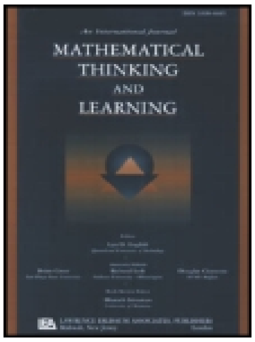


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# Communicating Experience of 3D Space: Mathematical and Everyday Discourse

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# Communicating Experience of 3D Space: Mathematical and Everyday Discourse

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In this article we consider data arising from student-teacher-researcher interactions taking place in the context of an experimental teaching program making use of multiple modes of communication and representation to explore three-dimensional (3D) shape. As teachers/researchers attempted to support student use of a logo-like formal language for constructing 3D trajectories and figures in a computer microworld, a system of gestures emerged. Observations of multimodal classroom communication suggest that teachers/researchers and students used similar words and gestures to represent different types of movement. We discuss possible sources of these differences, contrasting formal mathematical and everyday systems of representation of 3D space. More generally, we argue that understanding the structures of everyday discourse and their relationships to the structures of specialized mathematical discourse can provide insight into student interactions.

As human beings living in a three-dimensional (3D) world, we continually experience shape and motion within that world. Yet the mathematical description and analysis of these aspects of our experience appear to be exceptionally difficult for learners. In particular, we note issues identified by research in relation to identification and operation with angles (Clements & Battista, 1992) and recognition of connections between the physical contexts of corner, turn, slope, and bend (Mitchelmore & White, 2000), although Magina and Hoyles (1997) pointed out that some commonly identified errors, such as confusing the size of an angle with the length of the rays it is formed by or failing to recognize angles presented in nontypical orientations, may be problems with making sense of paper-and-pencil representations of angle rather than with the concept itself. A common approach to explaining such difficulties has been to identify conflicts between students' intuitions, built on their everyday experiences, and the formal definitions of such concepts and operations on them, underpinned by mathematical principles. Mitchelmore and White (2000) suggested a process of gradual development of a general "standard angle concept" by abstraction

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from the range of contexts—a process that is still incomplete for a substantial number of children at the end of primary school. They further argued that this abstraction will only occur as a result of systematic mathematical activity as there is no everyday reason to make connections between such disparate contexts as the corner of a tile, the opening of a door, and the slope of a hill.

Our experience of the world is not a purely physical experience but is mediated by language and other semiotic resources. Moreover, scientific concepts such as the abstract and general mathematical notion of angle are not directly related to empirical experience but only exist in relation to a system of other concepts. They cannot therefore develop spontaneously from everyday experience but must be the product of instruction (Vygotsky, 1986, pp. 172–173). In the classroom, teachers attempt to structure the mediating resources available to students in order to shape their experience and hence their learning in particular ways. Mathematics teachers thus provide forms of language, visual and physical resources, digital technologies, etc. that are designed to offer mathematical ways of experiencing the world (i.e., attending to specific types of objects, patterns, and relationships). Yet students come to the classroom with discursive as well as physical experience. The lexis, grammar, and ways of reasoning used outside the mathematics classroom also shape the ways in which they experience the world, including the world of the mathematics classroom itself. Learning mathematics can be thought of as learning to participate in specialized mathematical forms of discourse (cf. Sfard, 2008). We use the term discourse to refer to the linguistic element of a social practice (Fairclough, 2003). Participating in discourse is thus not simply using the words or other signs of the discourse but using them to engage with the types of knowledge and relationships that form the reality of the practice. An important part of such participation is recognizing how the specialized discourse is distinct from others, including the everyday. This is a prerequisite for developing the ability to produce reliably legitimate mathematical texts.

Our overall research purpose is to understand the ways in which interactions between various sets of discursive resources may affect students' opportunities for learning to participate in specialized mathematical discourse. In order to address this purpose, we seek to analyze how students make use of the resources available to them, drawn from everyday and specialized discourses, as they engage in activities in the mathematics classroom. In this article we draw on data from an experimental teaching program, conducted as part of the ReMath project,[1] focusing on 3D shape. The sequence of lessons was designed to provide students with multiple modalities of mediating resources, including digital technologies, to represent and to operate with the objects of study. The software used, described more fully next, also provided opportunities for activities in which students might make connections between static and dynamic contexts for experiencing angle. The objective guiding our research in this context was to investigate the meanings students made in relation to 3D geometry through their semiotic activity in the context of working with multiple modalities of resources. It is important to note at this point that our interpretation of "meaning," adopting a social semiotic approach (Hodge & Kress, 1988), is not related to individual cognition but to the way that communication functions in interaction between individuals to establish shared orientations. Analytically, meanings are characterized through examination of episodes of interaction, tracking the functioning of particular forms of representation in relation to others, and to the multimodal interaction as a whole.

[1]The ReMath project (Representing Mathematics with Digital Technology), was funded by the European Commission Framework 6 Programme IST4–26751

In this article we aim to consider the ways the students coordinated the resources provided in the classroom with those brought from their everyday discourses. In particular, we focus on the development and use of a system of gestures used to represent the various types of turn in 3D space. We address the specific questions:

1. How did teachers and students make use of gestures to communicate about motion in 3D space?
2. How did the use of gestures relate to the use of other modalities of resources available in the classroom?
3. How did students' use of gestures relate to the specialized mathematical discourse and to everyday discourses of motion in 3D space?

In addressing these questions in the specialized context of a single classroom working on a small part of 3D geometry, we hope to illuminate not only specific issues about communication within this topic domain but also more general issues about the interactions between specialized and everyday discourses.

## THE LANGUAGE OF MATHEMATICS—EVERYDAY AND MATHEMATICAL DISCOURSES

While our particular interest lies in the relationship between mathematical discourse and everyday discourses, our understanding of this is informed by more general distinctions between the everyday and the specialized. It is widely recognized that the forms of language used in school are different from those used in informal everyday situations. For example, we know that migrant children starting to learn the language of their new country achieve communicative competence in the social environment of the playground significantly faster than they acquire a similar level of competence in the language required for participation in academic studies (Cummins, 1981). Bernstein argued that the culture of the home influences both grammatical and semantic patterns of the everyday language experienced by children, socializing them toward particular forms of knowledge. For many children of working class backgrounds, this primary socialization privileges "particularist," context-dependent orders of meaning rather than the "universalist," abstract forms favored in the school (Bernstein, 1973, p. 199). Empirical studies by Hasan (2002), among others, confirm the existence of systematic variation, associated with social positioning, in the forms of communication in the home, consistent with Bernstein's distinction between orientations toward particularist or universalist orders of meaning. This is not to suggest essential cognitive differences between children of different backgrounds or to suggest that one form of language is "superior" to any other but to point out that school practices embody the linguistic and cultural norms, practices, and expectations of dominant social groups (MacSwan & Rolstad, 2003) and that, without explicit induction into the discursive practices of the school, some groups of students are likely to be disadvantaged.

