the vehicle's speed on the dynamic response is studied.

**Keywords**—vehicle bridge interaction, sensitivity analysis, road unevenness, random processes, critical speeds

## I. INTRODUCTION

HERE has been an increasing attention to solve the bridge-vehicle interaction, which is encouraged by the advent computational power of digital computers and the increasing number and weights of vehicles traveling on bridges. Therefore, researchers and modelers had been concerned with deriving solutions of the dynamic problem of bridge-vehicle interaction. *F. Yang et al.* [1] and *Yang et al.* [2] reviewed the different methods with their corresponding mathematical and computational descriptions. The solution of the dynamic response of the bridge and/or the vehicle starts by writing the equations of motion for both subsystems. These equations are either written in an integrated (coupled) form or left separate prior to the solution. The integrated (coupled) equations for the bridge-vehicle interaction are built by substituting for the dynamic interaction forces, e.g. *Cheung et al.* [3]. The coupled set of equations are often solved using direct time integration methods. For the latter, the two sets of differential equations of the vehicle and the bridge are left uncoupled contingent on satisfying the compatibility constraints at the contact points. The solution of the differential equations can be also determined by using direct integration methods in an iterative or non-iterative procedure, e.g. *F. Yang et al.* [1] and *Liu et al.* [5], respectively.

In examining some of the studies concerned with bridge-vehicle interaction, one can detect attempts to assess the effects of road unevenness on the dynamic response through probabilistic studies. *Hwang and Nowak* [6] presented a procedure to calculate statistical parameters for the dynamic loading of bridges. These parameters were based on surveys and tests

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and included vehicle mass, suspension system, tires and road roughness, which were simulated by stochastic processes. *Kirkegaard and Nielsen* [7] studied the effects of random road profiles on the dynamic response of highway bridges by examining the variance of the output. A recent study by [4] attempted to identify the wavelengths of unevenness that affect the dynamic response at different speeds. The study considered the irregularity as a sinusoidal wave corresponding to one wavelength and ran the dynamic analysis for variant speeds. Then the authors repeated the procedure for the different wavelengths. Such a procedure overlooked the realistic and random nature of unevenness. Moreover, solutions for the statistical characteristics of a bridge's response to the passage of a vehicle over a random rough surface have been of interest in a number of research works, such as *Lin et al.* [8], *Lombaert and Conte* [9], and *Wu and Law* [10].

This study attempts to go further and quantify in mathematical expressions the effect of random road unevenness in terms of its wavelengths using a proposed setting of a variance based sensitivity analysis. The effect of the vehicle's speed is also considered and examined. The outcome of such a study would help the state authorities and regulators in assessing the level of maintenance of roadways and deciding on the speed limits for heavy vehicles crossing short to medium highway bridges.

The first section of the paper deals with the general description and implementation of the sensitivity analysis followed by presenting the main solution algorithm of the bridge-vehicle interaction. A numerical example is used first to examine the dynamic responses and later to illustrate the application of the sensitivity analysis to identify the contribution of the roadway wavelengths to the response's variance.

## II. SENSITIVITY ANALYSIS

Sensitivity analysis is the study of how uncertainties or variances in the output of a model is apportioned to uncertainties or variances of the inputs. Variance based methods have been chosen due to their independence from the investigated model, and the influence of groups or sets of input parameters may be examined. Moreover, such an analysis provides the importance ranking of the input parameters as well as quantifying their contribution to the output variance [12].

The main idea of variance-based methods is to estimate the amount of variance that would disappear if the true value of the input parameter  $X_i$  is known. This can be described

by the conditional variance of  $Y$  fixing  $X_i$  at its true value  $V(Y|X_i)$ , and is obtained by varying over all parameters, except  $X_i$ . Since the true value of  $X_i$  in complex engineering problems is unknown, the average of the conditional variance for all possible values of  $X_i$  is used, i.e.  $E(V(Y|X_i))$ . Having the unconditional variance of the output  $V(Y)$  and the expectation of the conditional variance  $E(V(Y|X_i))$ , the following relation holds, which is known as the law of total variance:

$$V(Y) = V(E(Y|X_i)) + E(V(Y|X_i)), \quad (1)$$

From equation (1) the variance of the conditional expectation  $V(E(Y|X_i))$  is determined. This term is often referred to as the main effect, as it estimates the main effect contribution of the  $X_i$  to the variance of the output. Normalizing the main effect by the unconditional variance  $V(Y)$  results in:

$$S_i = \frac{V(E(Y|X_i))}{V(Y)} \quad (2)$$

The ratio  $S_i$  is known as a first order sensitivity index [13], which is also known as the importance measure [14]. The value of  $S_i$  is less than 1, further the sum of all first order indices corresponding to multiple input parameters is an indicator of the additivity of the model. The model is considered additive when the sum equals to one (no interactions between the input parameters), and non-additive when the sum is less than one. Hence, the difference  $1 - \sum S_i$  is an indicator for the presence of interactions between the input parameters. Total effect index  $S_{Ti}$  is used to present the total contribution of the input parameter  $X_i$  to the output, i.e. first order effects in addition to all higher order effects. A total effect index is defined as

$$S_{Ti} = 1 - \frac{V(E(Y|X_{-i}))}{V(Y)}, \quad (3)$$

where  $V(E(Y|X_{-i}))$  is the variance of the expected value of  $Y$  when conditioning over all except for  $X_i$ . The difference  $S_{Ti} - S_i$  is a measure of how much  $X_i$  interact with other input parameters.

