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Light cluster formation in low density nuclear matter and the stability of hot nuclei

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Abstract

We calculate the limiting temperature for the stability of hot nuclei assuming the hot nucleus to be a liquid drop in thermal, chemical and mechanical equilibrium with the surrounding vapour. Following recent theoretical and experimental results, the vapour is assumed to consist not only of nucleons but also clusters of nucleons up to $A = 4$ that are in chemical equilibrium with the nucleons. It is found that the presence of the clusters in the vapour reduces the limiting temperature by several MeV and makes it almost independent of the interaction used to determine the equation of state of the nuclear matter inside the drop.

1. Introduction

Theoretical studies [1–5] have predicted the abundance of light clusters in nuclear matter at very low densities and moderate temperatures. There has also been definite experimental evidence [6, 7] for a large degree of light cluster formation at these densities and temperatures. The large degree of clustering that occurs in nuclear matter at very low densities is expected to have a noticeable effect on the equation of state of nuclear matter in the vapour state even when only alpha clusters are included [8]. The clustering is limited to low densities because when the cluster is embedded in dense nuclear matter its binding energy decreases and it finally dissolves in the medium above a certain critical density (the Mott density) [9]. Clustering leads to a reduction of about 2.4 MeV in the value of the critical temperature of infinite uncharged nuclear matter [1].

In the present work we investigate the effect of the occurrence of light clusters and the corresponding modification in the equation of state of the vapour on the stability of hot nuclei. This investigation is carried out within a model in which the hot nucleus is treated as a spherical liquid drop with uniform density and temperature, a sharp edge and a surface tension. The liquid drop is in thermal, mechanical and chemical equilibrium with the surrounding vapour. In earlier studies [10–15] the vapour was considered as consisting of only single nucleons and it was found that the Coulomb force leads to the instability of hot nuclei above a limiting temperature T_L which is much lower than the critical temperature [1, 16, 17] of infinite (uncharged) symmetric nuclear matter. Here we examine how the value of T_L is affected by the inclusion of light clusters in the vapour state.

2. The equation of state of the vapour phase in the presence of clusters

The vapour's equation of state is determined using the nuclear statistical equilibrium (NSE) model [18–20] in which the only interaction between the nucleons is through the formation of clusters. The starting point in determining this equation of state is the evaluation of the pressure $P_{\rm id}$ and chemical potential $\mu_{\rm id}$ of an ideal quantum gas of nucleons at a low density ρ and temperature T. These are given by [11]:

$$P_{\rm id}(T,\rho) = T\rho\left[1+\sum_{n=1}^{\infty} b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n\right] \tag{1}$$

$$\mu_{\rm id}(T,\rho) = T\left[\ln\left[\frac{\lambda_T^3\rho}{g}\right]+\sum_{n=1}^{\infty} b_n\frac{n+1}{n}\left[\frac{\lambda_T^3\rho}{g}\right]^n\right] \tag{2}$$

where g is the nucleon's spin–isospin degeneracy factor and $\lambda_T = \left(\frac{2\pi^2}{mT}\right)^{1/2}$ is the thermal wavelength of the nucleons. The virial coefficients in the summations in equations (1) and (2) reflect the higher order degeneracy corrections that significantly modify the pressure and chemical potential of an ideal Fermi gas as compared to those for an ideal classical gas (given by the first term before the summation). The b_n coefficients are determined by the method outlined in [17] and the values of the first six coefficients are given by:

$$b_1 = 0.1767\,766\,952\,966$$

$$b_2 = -0.0033\,000\,598\,199$$

$$b_3 = 1.112\,893\,285\times10^{-4}$$

$$b_4 = -3.5405\,041\times10^{-6}$$

$$b_5 = 8.38\,635\times10^{-8}$$

$$b_6 = -3.662\times10^{-10}.$$

The rapidly decreasing value of these coefficients and their alternating signs ensure that summing up to $n = 6$ yields fairly accurate results. Even at a temperature as low as 3 MeV the contribution of the $n = 6$ term modifies the summation by about 5%, and is almost negligible at $T = 4$ MeV.

The nucleons in the vapour are in chemical equilibrium with the light clusters, where we include only clusters up to $A = 4$ (deuterons, tritons, helions and alphas) which is in line with similar studies [1, 3, 21]. Some calculations [2, 8] include only alpha clusters. The neglect of the effect of clusters with $A > 4$ is a limitation of the present work. In particular, as the density increases the heavier clusters become more important. This can be the subject of future investigation.