The extent of the gap between everyday forms of language and those forms used in the school thus varies between different groups of learners. However, entering a specialized academic discipline involves all students in learning new ways of using language. The magnitude of the teaching and research fields of English for Academic Purposes and English for Specific Purposes is evidence of the practical need experienced by many students attempting to engage in specialized disciplines, whether in their home language or in an additional language (see the *Journal of*

*English for Academic Purposes*; Hyland, 2007). The difficulty of specialized academic language is not, however, an arbitrary or elitist obstacle to participation. Rather, specialized forms are necessary to express the concepts and forms of reasoning peculiar to the field.[2] In the field of the natural sciences, Halliday and Martin (1993) provided analyses of written texts to support their argument that there is an intimate connection between the nature of scientific activity and the forms of language used in scientific texts. Considering the multimodal nature of mathematical communication, in which different modes of representation are combined within a single text, O'Halloran (2005) identified the ways in which specialist modes such as algebraic notation and graphical representation enable particular forms of mathematical activity.

Although mathematical communication is perhaps most notably characterized by its use of formal notations and uniquely mathematical vocabulary, this is only part of the story. As in many other specialized domains, the mathematics register[3] incorporates considerable use of what may be termed "ordinary" or everyday language in order to make mathematical meanings (Halliday, 1974; Pimm, 1987). While some parts of this ordinary language are used in ways compatible with everyday usage, others are used in ways that diverge subtly or, in some cases, radically. In English, for example, it is hard to see the connection between everyday uses of the word *right* and its use in the mathematical term *right angle*. In Arabic the word for the mathematical object angle, زاوية Zāwyah, has various meanings in everyday language, including, for example, a corner of a room but not a turn or slope. Even the subtlest of differences in usage can be a source of misunderstanding between novices, drawing on their everyday experience, and those with more experience of mathematical discourse. Importantly, substituting everyday meanings for apparently similar mathematical terms can change the mathematical structure of an object or problem (Mitchell, 2001). Many instances, arising at different levels of the education system, are identified in the literature. For example, Walkerdine (1988) noted unexpected (from a mathematical point of view) ways in which very young children use *more* and *less*, suggesting that this arises from the lack of symmetry in the ways these words are used in the home. Durkin and Shire (1991) noted the multiple meanings of a number of words that play central roles in learning mathematics at primary level. Pimm (1987) provided an example showing how a secondary school student's use of the word *diagonal* in an everyday way (equivalent to *oblique*) leads to a mathematically incorrect answer to a simple problem. Even at undergraduate level, students are observed to have difficulties making use of logical connectives such as *if . . . then . . .* in formally mathematical ways (e.g., O'Brien, 1973; Selden & Selden, 1995). One approach to the interpretation of such results is in terms of problems with understanding the mathematical concepts or logic (for example,

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[2]This "necessity" is, of course, open to challenge and resistance. The forms of scientific and mathematical writing have evolved over time, just as the nature of scientific and mathematical activity has changed. Even in the relatively narrow field of academic research papers in mathematics, there is considerable variation in the language used (Burton & Morgan, 2000).

[3]We use register in Halliday's (1974) sense:

> a set of meanings that is appropriate to a particular function of language . . . [thus we] can refer to a "mathematics register," in the sense of the meanings that belongs to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it used for mathematical purposes. (p. 65)

This is different from Duval's (2006) use of register to denote a specialized mode of representation (e.g., algebraic notation or Cartesian graphs).

using "child logic" in O'Brien's terms). From this perspective, persistent use of everyday forms of language is thus seen as evidence of continued reliance on everyday forms of cognition and failure to develop scientific concepts. We regard this approach as too simplistic, taking language as a transparent representation of thought. Rather than assuming a cognitive source for failure to use language in legitimately mathematical ways, we wish to consider possible explanations that propose a social or affective source. In doing so, we do not propose that the social and affective should be considered as domains separate from the cognitive. Indeed we believe that they are all intimately connected. Rather, we would suggest a need to adopt a broader range of explanatory frames. A possible alternative is to interpret "misuse" as a consequence of students continuing to make use of "everyday" linguistic patterns in circumstances when specialized mathematical usage is more appropriate. Or, to adopt a discursive perspective (see Evans, Morgan, & Tsatsaroni, 2006), students draw on the resources of an everyday discourse while their teachers expect them to be situated within a specialized school mathematics discourse.

School mathematics discourse is not a "pure" mathematical discourse but recontextualizes mathematics for pedagogic purposes (Bernstein, 2000). A consequence of this is that a mixture of everyday and specialized language is present in any classroom. Everyday forms of language may be used by teachers both for regulatory purposes and to support student learning (see, e.g., Kyriakides, 2009; Riesbeck, 2009), and the extent to which specialized forms are used varies between classrooms (Atweh, Bleicher, & Cooper, 1998; O'Halloran, 2004). The relationship between the everyday and specialized lexis in the mathematics classroom is thus complex: a term familiar to the everyday lexis may be used by teachers in a familiar way as part of the regulative discourse; it may be used in its everyday way to provide a student with support in making sense of a mathematical concept; it may be used in a specialized way as part of the mathematical instructional discourse and this specialized use may be close to or distant from everyday use. And of course students may also use everyday terms in a further range of ways and for various reasons that may or may not be considered legitimate by their teachers: copying the teacher's use; ignorance, inability, or discomfort with specialized forms; "off task" engagement with other students; or an attempt to establish a particular identity or position within the classroom or in a peer group.