For the engineering problem at hand, the sensitivity analysis is used to give a better understanding of the contribution of roadway wavelengths to the variance of the model response. The power of such an analysis can be attributed to the fact that the model itself is exercised, therefore, measurements or reference models are not needed. Furthermore, its implementation and simulation can be done regardless of the model type. As long as the input-output mapping is available, the analysis can be performed.

A sampling based numerical procedure is employed to compute first order and total effect indices for a model of  $k$  input parameters [15]. This procedure is thought to be benignant and best suited to estimate sensitivity indices that are based only on model evaluations.

### III. MODELING & ANALYSIS

The engineering problem of interest is the vibration of bridges caused by a moving heavy vehicle. A general description of the vehicle and the bridge models as well as the used solution algorithm are explained.

#### A. Vehicle model

The equations of motion for the vehicle can be written in the following general form:

$$\mathbf{M}_v \ddot{\mathbf{U}}_v + \mathbf{C}_v \dot{\mathbf{U}}_v + \mathbf{K}_v \mathbf{U}_v = \mathbf{P}_v, \quad (4)$$

where  $\mathbf{M}_v$  is the mass matrix of the vehicle,  $\mathbf{C}_v$  is the damping matrix of the vehicle,  $\mathbf{K}_v$  is the stiffness matrix of the vehicle,  $\mathbf{P}_v$  is the dynamic force vector of the vehicle, and  $\mathbf{U}_v$  is the generalized coordinate vector describing the dynamics of the vehicle model (degrees of freedom).

Researchers have used different suspended mass models to represent heavy vehicle systems varying from single to multiple degrees of freedom depending on the level of modeling [6], [16], [17]. The chosen vehicle model is an eight-degree-of-freedom model representing a typical configuration of a common heavy truck traveling on road networks [17]. The vehicle consists of a two-axle tractor and a three-axle semi-trailer linked by a hinge. The tractor and the semi-trailer are assumed to be rigid components and are characterized by their masses and moments of inertia. The vehicle model is excited at five points, which are the contact points between the tires and the roadway. It is also assumed that the three axles of the semi-trailer share the rear static load equally since load-sharing mechanisms are common in multi-axle heavy vehicle suspensions [18]. The generalized coordinates used to describe the vehicle dynamics are tractor vertical displacement  $y_T$ , tractor pitch angle  $\theta_T$ , semi-trailer vertical displacement  $y_S$ , semi-trailer pitch angle  $\theta_S$ , tractor front unsprung mass vertical displacement  $y_1$ , tractor rear unsprung mass vertical displacement  $y_2$ , and semi-trailer unsprung masses vertical displacements  $y_3$ ,  $y_4$ , and  $y_5$ , as shown below:

$$\mathbf{U}_v = \{ y_T \quad \theta_S \quad \theta_S \quad y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \}^T \quad (5)$$

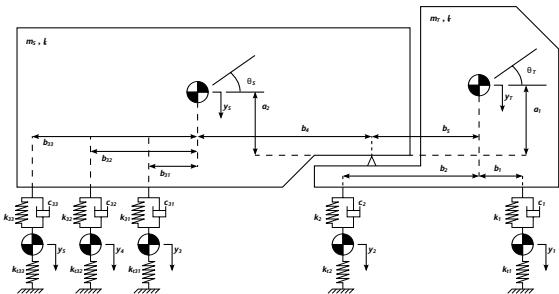


Fig. 1. Schematic for the five-axle vehicle model

Due to the articulation of the truck and the semi-trailer, a kinematic constraint can be written for the semi-trailer vertical displacement  $y_S$ , [16]:

$$y_S = y_T + b_5 \theta_T + b_4 \theta_S. \quad (6)$$

After fulfilling this constraint equation, the vehicle model has eight independent degrees of freedom. The equations of motion are based of the derivation provided by [16] for the ride behavior of a three-axle tractor and semi-trailer truck. Such a formulation has been also used in other studies [17].

The mass, damping and stiffness matrices can be found in the aforementioned studies.

The interaction force  $F_i^{int}$  can be expressed as:

$$F_i^{int} = k_{ti} [y_i(t) - y_b(x_i, t) - r_i(t)], \quad i = 1, 2, 3, 4, 5 \quad (7)$$

where  $y_b(x_i, t)$  and  $r_i(t)$  are the displacements of the bridge and road unevenness respectively, at the contact point corresponding to the  $i^{th}$  axle at instant  $t$ .

The vibration of such a heavy vehicle has two distinctive frequency ranges; the first range is 1.5 Hz to 4 Hz, representing the sprung mass bounce involving some pitching, and the second range is 8 Hz to 15 Hz, representing the unsprung mass bounce involving suspension pitch modes [18].