Chemical equilibrium in the clusterized vapour is guaranteed by having the chemical potential μ_C of cluster type C (containing Z protons and N neutrons) satisfy the relation:

$$\mu_C = Z\mu_P + N\mu_n = A\mu_{\rm id} \tag{3}$$

where we treat the protons (p) and neutrons (n) equally and the Coulomb force is switched off. The partial pressure and density of each type of cluster are then evaluated and added to the pressure and density of the nucleons to form the vapour's total pressure and total density respectively.

For the fermionic clusters (tritons and hellions) the partial pressure is given by an equation for the ideal Fermi gas similar to that used for the nucleons with the appropriate masses and degeneracies. For the bosonic clusters (deuterons and alphas) the partial pressure is given by an expression for the ideal Bose gas which is similar to equation (1) for an ideal Fermi gas but with b_{n} replaced by a_{n} where

$$a_n = (-1)^n b_n. \tag{4}$$

The contribution to the total nuclear density of each type of cluster is determined by the equation:

$$\rho_C = \frac{g}{(2\pi)^3} f\, \mathrm{d}^3 q n_C \tag{5a}$$

where

$$n_C = \left\{\exp\left[\beta\left(\varepsilon_C^0 - \mu_C - B_C\right)\right] \pm 1\right\}^{-1} \tag{5b}$$

with the (+) sign used for the fermions (helions and tritons) and the (−) sign used for the bosons (deuterons and alphas). Here ε_C^0 and B_C are the kinetic and binding energies of cluster C, where $C = d,\ t,\ h,\ \alpha$ refers respectively to the deuteron, triton, helion and alpha clusters. The total nuclear density in the vapour phase ρ_v is the sum of the density of the free nucleons ρ_{free} and the densities of the nucleons bound in the clusters:

$$\rho_v = \rho_{\mathrm{free}} + 2\rho_d + 3\rho_t + 3\rho_h + 4\rho_\alpha \tag{6}$$

where ρ_d, ρ_t, $\rho_h$andρ_α are the densities of the clusters.

The NSE model is only valid at very low densities because it ignores medium effects on the binding energies of light clusters and cannot describe their dissolution at high densities. To remedy this deficiency of the model we use density-dependent cluster binding energies. The reduction in the binding energy of the clusters with the increasing density of the surrounding medium is mainly due to Pauli blocking, reflecting the fact that the nucleons inside the clusters are indistinguishable from the free nucleons in the vapour and that the total wavefunction must be antisymmetric. Following [3] we continue to treat the clusters as non-interacting but with the medium-modified binding energies. Typel *et al* [3] evaluated the change in the binding energy at zero cluster momentum and found that it decreases almost linearly with density and vanishes at the Mott density ρ_M (0)whose value is temperature-dependent. At nonzero cluster momentum P the cluster can survive up to a higher Mott density ρ_M (P). More recently a fit for the momentum dependent shifts of the binding energies of light clusters has been published [21]. To simulate these effects we have used a density-dependent cluster binding energy of the form:

$$B_C = B_{C0} \exp(-\rho/\rho_M(0)). \tag{7}$$

The ρ_M (0) were obtained by a linear fit of the results of Typel *et al* [3]. A summary of the values of ρ_M (0) for the various clusters and at some typical temperatures is given in table 1.

Here it must be noted that the binding energy depends on the total density whose value is determined by adding the contributions of the various clusters, given by equations (5*a*) and (5*b*), to the density of the free nucleons, as in equation (6). Since equations (5*a*) and (5*b*) need the binding energies as input, the calculation has to be carried out iteratively until self-consistency is achieved. The abundance of the light clusters obtained in this way at a temperature of 4 MeV is shown in figure 1. At this temperature the deuterons are dominant, while the alpha clusters are dominant at temperatures below 2 MeV.

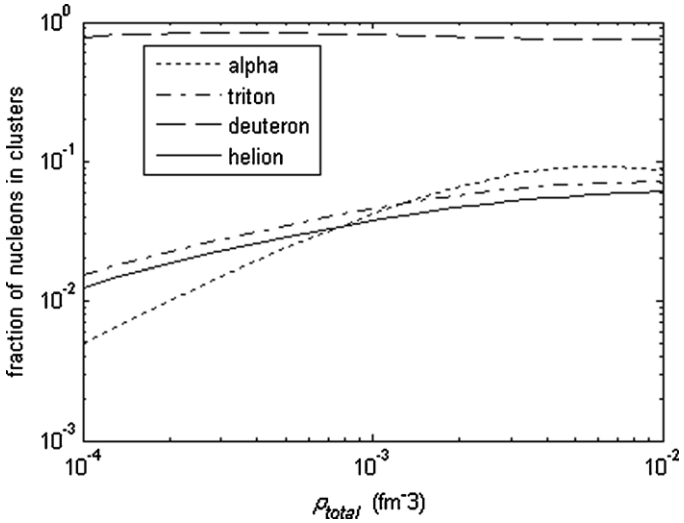

Figure 1. The fraction of nucleons existing in clusters at a temperature of 4 MeV.