Kress, Jewitt, Ogborn, and Tsatsarelis (2001) proposed an alternative to the cognitive approach to error, drawing on social semiotics and Kress's theory of the motivated sign (Kress, 1993). Rather than identifying error or failure to use correct mathematical language as a sign of a cognitive or linguistic weakness that needs remediating, any student production may be considered as a choice made by the student between the various possibilities available to them. Although we (and the student) may not be aware either of the full set of available alternatives or of the reasons for a particular choice, this choice is motivated. That is, it arises from the student's focus at that point, drawing on their previous experiences and their current positioning with respect to the activity. Thus, for example, a student may produce an "error": because they perceive the situation they are in as similar in some way to one in which they have had success with a comparable strategy; because they pick up clues from the teacher or from classmates that suggest this response will be correct; because they are focusing on parts of the task or context that are familiar while ignoring other parts; or because they are engaging in an activity with parameters and objectives that differ from those anticipated by the teacher. From a pedagogic perspective, Kress and colleagues (2001) argued that rather than simply identifying errors in students' texts, the teacher should be asking what the students' interests are "which underlie and motivate this specific representation of the issue at hand" (p. 118). In this article we seek to provide one means of addressing this challenge by drawing attention to the ways in which students' productions may be motivated by their experience of everyday discourses and the relationships between these discursive resources and those of mathematics.

## MULTIMODALITY—DIFFERENT MEANING POTENTIALS

The bulk of existing research into mathematical communication has focused on the semiotic systems of language and algebraic notation. Research concerned with students' use of diagrammatic and graphical forms has tended to approach these as vehicles for access to students' understanding of mathematical concepts rather than as forms of communication (although there are exceptions to this, e.g., Chapman, 2003, Alshwaikh, 2011). Even where the communicative role of visual forms is recognized, this tends to be subordinated to the linguistic. Thus Sfard (2008), for example, included "visual mediators" as a component of her characterization of mathematical discourse but, by labeling multiple nonlinguistic systems of representation in this nonspecific way (rather than recognizing that each of the various systems, e.g., geometric diagrams, Cartesian graphs or gestures, just as much as language or algebraic notation, contributes a distinctive communicative function), continues to privilege language as the primary mode. There has, however, been a major trend toward recognizing the multimodal nature of communication and the importance of studying the contributions made by different modes of communication and representation of such as images and gestures as well as language. Each of the various available semiotic systems provides a different range of meaning potentials enabling them to be used to effect different functions in mathematical and other activities (Kress & van Leeuwen, 2001; O'Halloran, 2005). Duval (2006) analyzed some of the differences in structure and potential use between various mathematical semiotic systems (which he termed registers) and argued that converting between these has an important cognitive function. While this function is beyond the concerns of the current article, Duval's argument strengthens the case for developing our understanding of multiple modes of representation and communication. In investigating the meanings that students make within a multisemiotic environment, it is thus important to consider their use of all these modes and the relationships between them.

Moreover, wherever communication takes place in face-to-face contexts, gesture and other physical actions also play a part. Kress and colleagues' (2001) multimodal analysis of communication in science classrooms shows teachers and students making use of a "complex ensemble" of modes, including gesture alongside speech, writing, images, etc. There has been recent research interest in the use of gestures in mathematics teaching and learning. Much of this has focused on the gestures used by students, analyzing the contribution made by gesture to learning and mathematical meaning making (e.g., Arzarello & Robutti, 2008; Bjuland, Cestari, & Borgersen, 2008; Edwards, 2009; LaCroix, 2009; Radford, 2009; Radford & Bardini, 2007). In considering gestures used by teachers, studies have shown teachers and students making shared use of gestures initiated by student communication efforts (Arzarello, Domingo, Robutti, & Sabena, 2009; Maschietto & Bartolini Bussi, 2009) and teachers using deictic gestures as mediating resources (Bjuland, Cestari, & Borgersen, 2009).

In this article we do not seek to address any relationship between students' gesture use and their mathematical thinking but are concerned with understanding the "interests" that may underlie their choices of forms of communication, including those arising from their experience of participation in everyday forms of discourse. In particular, we consider the evolution and use of a system of gestures for communication about movement in 3D space. Our interest is in how students adopted and adapted the system of gestures offered by the team of teacher-researchers (research question 1). In seeking to understand how students' use of these gestures may have been motivated, we focus on the ways in which they relate to other semiotic systems (research question 2), considering especially the relationships between specialized mathematical systems and those of everyday communication (research question 3).

## GESTURES, LANGUAGE, AND A 3D TURTLE WORLD

The episodes we discuss here arose during an experimental teaching program, conducted as part of the ReMath project, involving a multisemiotic interactive learning environment, MachineLab Turtleworld (MaLT). This environment, designed by the University of Athens Educational Technology Lab (ETL) project partners, incorporates a 3D turtle geometry (represented on the two-dimensional computer screen), driven by a Logo-like language. It also includes variation tools for direct manipulation of variables, although we do not discuss this component of the software in this article (see Kynigos & Latsi, 2007). In Figure 1, the Turtle Screen on the left shows the path travelled by the turtle following the instructions in the Logo Editor screen on the right. The commands used in this sequence of instructions include: *forward* (fd) – move forward (in the direction of the current heading) given number of steps; *up_pitch* (up) – pitch upwards (relative to the current heading) given number of degrees; *right_turn* (rt) – turn right (relative to the current heading); *left_turn* (lt) – turn left; *left_roll* (lr) – roll left (relative to the current orientation).

### Methodology

The study was conducted as part of a program of cross-experimentation (Artigue, Cerulli, Haspekian, & Maracci, 2009; Bottino & Kynigos, 2009) in which the designers of the software (in this case ETL) and another "alien" research team (in this case the authors) designed and conducted separate teaching experiments in their local contexts. The aims of the cross-experimentation were on the one hand to study the ways in which the representations offered by the software related to student learning and on the other hand to understand how the local context

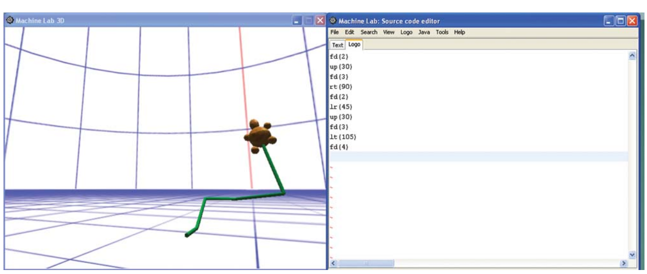


FIGURE 1 MaLT screenshot. (Color figure available online.)

(comprising the research and school cultural and institutional contexts and the theoretical frameworks of the two research teams) influenced use of the software (Lagrange & Morgan, 2010).

The pedagogical plan used in the London-based teaching program[4] reported here was designed to allow us to investigate the meanings students would make in relation to 3D geometry through their semiotic activity in the context of working with MaLT and other modes. Interacting with MaLT itself involves making use of several inter-related systems of representation, including turtle movement and its traces formed in the Turtle Screen (dynamic and static graphic representations), instructions and procedures constructed in the Logo Editor (symbolic), and the variation tools (dynamic visual and symbolic). Moreover, the social environment of the teaching program was intended to allow, and indeed encourage, communication through talk and various paper-and-pencil based forms of representation and the use of physical manipulatives as well as through the computer software itself.