### B. Bridge model

The equations of motion of the bridge considering time varying forces can be expressed in the following matrix notation:

$$\mathbf{M}_b \ddot{\mathbf{U}}_b + \mathbf{C}_b \dot{\mathbf{U}}_b + \mathbf{K}_b \mathbf{U}_b = \mathbf{P}_b, \quad (8)$$

with  $\mathbf{M}_b$ ,  $\mathbf{C}_b$ ,  $\mathbf{K}_b$  are the mass, damping and stiffness matrices of the bridge,  $\ddot{\mathbf{U}}_b$ ,  $\dot{\mathbf{U}}_b$ ,  $\mathbf{U}_b$  are the accelerations, velocities and displacements of the bridge, and  $\mathbf{P}_b$  is the vector of forces acting on each bridge node at time  $t$ , which has two components, as shown below:

$$\mathbf{P}_b = \mathbf{F}^g + \mathbf{F}^{int}, \quad (9)$$

where  $\mathbf{F}^g$  is the force acting on the bridge due to the weight of the vehicle, which is independent of the interaction, and  $\mathbf{F}^{int}$  is the time-variant force acting on the bridge, which depends on the interaction between the bridge and the vehicle. The damping of the bridge is assumed to be viscous, which means that it is proportional to the nodal velocities.

### C. Bridge-vehicle interaction

During the passage of a vehicle across a bridge, the dynamic tire forces of a vehicle can lead to additional dynamic effects on the bridge. These effects are mainly due to the excitation of the vehicle by the dynamic deflection of the bridge and by the initial road unevenness. Models that consider bridge-vehicle interaction are often derived to consider these sources of excitation. The equations of motion for the vehicle and the bridge are written as (4) and (8), respectively. Assuming perfect contact, the solution of these equations is governed by satisfying the compatibility equation and imposing the equality of displacement at the contact point, as expressed below:

$$y_w(x_i, t) = y_b(x_i, t) + r_i(t), \quad (10)$$

where  $y_w(x_i, t)$  is the displacement of the tire of the vehicle at  $i^{th}$  contact point at instant  $t$ . In addition, the force equilibrium conditions at the contact point  $i$  must be satisfied, which can be shown as:

$$P_b^i = F_i^g + F_i^{int}, \quad (11)$$

where  $F_i^g$  is the static weight of the  $i^{th}$  axle and  $F_i^{int}$  is the interaction force at the  $i^{th}$  axle. The  $i^{th}$  contact point

usually does not coincide with the a DOF of the bridge model. Therefore, the forces  $F_i^g$  and  $F_i^{int}$  are converted to equivalent nodal forces associated with the bridge's DOF.

The analysis starts by assuming the initial displacements of the bridge for the time step  $t$ . The displacement of the vehicle's tire at the contact point is computed following the compatibility condition in (10). The vehicle equations (4) are solved using a numerical integration method for its displacements ( $\mathbf{U}_v$ ). The determined displacements of the vehicle are replaced into (7) to calculate the interaction forces ( $F_i^{int}$ ). Satisfying the equilibrium conditions at the contact point as in (11) and converting the forces to the associated DOFs of the bridge results in the equivalent bridge forces ( $\mathbf{P}_b$ ). These forces ( $\mathbf{P}_b$ ) are then used to solve the bridge equations (8) using a numerical integration method to compute the improved displacements of the bridge ( $\mathbf{U}_b$ ). This procedure is repeated till a tolerance assigned to the difference between the outputs is met for the analyzed time step. Then the same iterative analysis is repeated to  $t + \Delta t$  till the desired period of time is reached.

An alternative, which is described as non-iterative algorithm is proposed by [5] for the above solution procedure, which is used in the analysis. It is non-iterative conditioning over a sufficiently small time step. With such a time step, the force acting on the vehicle at the current time step is estimated from the previous step. According to [5], the choice of the time step should be *small enough* to capture the highest desired frequency of the bridge, the vehicle passage, and the excitation from road unevenness. Moreover a factor of  $\frac{1}{10}$  is introduced into the  $\Delta t$  selected to secure reasonable integration accuracy and is expressed as:

$$\Delta t \leq \frac{1}{10} \times \min \left\{ T_f = \frac{1}{f_{\max}}, \quad T_s = \frac{L}{v}, \quad T_r = v \kappa_r^{\max} \right\}, \quad (12)$$

where  $f_{\max}$  is the upper frequency of interest for the bridge, and  $\kappa_r^{\max}$  is the largest wavenumber of the road unevenness corresponding to the minimum wavelength. The numerical integration procedure used to solve the system of differential equations is the Newmark- $\beta$  method.

In general, many DOFs are involved in the FE model of the bridge system, but only the first modes of vibration make the significant contribution to the dynamic response. Therefore, the modal superposition method has been used to solve the equations of motion of the bridge, which reduces the computational effort considerably, which is regarded as advantageous [19].