Table 1. The Mott densities ρ_M (0) for various clusters at typical temperatures.

	$T = 2$ MeV	$T = 4$ MeV	$T = 5$ MeV	$T = 6$ MeV
Alpha	0.0059 fm^{-3}	0.0073 fm^{-3}	0.0080 fm^{-3}	0.0088 fm^{-3}
Deuteron	0.00148fm^{-3}	0.00216 fm^{-3}	0.0025 fm^{-3}	0.0029 fm^{-3}
Helion	0.0023 fm^{-3}	0.0031 fm^{-3}	0.0035 fm^{-3}	0.0040 fm^{-3}
Triton	0.0028 fm^{-3}	0.0036 fm^{-3}	0.0036 fm^{-3}	0.0046 fm^{-3}

3. The nuclear equation of state of the liquid phase (without clusters)

Assuming nuclear matter in the liquid phase to consist of only nucleons interacting through a zero-range Skyrme interaction [22] of the form:

$$\upsilon_{12} = -\frac{8}{3}a_0(1 + x_0P_\sigma)\delta(\vec{r}_1 - \vec{r}_2) + \frac{8}{3}a_3(1 + x_3P_\sigma)\rho^\sigma\left[\frac{\vec{r}_1 + \vec{r}_2}{2}\right]\delta(\vec{r}_1 - \vec{r}_2). \tag{8}$$

Its pressure and chemical potential are given by [11]:

$$\tilde{P}(T, \rho_l) = -a_0\rho_l^2 + a_3(1 + \sigma)\rho_l^{(2+\sigma)} + P_{\mathrm{id}}(T, \rho_l) \tag{9}$$

$$\tilde{\mu}(T, \rho_l) = -2a_0\rho_l + a_3(2 + \sigma)\rho_l^{(1+\sigma)} + \mu_{\mathrm{id}}(T, \rho_l) \tag{10}$$

where the ideal pressure and chemical potential of a gas of nucleons are given by equations (1) and (2). These expressions will be used only for the liquid phase whereas they have been previously used [11, 12] for both the high-density liquid state as well as for the low-density vapour state. Their use for the vapour state is however questionable in light of the considerable evidence mentioned above for the abundance of clusters at low densities. On the other hand their use for the liquid state is acceptable since the clusters disappear at high densities and the liquid phase consists only of nucleons.

Table 2 summarizes the values of the parameters of the two Skyrme forces used in the present calculation as well as in [11]. The parameter x_3 is usually given [22] the value

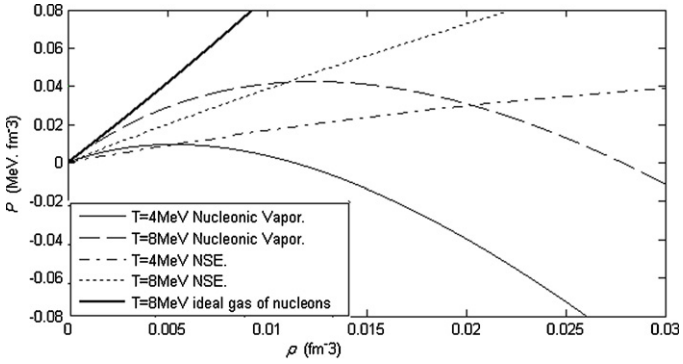

Figure 2. Comparison between the pressures of the vapour state calculated with and without clusters. The NSE model assumes the formation of clusters that are in chemical equilibrium with the nucleons in the vapour, while the nucleonic vapour model assumes there are only nucleons interacting via the Skyrme force.

Table 2. Parameters of the Skyrme forces and the critical temperatures calculated with them.