The teaching program was conducted with a Year 8 class (aged 12–13 years) in a state-maintained secondary school in London. The school grouped students by attainment for their mathematics lessons; this class was ranked 4th out of the 5 in the age cohort, and their achievement in national tests to be taken in the next year was anticipated to be below average for their age group. The students had no previous experience with MaLT or with other forms of Logo and had little experience of using computers in their mathematics lessons, although most were confident users in other contexts, including social contexts. A sequence of nine lessons was taught collaboratively by the class teacher, the researchers, and a student teacher attached to the class. While the content and activities to be used in each lesson were planned in advance of the beginning of the whole teaching program, review and reflection by teachers and researchers following each lesson affected interactions in subsequent lessons.

In each lesson a video record was made, focusing primarily on the teacher or teacher-researcher during teacher-led whole class interaction and on a selected group of students or an individual student during student work on group or individual tasks. The video aimed to capture gestures and the various visual and physical resources available, including the computer screen when in use, but did not capture other interactional components such as gaze. Microphones attached to the video camera similarly captured teacher talk and most student contributions during whole class interactions and talk within a group of students or between students and teacher/researcher during group or individual work. Writing or drawing using paper-based media and saved records of computer-based work were also collected.

Both authors viewed the videos and independently identified extracts of video that were of interest. All these extracts were transcribed and associated with paper-based work or computer output arising within the same time frame. By selecting extracts identified by either author, we sought to make use of as much relevant data as possible. In accordance with our research focus on multiple semiotic resources, extracts chosen for transcription included, in particular, those where several modes of communication were being used together.

We consider the form of transcription to be part of the analytic process as a preparation for the multisemiotic analysis needed to address our research questions. The use made of each mode of communication (spoken language, gesture, computer screen image, written language, drawing, manipulation of physical apparatus) was thus recorded (by transcription of linguistic data,

[4]The pedagogic plans used in the ReMath project may be found at http://remath.itd.cnr.it/index.php.

description or sketch of visual data, or insertion of video stills) in a separate column of a spreadsheet, allowing both horizontal (a snapshot of all simultaneous semiotic activity at each moment) and vertical (an overview of semiotic activity within a particular mode through the whole period of the extract) examination of the data. The transcript was divided into moments of communication (including all modes of communication used during the moment) that were considered to have some functional coherence; this division was a pragmatic consideration without an explicit theoretical basis. The process of transcription was recursive: transcripts initially produced by the second author were reviewed and revised by the first author; as analysis progressed, the original video was revisited and transcripts augmented as we sought to gain a deeper understanding of the data. In particular, descriptions of gestures became more detailed and more systematic during the analytic process.

Our approach to analysis involved both the application of a priori categories and the iterative definition and refinement of categories derived from the data. Our initial categories included the modes used (already coded through the transcription process in different columns), together with some a priori subdivisions:

- spoken language (subdivided into everyday/ mathematics/ MaLT registers)
- gesture (pointing/ mimicking motion/ other)
- written language (natural language/ conventional mathematics notation/ MaLT notation)
- drawing (outcome of MaLT programming (desired or actual)/ aid to problem solution)

Initial coding of the data using these a priori categories was conducted by the second author and verified by the first author. Disagreements or difficulties were resolved in discussion between the authors, and these discussions often gave rise to more refined definition of subcategories derived from interaction with the data. In what follows, we describe the development of a more delicate coding of some specialized forms of gesture, arising from difficulty in applying the code "mimicking motion" in a way that would distinguish different communicative functions.

A detailed account of coherent moments of communication was then constructed, identifying the ways in which the specific conjunction of modes of communication functioned to construe the nature of motion in 3D space. At each stage in the analysis, descriptions, codings, and interpretations were initially made by one of the authors, and then reviewed by the other with subsequent discussion to resolve issues arising and to develop more refined analytic tools. See Morgan and Alshwaikh (2009) for a detailed example of similar analytic methods used with multimodal data arising from an experimental teaching program with a different interactive learning environment, also conducted as part of the ReMath project, focusing on constructs of velocity and acceleration.

This formal transcription and analysis was conducted after the completion of the entire program of teaching. However, the researchers viewed some parts of the video recordings after each lesson and this contributed to the informal review of the lesson and hence affected forms of interaction in subsequent lessons.

## Identifying a New System of Gestures and Its Use by Teachers and Students

As we started to view the video data collected during use of MaLT, it was noticeable that the teachers and researchers made extensive use of gestures in an apparent attempt to support students'

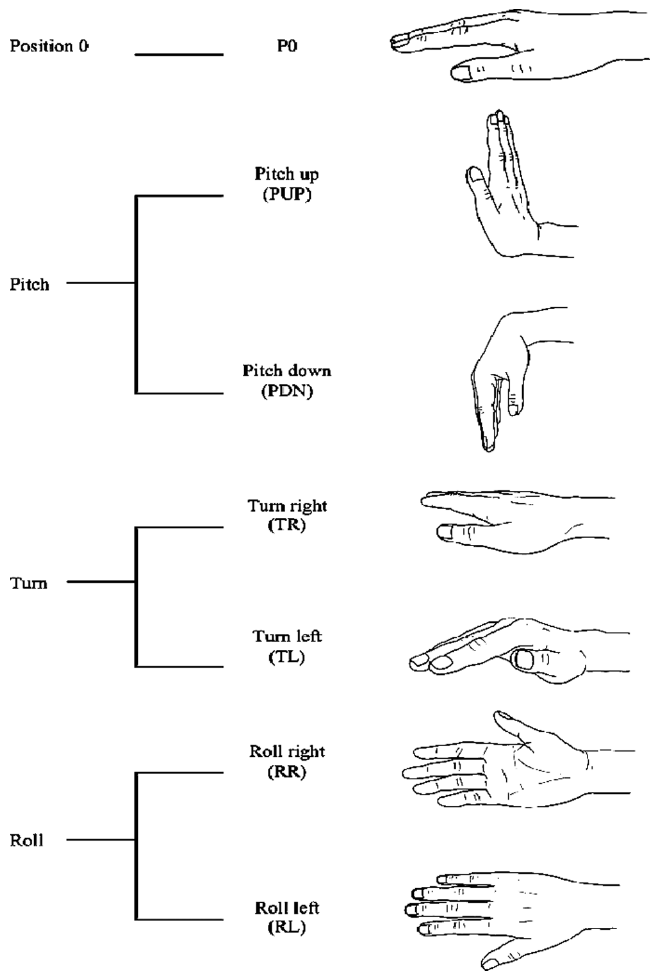


FIGURE 2 3D turn gestures.

planning and execution of constructions in MaLT. One significant type of gesture was a set of stereotyped hand and/or arm movements, often associated with use of the terms *turn*, *pitch*, and *roll* and the associated Logo instructions (see Figure 2 for the codes used in transcription of these gestures).