### D. Road unevenness

The road unevenness is often obtained from measuring existing roadways which is a laborious procedure. Therefore, [20] suggested simplified means to describe the road surfaces. The authors attested the treatment of road unevenness as a realization of a stationary Gaussian homogeneous random process described by its power spectral density function in space domain  $S_{f_0, f_0}(\kappa)$  with  $\kappa$  as the wavenumber. However, the dynamic analysis is performed in time domain, and a description of the road unevenness in time domain is needed. Therefore,

the temporal power spectral density function  $S_{f_0 f_0}(\omega)$  is to be computed. Assuming a constant speed for the vehicle  $v$ ,  $S_{f_0 f_0}(\omega)$  and  $S_{f_0 f_0}(\kappa)$  can be related using the following:

$$S_{f_0 f_0}(\omega = v\kappa) = \frac{1}{v} S_{f_0 f_0}(\kappa) \quad (13)$$

When performing the analysis in time domain, one can deduce that the excitation of the vehicle due to road unevenness can be described as non-stationary when the vehicle speed is time dependent [21]. Even when the speed is constant and the vehicle excitation is stationary, the dynamic responses of the bridge are non-stationary due to the movement of the vehicle [8]. This observation is of importance in deriving the stochastic characteristics when the dynamic problem is solved in frequency domain.

In most engineering applications, the one sided spectral density function  $S_{FF}(\kappa)$  is derived from measurements for which the following relation holds,

$$S_{FF}(\kappa) = 2S_{ff}(\kappa). \quad (14)$$

There are two main models for generating realizations of random processes going back to the work of Rice and Shinozuka [22], [23]. One consists of a series of sines and cosines with random amplitudes and the other consist of a series of cosine terms with random phase angles. The latter is adopted for the realizations of road profiles and is modified by assuming random amplitudes. This modified model can be found in [7], [29], and described in (15).

$$\begin{aligned} f(t) &= \sum_{k=0}^{N_d-1} [C_k \cos(\omega_k t + \Phi_k)] , \\ \omega_k &= \omega_l + k\Delta\omega , \\ k &= 0, 1, 2, \dots, N_d - 1 , \end{aligned} \quad (15)$$

where  $\Phi_k$ s are independent random phase angles uniformly distributed in the range  $[0, 2\pi]$  and  $C_k$ s are random variables following Rayleigh distribution with a mean value of  $\beta_k \sqrt{\frac{\pi}{2}}$  and a variance of  $\beta_k^2(2 - \frac{\pi}{2})$  taking  $\beta_k$  as  $\sqrt{S_{FF}(\omega_k)\Delta\omega}$ .  $S_{FF}$  is the one sided power spectral density function (PSD) used to describe the road unevenness. Further, the realized road surfaces reflect the prescribed probabilistic characteristics of the random process accurately as the number  $N_d$  gets larger.

It is noticed from (15) that the PSD is discretized into temporal frequency bands of a width of  $\Delta\omega$ , and the corresponding discretized frequencies are used in the realization of the stochastic process. However, the entire frequency domain of the PSD cannot be used in the realization for mathematical and physical reasons [23]. For the realizations of road surfaces, cut-off frequencies are needed. The discretizing frequency band is defined as

$$\Delta\omega = (\omega_u - \omega_l)/N_d , \quad (16)$$

with  $\omega_u$  and  $\omega_l$  (rad/s) as the upper and the lower cut-off frequencies. The long wavelength irregularities correspond to low frequency components in the time domain and short wavelength irregularities correspond to high frequency components [24]. The different wavelengths and their corresponding temporal frequencies excite different vibrational modes of

the heavy vehicle; the bouncing mode of the sprung mass is more of a low frequency mode while the axle hop and pitching modes are of higher frequencies [18]. Further, when the wavelengths of the irregularities are too small compared with the dimensions of the contact patch between the tire and the roadway, the tire due to its flexibility absorb these irregularities. This phenomenon is referred to as tire envelopment which reduces the excitations of the axle of the vehicle. Therefore, filtering or smoothing algorithms depending on the dimension of the contact patch are recommended [25]. Often a moving averaging filter is employed for such purposes [26].

#### IV. NUMERICAL EXAMPLE

##### A. Description of subsystems

For the numerical verification, the vehicle model presented by [17] is used. The characteristics of the vehicle are found in [28]. The bridge model is a single span simply supported beam model for the Pirton Lane Highway bridge in Gloucester (United Kingdom) [18]. The bridge has a length  $L = 40\text{m}$ , an estimated mass per unit length of  $m = 12000 \text{ kg/m}$  and a bending stiffness of  $EI = 1.26 \times 10^5 \text{ MNm}^2$ . The bridge's first natural frequency is  $f_1 = 3.20 \text{ Hz}$  with a modal damping ratio  $\zeta_1 = 0.02$ .