σ	x_0	$a_0\rho_0$(MeV)	$a_3\rho_0^{1+\sigma}$(MeV)	Critical temperature
0.25	0.75	136	96	17.3 MeV
1	0.47	64	24	22.9 MeV

of 1. One of the forces has a value of $\sigma = 0.25$ and the second one has a value of $\sigma = 1$.These parameters are given in terms of the nuclear matter saturation density $\rho_0 = 0.17$ nucleons fm^{-3}. Also given in the last column of table 2 are the values of the critical temperature of infinite (uncharged) symmetric nuclear matter calculated with the use of these forces. Because of the Coulomb force, hot real nuclei however cannot survive up to the critical temperature and become unstable at the limiting temperature. The values of the limiting temperature corresponding to these two forces will be evaluated in the following section.

Figure 2 compares the pressure of the vapour state in the two approaches (with and without clusters) at two different temperatures and for densities that are much less than nuclear saturation density. It is seen that there is a significant difference between the two approaches. Of particular interest is the behaviour at very low densities (less than 0.01 nucleons fm^{-3}) where the presence of clusters in the NSE model lowers the pressure below that obtained with the assumption of a vapour of interacting nucleons. The latter approaches the pressure due to an ideal Fermi gas of nucleons at extremely low densities. It must be noted that the pressure isotherms calculated in the NSE model are qualitatively similar to those obtained by Samaddar and De [8] in a nucleon–alpha model.

4. The limiting temperature T_L

We treat the hot nucleus as a spherical liquid drop with uniform density and temperature, a sharp edge and a surface tension in thermal, mechanical and chemical equilibrium with the surrounding vapour along the lines followed earlier [10–12]. We will carry out calculations for the same cases treated in [11] namely the hot nuclei of ^{109}Ag and ^{208}Pb and using the same Skyrme interactions and the same approximations. The only difference is that in the present

Table 3. Comparison between the results with and without clusters included in the vapour. T_L is the limiting temperature (in MeV), ρ_l and ρ_v are the equilibrium liquid and vapour densities (in nucleons fm^{-3}) and α_v is the vapour asymmetry, all evaluated at the limiting temperature.

		Without clusters				With clusters				
Nucleus	σ	T_L	ρ_l	ρ_v	α_v	T_L	ρ_l	ρ_v	α_v	ρ_{free}
^{208}Pb	1	7.60	0.1652	0.01370	0.15000	2.8	0.153	0.0376	0.599	0.0003
	0.25	5.49	0.1689	0.00772	0.17000	2.8	0.172	0.0376	0.545	0.0003
^{109}Ag	1	9.22	0.1642	0.01678	0.05220	3.3	0.169	0.0405	0.187	0.00035
	0.25	6.80	0.1682	0.01000	0.05147	3.3	0.170	0.0405	0.190	0.00035

work we include the effect of the existence of light clusters in the vapour state. The results obtained for these nuclei without the inclusion of light clusters [11] are summarized in the first half of table 3. Comparing these earlier results with the results to be obtained in the present work will allow us to elucidate the effect of the presence of clusters in the vapour.

The surface tension of the liquid drop $\gamma(T)$ is given by [23]:

$$\gamma(T) = 1.14\ \text{MeV.fm}^{-2}\left(1+\frac{3}{2}\frac{T}{T_c}\right)\left(1-\frac{T}{T_c}\right)^{3/2} \tag{11}$$

and its contribution to the pressure inside the liquid drop is given by:

$$P_{\text{surf}}(T,\rho_l) = -2\gamma(T)\left[\frac{4\pi\rho_l}{3A}\right]^{1/3}. \tag{12}$$

We also include the Coulomb force contribution to the pressure and the proton chemical potential inside the liquid drop in an approximate manner as in [11, 12]:

$$P_{\text{Coul}}(\rho_l) = \left[\frac{4\pi\rho}{3A}\right]^{1/3}\frac{Z^2e^2}{5A}\rho_l \tag{13}$$

$$\mu_{\text{Coul}}(\rho_l) = \frac{6}{5}Ze^2\left[\frac{4\pi\rho_l}{3A}\right]^{1/3}. \tag{14}$$