In each case, the dynamic gesture consists of movement from position 0 (with forearm and hand both oriented in the same direction) to another position. Consistent with the relative nature of the turning commands in the Logo language, the significant characteristic of each of these gestures is the relationship between the (static) orientation of the forearm (representing the direction of motion prior to turning) and the new orientation of the hand (representing the direction of motion after turning). The starting orientation is not significant: for example, whichever direction the hand and arm are pointing initially, the "pitch up" gesture involves turning the hand so that the back of the hand forms an "upward" angle with the back of the arm (see Figure 3).

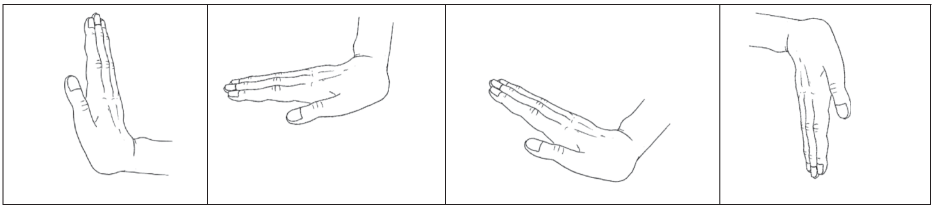

FIGURE 3 All these gestures indicate “pitch up.”

This set of gestures constitutes a new semiotic system, i.e., a set of signs together with a set of rules for using and combining them, linked with, but not identical to, both the everyday linguistic description of 3D movement and the symbolic system of Logo. They may be considered iconic gestures (McNeill & Levy, 1980; Roth, 2001), in that each bears a visual resemblance to the anticipated trajectory of an object moving in 3D space (or a turtle moving in the simulated 3D space of the MaLT display). Students also made use of these and other gestures to support their communication about turtle movement. We have said that the students used “these” gestures in order to indicate that their hand and arm movements resembled those used by the teachers/researchers. However, our coding of the gestures was initially related only to their physical characteristics, not to the ways they functioned communicatively. It was only by considering the complete multimodal episodes of communication that we were able to make such functional interpretations. We then related gestures to accompanying words and other symbolic, graphic, or physical components of communication, taking into account the nature of the task and other contextual features that might affect the ways the gestures were used, including, for example, whether the communication was with a teacher or a student and the relationship of the episode to previous episodes. As will become apparent, we believe that the students made use of the gestures in different ways, thus construing different experiences of 3D space and movement. We became interested in the evolution of these signs, their adoption by students, and the relationships between the semiotic activity of teachers and researchers and that of the students.

For the teachers and researchers, using these gestures as ways of thinking and communicating about movement of the turtle within MaLT seemed to emerge as a natural consequence of our experience with two-dimensional versions of Logo, both as users and as teachers. In Papert’s seminal *Mindstorms* (1980), he argued that turtle geometry is useful for learning because it is body syntonic, “firmly connected to children’s sense and knowledge about their own bodies” (p. 63). This connection to personal bodily knowledge may be operationalized through playing turtle, either literally by walking along a path, enacting the instructions given to the turtle, or metaphorically in the imagination. Encouraging and supporting students to play turtle has become a standard part of Logo pedagogy. The metaphor of playing turtle thus formed part of our experience of Logo culture and constituted for us a more or less implicit theory about learning with Logo.

Our Greek partners ETL incorporated the idea of body syntonicity as an explicit theoretical justification for their own pedagogical plan, implemented in Athens:

> This is a sequence of tasks for students, taking them from an initial introduction to the software and its functionalities through to a number of geometrical simulation challenges in the 3d space and opportunities for creative exploration through body syntonic activities. Initially students will be asked

> to explore turtle's turns and moves by using different sets of 3d Logo commands and then to use them to demonstrate an aeroplane taking-off with the use of a relevant tangible concrete object (e.g. a model of a 3d aeroplane).[5]

In the 3D context, it is not possible to act out turtle movements physically with the whole body. Instead, the hand (or in this case a toy aeroplane held in the hand) substitutes for the body. We adopted a similar initial activity in our own introduction of MaLT to London students, using a model aeroplane to demonstrate a trajectory of turns and moves. Having done so, we substituted gestures (hand and arm movements without holding a model) for the movement of the aeroplane and incorporated these into our further communications about 3D movement throughout the teaching experiment. Initially, this use of gesture was spontaneous as we talked with students about their attempts to construct trajectories in Logo. At this stage, the hand substituted directly for the model aeroplane as students grappled with the task of constructing take-off paths on their screens. Our spontaneous association of specific gestures with types of turning in 3D space became an object of reflection after this initial lesson and was later incorporated more consciously into our communications with students in an attempt to encourage students to associate a sense of their bodily movement with the Logo symbolism. As the students moved on to other tasks in which they were drawing 3D objects, no longer connected with the context of aeroplane trajectories, the hand and its movements came to be used as representations of the Logo turtle.

## ILLUSTRATIVE EPISODES

We now present two episodes from the experimental teaching program in which the teachers and researchers modeled use of gestures to play turtle. The first of these constitutes the first occurrence of the new system of gestures, arising spontaneously from the planned "aeroplane" activity described previously. The second episode is from a subsequent lesson in which the teachers/researchers explicitly planned to use the gestures to support student use of MaLT. We have chosen to present these two episodes in order to illustrate the evolution of the use of the system of gestures in combination with specialized and everyday linguistic resources. We then present an analysis of an episode of a student's use of similar gestures. While we do not claim that this episode is representative of the entire data set, it is not atypical. We have chosen it in order to illustrate some of the differences in the ways gestures were used by teachers, researchers, and students. These differences and their possible origins are discussed in the following section.