TABLE I  
EIGENFREQUENCIES OF THE SINGLE SPAN BRIDGE

Bridge mode	Natural freq. [rad/s]
1 <sup>st</sup>	20
2 <sup>nd</sup>	80
3 <sup>rd</sup>	180

The eigenmodes with their corresponding eigenfrequencies are computed and presented in Table II for the vehicle model. It is noticed that the heavy vehicle has two ranges of vibrational frequencies, the first range is (1.5-5 Hz) representing the sprung mass bounce involving pitching and axle hop movements, the second range is (8-12 Hz) representing the axle hop involving slight suspension pitch modes.

TABLE II  
EIGENMODES AND EIGENFREQUENCIES OF THE (8DOF) VEHICLE

Vehicle mode	Freq. (Hz)
Bounce of body mass	1.4
Bounce and pitch	1.6
Pitch, axle hop, and bounce	4.6
Axle hop (tractor-front)	8.9
Axle hop (tractor-rear)	10.4
Axle hop (tridem-middle)	12.0
Axle hop (tridem-front)	12.0
Axle hop (tridem-rear)	12.1

Each realization of road profiles follows (15) where  $\omega_l = 1.74 \text{ rad/s}$  and  $\omega_u = 75.54 \text{ rad/s}$  with  $\Delta\omega = 0.104 \text{ rad/s}$ . Furthermore, the road profile is passed through a moving averaging filter.

The dynamic estimates used for the assessment are the dynamic incremental factor for the displacements ( $DIF_u$ ) and strains ( $DIF_e$ ) and the normalized accelerations. These estimates are determined for the mid-span.

### B. Effects of random road processes

The focus of the dynamic problem studied is the interaction between the bridge and vehicle when road unevenness is considered. As the vehicle travels over the bridge, its dynamic tire forces introduce dynamic effects on the bridge. These effects are mainly due to the excitation of the vehicle caused by the dynamic deflection of the bridge (*Source I*) and the initial road unevenness (*Source II*). The combination of these two excitations describes the dynamic effect of the coupling of the vehicle and bridge on the response of interest. Fig. 2 depicts the mid-span displacements of the single-span system, i.e., the total response and its constituents due to a vehicle model traveling at a speed of 25 m/s (90 km/h). The total response is the moving weight solution combined with the interaction of the two sources of excitation. The interactions between the excitations caused by the dynamic deflection of the bridge and road unevenness vary, thereby having different effects on the moving weight solution. In addition, the amplitudes of the excitations due to road unevenness have a great influence on the additional dynamic effects caused by bridge-vehicle interaction as these excitations may render the ones due to the dynamic deflection of the bridge to secondary. Fig. 2 depicts two examples for the one axle of the vehicle model excited by two different road profiles while traveling over the bridge. Fig. 2(a) shows the excitations to be significant enough to amplify the dynamic response. This case is attributed as “greatly amplified”. Whereas, Fig. 2(b) shows the excitations being out-of-phase with approximately equal amplitudes to cancel each other, thereby having no significant effect on the moving weight solution. This case is attributed as “slightly amplified”. This observation depends on the interaction between the excitations due to the dynamic deflection of the bridge and road unevenness as well as the internal interaction between the excitations caused by the frequencies constituting the road unevenness.

The interaction forces corresponding to the displacements in Fig. 2 are retrieved and plotted for both cases in Fig. 3. It can be seen that the maximum of the interaction forces for the greatly amplified case is larger by a factor of 1.95 when compared to the slightly amplified case as shown in Fig. 3(a). In studying the amplitude spectra shown in Fig. 3(b), it is observed that the amplitudes around the bouncing frequency of the vehicle and the bridge's first natural frequency (3.2 Hz) are higher for the greatly amplified case.

The realization of road profiles is described as a summation of cosine waves with random phase angles. Hence, the main difference between a realization and another is the randomly selected phase angles, which are introduced to ensure the randomness of the generated road profiles. Therefore, it is logical to assume that these phase angles are the reason behind the variation of the effects caused by road unevenness on the dynamic response. In order to examine this, the interaction

forces caused by a single harmonic excitation related to one of the frequencies describing the road unevenness are determined and shown in Fig. 4. The frequencies are selected around the bouncing frequency of the vehicle, and the same set of data employed in the realizations of the road profiles for the analyses in Fig. 2 are used. It can be seen from Fig. 4, that the interaction forces caused by single harmonic excitations vary from being in-phase and out-phase. Therefore, the forces in Fig. 4 may amplify or cancel each other depending on the phase angles assigned to the exciting frequencies of the road profile. Also in Fig. 4, the sum of the interaction forces due to the single harmonic excitations is computed to show the amplification (Fig. 4(b)) or the cancellation (Fig. 4(b)) of the output; the sum holds as linear systems are assumed. The same analysis can be performed for all frequencies of the road profile and this amplification or cancellation of the exciting forces would explain the difference in the total contact forces observed in Fig. 3 and in the responses in Fig. 2. The above are only examples of what can be expected when performing the dynamic analysis considering road unevenness, which may explain the output of the dynamic analysis and its scatter when a probabilistic analysis is run.