Also included in the pressure inside the liquid drop is the contribution that comes from the asymmetry between the protons and the neutrons in the nuclei being studied [11]:

$$P_{\text{sym}}(T,\rho_l,\alpha) = \left[\frac{2}{3}\left(x_0+\frac{1}{2}\right)a_0\rho_l^2 - (1+\sigma)a_3\rho_l^{(2+\sigma)} + T\rho\sum_{n=1}^{\infty}\frac{n(n+1)}{2}b_n\left[\frac{\lambda_T^3\rho_l}{g}\right]^n\right]\alpha^2$$

where α is the neutron–proton asymmetry parameter $\alpha = \frac{N}{A} - \frac{Z}{A}$. The corresponding contribution to the nucleon chemical potential inside the drop is [11]:

$$\mu_{\text{sym}}(T,\rho_l,\alpha) = \left\{-\sigma a_3\rho_l^{1+\sigma} + T\left[-\frac{1}{2}+\sum_{n=1}^{\infty}\frac{(n^2-1)}{2}b_n\left[\frac{\lambda_T^3\rho_l}{g}\right]^n\right]\right\}\alpha^2.$$

Mechanical equilibrium requires that the pressure inside the liquid drop is equal to the pressure of the vapour:

$$\tilde{P}(T,\rho_l) + P_{\text{sym}}(T,\rho_l,\alpha_l) + P_{\text{Coul}}(\rho_l) + P_{\text{surf}}(T,\rho_l) = P_{\text{vapor}}(T,\rho_v) \tag{15}$$

where ρ_l and ρ_v are the densities of the liquid and vapour respectively and α_l is the asymmetry parameter of the liquid drop (the hot nucleus). The vapour pressure includes the sum of the contributions of the clusters and the free nucleons, with each species treated as an ideal quantum gas. The total vapour pressure is determined in the NSE model with the modifications

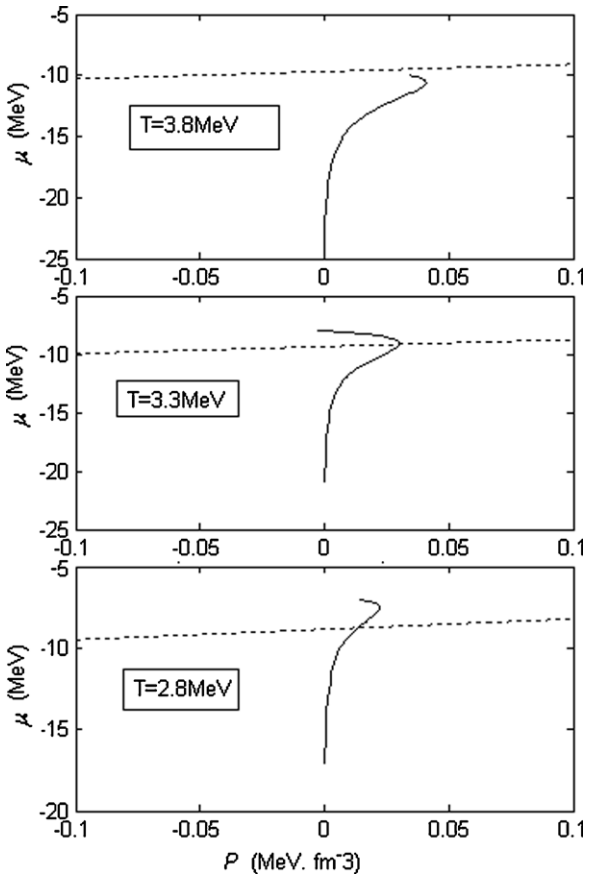

Figure 3. Determining the limiting temperature for a hot ^{109}Ag nucleus in the presence of clusters in the vapour state. The dotted line shows the nucleon chemical potential of the liquid drop given by the left-hand side of equation (16) versus the total pressure inside the liquid drop given by the left-hand side of equation (15). The continuous line shows the same quantities for the surrounding vapour which are given by the right-hand side of equations (16) and (15) respectively.

introduced by the use of density-dependent binding energies as given by equation (7). This is the only significant difference between the present work and [11] which assumed that the vapour consists of only nucleons.

Chemical equilibrium requires that the nucleons' chemical potential inside the drop is equal to the nucleons' chemical potential in the surrounding vapour [11]:

$$\tilde{\mu}(T, \rho_l) + \tfrac{1}{2}\mu_{\mathrm{Coul}}(\rho_l) + \mu_{\mathrm{sym}}(T, \rho_l, \alpha_l) = \mu_{\mathrm{id}}(T, \rho_v). \tag{16}$$

Here it must be emphasized that the clusters are already in chemical equilibrium with the nucleons in the vapour through equation (3). We also use the same approximation of neglecting the Coulomb and asymmetry contributions for the vapour in equations (15) and (16) as in [11].