### Episode 1: From Aeroplane Trajectory to Gesture System

In the introductory session with MaLT, the first author (CM) introduced the notion of turtle movement using a toy aeroplane as described in the ETL team's pedagogical plan. She accompanied the physical movement of the aeroplane with a verbal description, using and stressing the terms pitch (up/down), roll (right/left), and turn (right/left) in synchrony with the associated movement as shown in the following illustrative extract. For reasons of space, we do not

[5]"Programmable constructions in 3D geometrical space (familiar)" http://remath.itd.cnr.it/index.php?cmd=view.

include complete multimodal transcriptions of data extracts but abbreviated versions showing the components necessary for our current argument.

CM: and goes up up [*toy plane held in right hand; arm, hand and plane aligned (P0) at 45° angle upwards*] Anybody thought what happens when it goes up sort of when it is about there? How does it move?
Student: Doesn't it go straight?
CM: Yes, pitches down again doesn't it [*hand and plane moved to PDN position: arm still at 45°, hand and plane horizontal*]?

Following this initial introduction, students were set the task of using the 3D Logo language to construct the trajectory of an aeroplane taking off. After the lesson, on observing how we and other members of the teacher/researcher team supported students' attempts at this task, we noticed that we made use of iconic gestures in which the movement of the hand resembled the desired movement of the aeroplane or 3D Logo turtle. Sometimes a gesture was used synchronously with an equivalent word or Logo symbol; at other times a gesture appeared to be used, without equivalent verbal or symbolic language, in order to elicit such language from the student. Although this use of gesture had not been planned, in the course of the first lesson with MaLT a system of gestures emerged, supplementing the planned use of everyday and formal language. As the teacher/researcher team reflected after this first lesson, we became explicitly aware of our use of these gestures, although this was not as yet a focus for formal analysis.

## Episode 2: Explicit Focus on Gesture and Specialized Language

In a later lesson, recognizing that some students were still having difficulty distinguishing between the different kinds of turn as they attempted to construct 3D figures in MaLT, we planned an activity to make more explicit links between the gestures and the specialized language of 3D movement. At this stage, the system of gestures had become a code for us, mapping each change in the relationship between hand and arm in a one-to-one relationship to the language of 3D turns and hence to the formal Logo terms as shown in Table 1.

Of course, although at the time we attempted to use these modes as if they were equivalent, in practice the possibilities for meaning-making are different in each mode. As Kress and colleagues asserted, "a shift in mode never amounts merely to saying the same thing in a different mode, rather it involves a deep reshaping of the thing which is represented" (2001, p. 99). We will argue that one of the significant factors in this reshaping is the potential intertextuality and

TABLE 1
Mapping Gesture—Language—Logo Formalism

| Hand/Arm Relationship | 3D Turn Language | Logo Formalism |
|---|---|---|
|  | Down (more fully "Pitch down") | PD 90 |

interdiscursivity afforded by each mode, that is, the potential for forming connections with other texts (intertextuality) or with other discourses associated with different forms of social practice (interdiscursivity). In particular, in this article we focus on the ways in which the various modes used within specialized mathematical communication about 3D motion relate to the resources of everyday discourse and on the consequent potential for participants to make connections to (and use of) everyday forms of knowledge and reasoning.

In this activity the class teacher (GD) used her arm and hand to act out the role of the turtle drawing a "door" under instruction from the class. Once a movement was agreed to be correct, the corresponding instruction was entered into Logo and the consequent figure displayed to the class. The links between gestures, the language of turning and the Logo instructions were not stated explicitly but were enacted through the multimodal interaction. The teacher was careful to follow the conventions of the gesture system in order to emphasize the relative nature of turtle movement. Thus, for example, as seen in the partial multimodal transcript in Table 2, she turned her hand in a pitch down gesture when given the instruction to go down, even though, because her arm and hand were initially oriented horizontally with the back of the hand towards the class, this resulted in her hand pointing towards the wall rather than down towards the floor. Although student S1 used a term *down* that was part of the specialized code system, he used it in an everyday sense to indicate movement toward the ground. This resulted in conflict for students between their intended outcome and the visual feedback provided.

TABLE 2
"Down" Doesn't Mean "Down"

| | |
|---|---|
|  | CM: Ok. Look at the way that Miss's hand is pointing. Which way has she got to turn it now?<br>S1: Down.<br>CM: OK. Would you turn your hand down please? |
|  | S1: No.<br>Ss: [laugh]<br>CM: Was that right?<br>S1: No. |

The immediate problem was resolved as shown in the following extract.

CM: It wasn't was it? So which? Can you think about which way to turn?
S2: [indecipherable]
GD: What did you say S2?
S2: Sideways.
GD: Sideways? Which way? Right or left? [GD uses her left hand to point to the right and left sides of her right hand]
S2: Right.
CM: Ok, everybody agree with that? [*GD turns her hand TR*] Does that look right?
Ss: Yeah.

It is worth noting that student S2 again used an everyday term *sideways*, although in this case a term that was not also part of the formal system. The teacher GD revoiced this instruction, offering the terms *right* and *left*—terms with formal places in the code system as well as everyday meanings—accompanying this revoicing with pointing gestures toward the two sides of her hand (Figure 4).

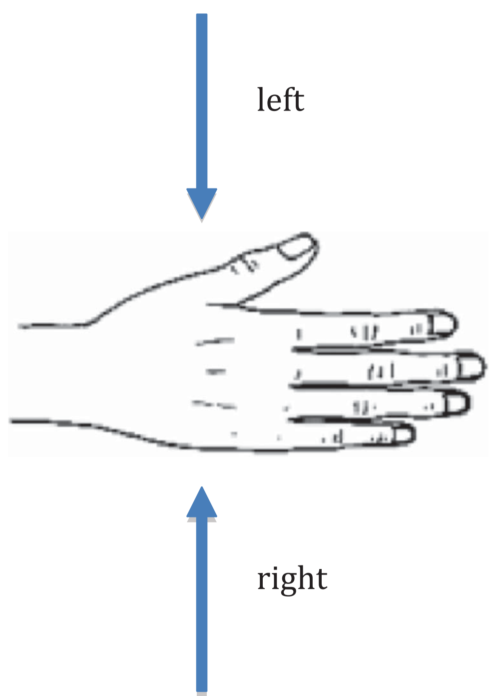


FIGURE 4 Pointing to left and right sides of the hand. (Color figure available online.)