### C. Effect of vehicle's speed

The dynamic responses determined considering bridge-vehicle interaction depend on the frequencies of the bridge, the vehicle and the driving speed used in the analysis. The influence of these frequencies and their interaction on the dynamic response is evident. A number of studies have focused on the relation between the dynamic response and speed. In the study done by [27], it was found that a unique function for the maximum dynamic response and the speed exists. This was calculated analytically for a simply supported beam using a moving weight model, in which the first beam mode was considered. Furthermore, [28] studied critical speeds in relation to the frequency of the bridge system; the examination was also done for a moving weight model on single-span bridges.

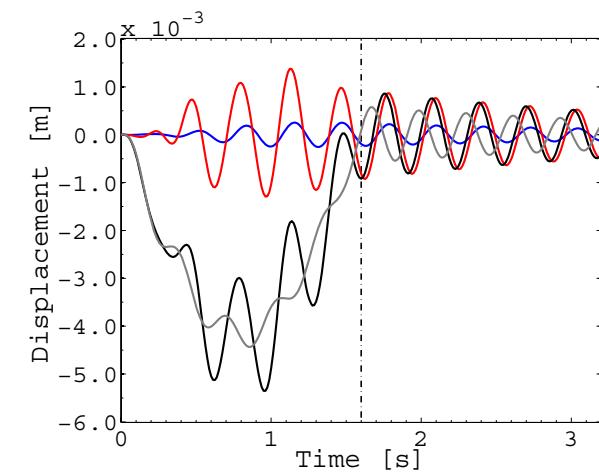
The relationship between the speed and first natural frequency of the bridge is examined here when the interaction between the bridge and the vehicle is considered. Fig. 5 depicts the dynamic response estimates due to the eight-degree-of-freedom vehicle model in relation to the speed circular frequency  $\omega_s$  and the bridge first natural frequency  $\omega_b$ . The speed circular frequency  $\omega_s$  is defined as:

$$\omega_s = \frac{\pi v}{L}, \quad (17)$$

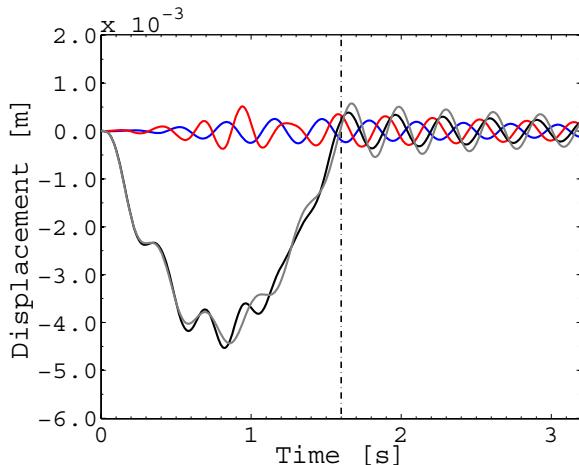
where  $v$  is the speed of the vehicle [m/s] and  $L$  is the span of the bridge [m].

The pattern for the displacements and the strains is clear and similar, while the accelerations show a slightly different trend as they tend to positively increase with the speed circular frequency with no distinguished pattern, Fig. 5.

Diagonal lines can be drawn to envelope the local peaks of the dynamic response relations in Fig. 5. The slopes of these lines represent the critical ratios which relate the speed and



(a) greatly amplified



(b) slightly amplified

Fig. 2. Decomposition of the mid-span displacement of the single-span system: (—) Source I, (—) Source II, (—) Total coupled response, (—) Moving weight

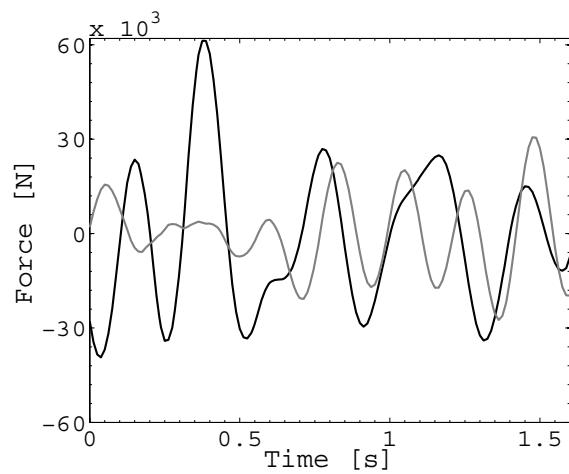
the bridge frequencies that envelope the maximum dynamic response estimate,

$$FR_c = \frac{\omega_s}{\omega_b}. \quad (18)$$

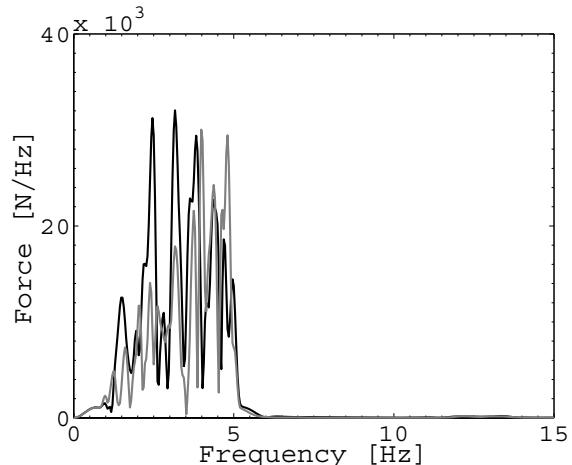
Some of these critical frequency ratios  $FR_c$  are computed and presented in Table III. The ratios apply to short to medium bridges that have a range of first bending natural frequencies between 10 rad/s and 40 rad/s (1.6-6.4 Hz) and to vehicle's speeds that range from 40 km/h to 130 km/h (10-36 m/s). The critical ratios for the accelerations are not computed since they positively increase with the speed circular frequency and there is no clear enveloping slopes for the response.