Using these equations we can determine the limiting temperature which is the temperature above which the equations for mechanical and chemical equilibrium between the drop and the surrounding vapour do not have a solution. The results for a hot ^{109}Ag nucleus are shown in figure 3 where we plot the chemical potential versus the pressure for the liquid drop and the vapour. The system is stable if the vapour and drop curves intersect guaranteeing that

a solution exists. Note that beyond the maximum value of the vapour pressure the system becomes mechanically unstable and so the condition for the stability of the hot nucleus is that the intersection occurs before the vapour pressure reaches its maximum value. The limiting temperature therefore occurs when the drop curve passes through the maximum of the vapour pressure curve. Figure 3 shows that the limiting temperature for ^{109}Ag is 3.3 MeV which is to be compared with a value of 6.8 MeV obtained under the assumption of a purely nucleonic vapour [11].

In the right-hand half of table 3 we show the results obtained in the present work for two hot nuclei (^{208}Pb and ^{109}Ag) embedded in a vapour containing clusters, calculated with the two Skyrme forces given in table 2. In each case we give the limiting temperature and the equilibrium liquid and vapour densities at this temperature. For comparison we also give in the left-hand half of table 3 the values obtained in [11] for a vapour without clusters, that is assuming the vapour to consist of only nucleons interacting via the Skyrme force. It is seen that the effect of including clusters in the vapour lowers the limiting temperature by several MeV. We also note that in the presence of clusters the values of the limiting temperature are almost identical for the two Skyrme forces. This is not surprising because in the NSE model the Skyrme force plays no role in the vapour state as the only interaction is that through the formation of clusters.

Also shown in table 3 are the values of the vapour asymmetry parameter α_v which is determined by the condition [11]:

$$\mu_1(T, \rho_l, \alpha_l) - \frac{1}{2}\mu_{\mathrm{Coul}}(\rho_l) = \mu_1(T, \rho_v, \alpha_v) \tag{17}$$

where:

$$\mu_1(T, \rho, \alpha) = \left\{ \frac{4}{3}\left(x_0 + \frac{1}{2}\right) a_0\rho - 2a_3\rho^{1+\sigma} + T\left[1 + \sum_{n=1}^{\infty}(n+1)b_n\left[\frac{\lambda_T^3\rho}{g}\right]^n\right]\right\}\alpha.$$

In evaluating the right-hand side of equation (17) for the case with clusters the Skyrme force parameters a_0 and a_3 are set to zero because the free nucleons in the vapour are assumed to be non-interacting in that case. Here it must be emphasized that a direct comparison between the vapour asymmetry parameters α_v given in table 3 for the cases with and without clusters is not meaningful. For the case where the vapour is assumed to have clusters, α_v refers only to the free nucleons which constitute less than 1% of the vapour as can be seen from the last column in table 3. Actually, since the vapour in this case consists mainly of deuterons the total asymmetry of the vapour is negligibly small.

In conclusion, the formation of clusters in the surrounding vapour has a profound effect on the stability of the hot nucleus, lowering its limiting temperature by several MeV. This lowering of the limiting temperature of hot finite nuclei is comparable to the lowering of the critical temperature of infinite uncharged nuclear matter due to the presence of clusters [1]. Also, because the NSE model assumes that there is no interaction between the nucleons in the vapour, aside from the formation of clusters, the values for the limiting temperature and the equilibrium vapour density are independent of the Skyrme interaction which is thus used only in determining the properties of the liquid phase. The use of the simple parametrization given in equation (7) for the density dependence of the cluster binding energy needs some further investigation. A more sophisticated calculation of the medium modification of the cluster binding energies may modify the magnitude of our results somehow. Similarly the inclusion of clusters heavier than the alpha particle will have some effect on the present results. Finally the Coulomb corrections due to the charged particles in the vapour [12, 24] have not been included here in order to simplify the comparison with the results of [11].

All the improvements mentioned in the previous paragraph can be the subject of future investigation but they are not expected to change the main result of the present work which is the lowering of the limiting temperature due to the presence of clusters in the vapour as compared to the case when the clusters in the vapour are neglected. The rather low limiting temperatures predicted for the nuclei ^{109}Ag and ^{208}Pb studied here indicate that intermediate mass fragments should be dominant among the products of relativistic heavy ion collisions as the heavier fragments have lower limiting temperatures and thus are less stable. This dominance of the intermediate mass fragments agrees with what is observed experimentally [25] and what is found theoretically in the statistical multifragmentation models [26–29]. These models may suggest generalizing the present work to a more ambitious calculation in which all clusters, including intermediate mass fragments, are taken into consideration. However such a generalization of the present work would be prohibitively complicated.

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