TABLE 3
Identifying the Direction of Turn

| | | |
|---|---|---|
|  | CM:<br>S3:<br>GD:<br>S3: | And which way—you're going to find this hard aren't you? Which way should that hand turn now?<br>That way [pointing].<br>Yes and what's the instruction?<br>Oh . . . left. |
|  | S3: | No, right. |

GD's additional deictic gesture, pointing to the right and left sides of her hand, served to indicate the desired plane of reference for right/left. This deictic gesture might be considered to serve as a form of scaffolding for developing the formal code, as all terms in the code must be interpreted relative to the current plane of the hand. On the other hand, by offering students a choice of just two acceptable answers, their attention may be focused very narrowly on that choice rather than on the underlying principle. In the subsequent interaction, shown in Table 3, one of the students (after seeing and rejecting the visual feedback of GD's TL gesture in response to his initial instruction "left") succeeded in providing an instruction "right," which was accepted by the teacher as correct, equivalent to the formal instruction "turn right." It is, however, unclear whether the student is basing his response on consideration of the full range of possible types of turn or whether his selection of left/right is based on the narrower choices suggested by the teacher for the previous turn.

## Episode 3: Changing Hands, Changing Sign Systems

Student T, having constructed a representation on the MaLT screen of one rectangular wall, was trying to construct a second wall perpendicular to the first. She explained what she was trying to draw using language and gesture. Her words are shown in Table 4, together with a verbal description and a sketch of the accompanying gesture.

Starting with her right hand and arm vertical (P0 vertical—see Figure 2) and with the inside arm and the palm of the hand facing away from her and toward the computer screen (line 1), she

TABLE 4
T draws a wall

| | | | |
|---|---|---|---|
| 1 | here | whole rt arm vertical p0, palm facing away from body, moves up in direction of fingers |  |
| 2 | turn here | TR, arm moved in direction of fingers (maintaining TR position) |  |
| 3 | turn here | attempt to move rt hand TR again (too difficult?) |  |
| 4 | | switch to it hand, arm horizontal pointing rt, hand PDN (fingers pointing down) |  |
| 5 | turn here | moves forearm clockwise, hand still PDN (fingers pointing left) |  |
| 6 | but I want it to come forward | turns arm (awkwardly) so that, hand still in PDN position, fingers point towards body | |

made a turn right gesture (TR) subsequently aligning her forearm horizontally with the direction of her fingers (P0 horizontal), with the inside arm and palm still facing away from her (line 2). She then attempted a further TR gesture with the same hand (line 3) but replaced this with her left hand and arm, positioning the left forearm horizontally (parallel to the previous position of her right forearm but oriented with the inside arm downwards) with her hand pointing downwards in a pitch down gesture (PDN) (line 4). Continuing with the left arm, she moved the whole configuration of forearm and hand, maintaining the PDN relationship between forearm and hand, so that the forearm was oriented vertically downwards and the hand pointed horizontally towards the left (line 5). Finally, she attempted to rotate the fixed PDN configuration of left forearm and hand so that the fingers pointed toward her and away from the computer screen (line 6).

The switch (lines 3–4) between use of right and left hands appears to be a response to the physical difficulty of achieving the desired position with the right hand (see Figure 5).

We consider what remains the same and what is changed with this switch of hand. The switch allows T to maintain the direction in which the fingers are pointing (down). This may be taken to represent the turtle heading within the vertical plane parallel to the screen. However, in switching arms, she changes the relationship between arm and hand from a turn gesture to a pitch gesture. We use turn and pitch within the conventions set up by the teachers/researchers and the Logo language, not to suggest that T associates her gestures with these terms. On the contrary, she does not appear to attach any significance to the distinction, using turn in a generic, everyday way, focusing solely on the position of her hand and the direction in which her fingers are pointing in order to describe the intended turtle movement. While she is to some extent playing turtle with her hand,

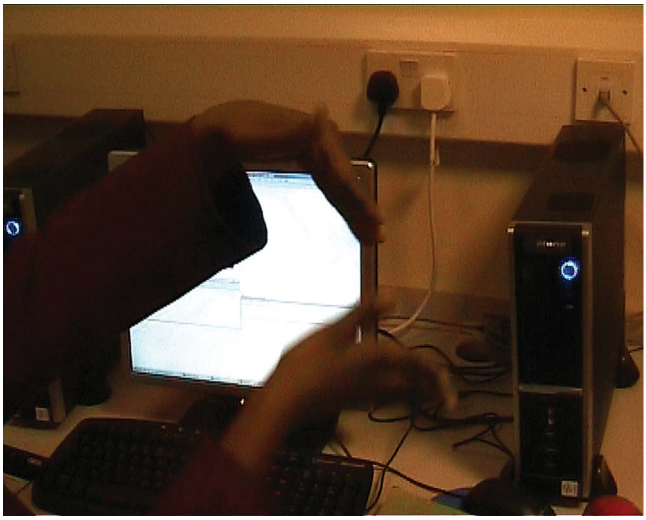

FIGURE 5 T switches to use her left hand. (Color figure available online.)

she is defining the turtle's movements by using position and heading at the corners of her imaginary wall rather than by using turn and distance as required by the Logo language. The use of the turn and pitch gestures is thus not supporting her move into using Logo code and may indeed have made her communication with teachers/researchers less effective. We contend that her ways of using the turn and pitch gestures is related more strongly to everyday ways of describing motion in 3D rather than to the specialized forms of description used by the teacher/researchers.

## CONTRASTING GESTURES: IMAGING PROCESS VS. IMAGINING OBJECT

In Morgan and Alshwaikh (2008) we considered the difference between the ways in which teachers/researchers and students were using the same gestures, and distinguished between the two notions of imaging and imagining. In this section, we revisit the ideas presented in that paper, relating them to our analysis of the three episodes presented previously. In the next section we develop the discussion further, elaborating a framework for understanding differences between teacher and student communications through considering their use of specialized mathematical and everyday discursive resources.

We define imaging as using an iconic gesture to create an image of the construction of the turtle path. The movement of the hand mimics the movement of the turtle: the forearm is held parallel to the current heading of the turtle and the hand is moved to define the next heading. Thus, as shown in Figure 3, the gesture indicating up pitch is always relative to the current heading of the turtle. In both episodes 1 and 2, the teacher/researcher gestures were imaging the process of construction of the turtle path.

In contrast, in episode 3 student T used apparently similar hand movements to construct a very different effect. For her, the relationship between forearm and hand did not appear to have significance, as she was willing to substitute a pitch down gesture with her left hand for a turn

FIGURE 6 "Pitch down" indicates "go up."

right gesture with her right hand. We characterize her use of gesture as imagining, referring to her mental image of the desired outcome of turtle drawing. Such use appears to have both iconic and deictic characteristics.[6] In this episode, as in several other episodes of student gesture within the data set, the gesture points to the desired direction of movement in order to draw the desired outcome, rather than mimicking the required type of turn. Thus, for example, a movement in the up direction (within the plane of the screen) might be indicated by use of the spoken word up accompanied by a pitch down gesture (Figure 6).