#### D. Identification of critical wavelengths

Sensitivity analysis is used to identify the temporal frequencies of road unevenness, which influence a dynamic output the most. The randomness of the realized road profiles has been considered in this investigation, and the dynamic response is determined for different speeds. Hence, the speed and



(a) Interaction forces



(b) Amplitude spectra of interaction forces

Fig. 3. Contact forces between 2DOF vehicle model and the bridge considering road unevenness: (—) greatly amplified, (—) slightly amplified

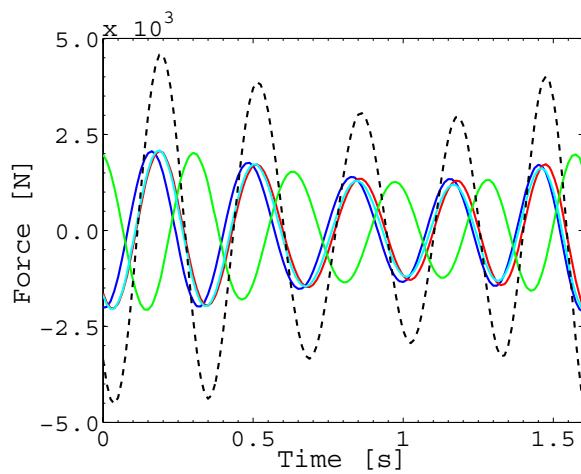
TABLE III  
 CRITICAL FREQUENCY RATIOS ( $FR_c$ )

$DIF_u$	$DIF_e$
0.090	0.092
0.144	0.146

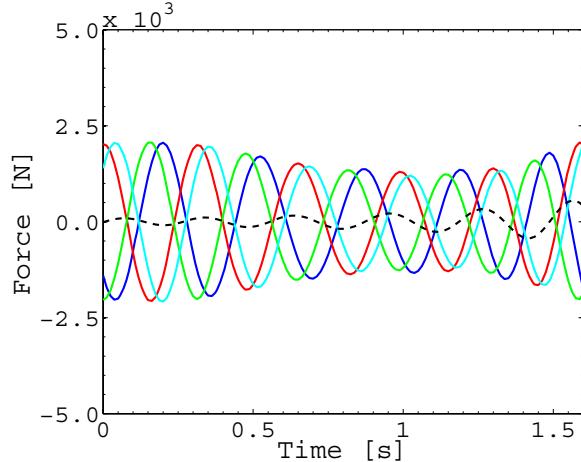
corresponding temporal frequencies of road unevenness are used for the sensitivity analysis.

The sensitivity analysis is run directly on the model output. The general scheme for running the sensitivity analysis is as follows:

- 1) One set of random phase angles is generated  $\Phi (1 \times N_d)$  with  $N_d$  as the number of discretized frequencies.
- 2) Random samples of the amplitudes are generated  $C (N_s \times N_d)$  with  $N_s$  as the number of samples.
- 3) The road profiles are generated according to (15) assuming the same set of phase angles for every realization.
- 4) The dynamic analysis is performed considering the sam-



(a) greatly amplified: In-Phase excitations



(b) slightly amplified: Out-of-Phase excitations

Fig. 4. Contact forces due to single excitations of road unevenness temporal frequencies: (—)  $\omega_1 = 19.87 \text{ rad/s}$ , (—)  $\omega_2 = 19.94 \text{ rad/s}$ , (—)  $\omega_3 = 20.03 \text{ rad/s}$ , (—)  $\omega_4 = 20.11 \text{ rad/s}$ , (---) Sum

ples of road profiles generated and the dynamic output is obtained.

- 5) The sensitivity analysis is applied and the sensitivity indices for the speed and the ranges of road frequencies studied are estimated.
- 6) All steps (1-6) are repeated in order to consider different random sets of phase angles and the average value of the sensitivity indices are calculated.

The proposed sensitivity analysis is preformed in such a way to overcome the challenge imposed by considering the random phase angles in the realization of road profiles.

The numerical example is illustrated for the dynamic response due to the eight-degree-of-freedom vehicle model traveling at different speeds considering road unevenness. The sensitivity indices  $S_i$  and  $S_{Ti}$  are given in Table IV. These indices provide quantitative measures for the influence of frequency ranges of road profiles on the variance of the dynamic response. The ranges of the frequencies of road profiles are chosen in the ranges of the eigenfrequencies of

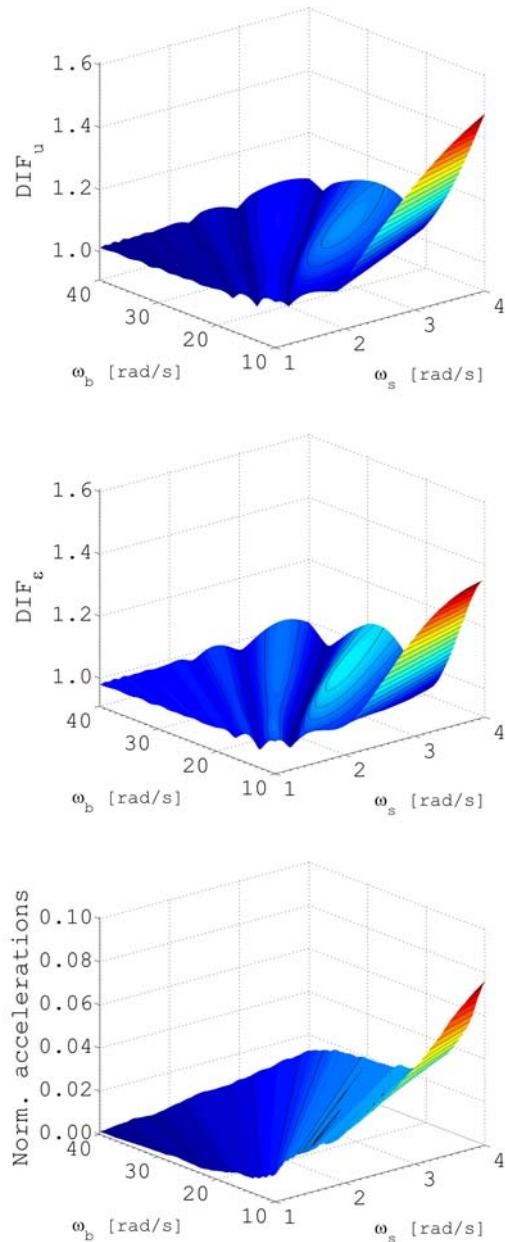


Fig. 5. Dynamic responses versus the bridge first natural frequency and the speed circular frequency for the 8DOF vehicle model

the eight-degree-of-freedom vehicle model. The first range (0.2 Hz to 4.6 Hz) corresponds to the bouncing of the tractor and the semi-trailer masses with some pitching modes; the second range (4.6 Hz to 10.5 Hz) corresponds to the axle hop of the tractor axles; and the third range (10.5 Hz to 12.5 Hz) corresponds to the axle hop of the semi-trailer axles. It can be observed from the indices in Table IV as expected that the displacements and strains are mainly affected by the temporal frequencies of road profiles, which coincide with the bouncing mode of the tractor and the semi-trailer, whereas the accelerations are affected by the ones that coincide with the axle hop modes, especially the ones

of the semi-trailer axles. The corresponding wavelengths of the identified frequencies of road unevenness can be calculated for the different speeds.

TABLE IV  
IDENTIFICATION OF THE FREQUENCIES OF ROAD UNEVENNESS WITH THE GREATEST IMPACT ON DYNAMIC RESPONSE

	$S_i$	$S_{Ti}$
<b>Displacements</b>		
Road temporal frequency range		
0.2 to 4.6 [Hz]	0.19	<b>0.28</b>
4.6 to 10.5 [Hz]	0.00	0.00
10.5 to 12.5 [Hz]	0.00	0.00
Speed	0.71	<b>0.81</b>
<b>Strains</b>		
Road temporal frequency range		
0.2 to 4.6 [Hz]	0.11	<b>0.18</b>
4.6 to 10.5 [Hz]	0.00	0.00
10.5 to 12.5 [Hz]	0.00	0.00
Speed	0.81	<b>0.89</b>
<b>Normalized accelerations</b>		
Road temporal frequency range		
0.2 to 4.6 [Hz]	0.08	<b>0.11</b>
4.6 to 10.5 [Hz]	0.02	0.04
10.5 to 12.5 [Hz]	0.04	<b>0.08</b>
Speed	0.78	<b>0.84</b>

## V. CONCLUSION

The study examines the effects of the interaction of different wavelengths of roadways on the dynamic response of bridges, and it goes further to employ a variance based sensitivity analysis approach to identify the contribution of these wavelengths to the variance of the response when considering bridge-vehicle interaction. It is found from the estimated sensitivity indices that the displacements and strains of the bridge are mainly affected by the temporal frequencies of road profiles that matches the bouncing mode of the tractor and the semi-trailer, whereas the accelerations are affected by the ones that coincide with the bouncing modes and the axle hop modes, especially the ones of the semi-trailer axles. The wavelengths corresponding to these frequencies can be determined for different speeds. The outcome of such a study is of help to the state authorities and regulators in assessing the level of maintenance for the roadways and deciding on appropriate speed limits for heavy vehicles crossing short to medium highway bridges.

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