Our conclusions in Morgan and Alshwaikh (2008) related to a disjunction between students' everyday experience of 3D space and the movement of a turtle in MaLT, interpreted as a cognitive difficulty in imagining one's body moving freely in that space:

> While we have extensive knowledge of our own body movement in the normal two-dimensional horizontal plane that can be connected to the movement of a turtle in the vertical plane of the computer screen, our experience and knowledge of movement in three dimensions is much more limited. Many of the movements required of a turtle constructing a path in the three-dimensional space of MaLT are impossible for the human body within its normal environment. The extra leap of imagination required to 'play turtle' as if in control of an acrobatic aircraft or perhaps in deep water with highly developed underwater maneuverability may be too great for genuine body syntonicity. (p. 141)

We noted at that time the possibility that students might be drawing on everyday communicative resources rather than on the formal systems proposed by the teacher-researcher team and by the Logo language but did not examine this possibility in detail. We now develop our analysis of the relationship between everyday and specialized discursive resources.

## DISCUSSION: EVERYDAY VS. SPECIALIZED RESOURCES

When movement is restricted to a 2D space, rotations are only possible around an axis perpendicular to (and outside) the plane. Our everyday experience is most commonly confined to movement experienced as more or less within a plane (i.e., traveling on the surface of the earth) and everyday English language reflects this, using the single word turn to denote any form of rotation. Changes of direction resulting from irregularities in the surface on which we are situated are generally

[6]See McNeill's (2005) development of his characterization of gestures, recognizing iconic and deictic as dimensions rather than categories of gesture.

referred to in terms of motion along a slope (e.g., “go up the hill”) rather than as a rotation of our own body or its trajectory (cf. Mitchelmore and White’s 2000 finding that students do not recognize connections between the concepts of slope, corner, and turn). Even when rotations out of this plane are experienced they are generally referred to using the generic turn, modified by a description of the sense of the rotation (e.g., clockwise) or of the heading following the rotation (e.g., up or down).

In contrast, rotations in 3D space are possible around any line in the space, although any rotation may be defined as a combination of rotations around a set of three mutually perpendicular axes.[7] Consequently, in order to specify rotations in 3D, three different words are used to distinguish between rotations around each of the axes. For this specialized purpose, in English (and Logo) the terms pitch and roll are adopted in addition to turn. Turn itself acquires a specialized use, referring to rotation around an axis perpendicular to the plane in which the moving object is currently located, while maintaining its everyday generic use. Thus it is possible to say: “In 3D space there are three types of turn [*everyday*]—pitch, roll and turn [*specialized*].” The additional terms are borrowed from the specialized discourse associated with travel on water and, more recently, air travel and are not widely used outside these contexts. As everyday discourse does not systematically distinguish different types of turning, it is common to omit specific reference to the process altogether, simply providing the heading following the turn.

Another difference between everyday language and the specialized language of Logo and of the introduced gesture system lies in the ways that directions are used. In Logo and the gesture system, right/left and up/down are always defined relative to the current heading. In everyday discourse while right and left are usually used in a similar, relative way, up and down are more commonly used to refer to absolute directions relative to the earth. Gestures used to indicate turns in everyday discourse may also tend to be deictic—pointing in the direction of the turn—or hybrid like student T’s gestures, rather than purely iconic—mimicking the trajectory of the movement.

Problems with shifting between relative and absolute directions are also evident in our experience of students working with two-dimensional Logo, but we would argue that, in two-dimensions, the conflict between everyday and specialized discourse is more explicit. We recognize two distinct types of apparently similar errors made as students direct the turtle’s two-dimensional trajectory. The first type is the use of left and right from the perspective of the student herself rather than the perspective of the turtle. We would contend that this is an error that is not specific to the Logo context or the development of the specialized language. Rather it reflects the common difficulty of projecting one’s personal experience of left and right onto the experience of another. This difficulty is also observed in everyday contexts when attempting to give directions to someone who is facing in another direction. It may thus be seen to be a conflict between perspectives rather than between discourses. Visual feedback is offered by the turtle moving in an unintended direction, providing support for debugging. Use of the playing turtle strategy addresses this difficulty directly or students may use a trial and error strategy similar to that observed in Episode 2 previously, substituting left and right until the desired outcome is observed.

The second type of error working with 2D Logo is the attempt to use down or up as instructions to make the turtle travel toward the bottom or the top of the screen. In this case, the student appears

[7]In fact, two axes is the minimum required to define any rotation, but using only this minimum can result in more complex sequences of turns. For example, any pitch can be achieved by a combination of rolls and turns. In the simplest case, a pitch up 90° can be substituted by the sequence roll right 90°, turn left 90°, roll left 90°.

to be imagining the picture produced as an outcome of the trajectory in the sense we use in the current article rather than imaging the process of construction. Unlike in the right/left case, there is conflict between the resources of the specialized and everyday discourses as the terms *up* and *down* are not part of the 2D Logo lexis and have no meaning within the 2D world of the Logo turtle. The student receives a verbal error message rejecting the instruction (e.g., “Logo does not understand DOWN”) rather than visual feedback showing the turtle’s interpretation of it. The error and the conflict between specialized and everyday resources are thus transparent as the everyday use of up/down as absolute directions is explicitly rejected (though this type of error message may not be very helpful in finding the “correct” instruction).

In the context of 3D Logo, up/down form part of the specialized lexis (in combination with pitch) but have a distinctly different form of use, indicating turn relative to the current plane of the turtle rather than absolute direction away from or toward the ground. There is thus a conflict between the specialized and the everyday use of these terms. Students who use up or down to attempt to make the turtle move toward the top or the bottom of the computer screen receive the visual feedback of movement in an unanticipated direction. However, a trial and error approach, such as that used with left and right in 2D, is unlikely to be successful as the desired turn most likely requires use of a turn or roll instruction rather than a simple substitution of pitch up for pitch down. We contend that, as in the 2D case, this type of use of up or down as absolute direction makes use of the resources of everyday discourse rather than those of the specialized Logo language. Unlike in the 2D case, however, the feedback does not reject the instructions but accepts them as legitimate. The conflict between the specialized and everyday discourses is thus not made transparent and the source of the error may be hard to identify.

As students talked about their work on tasks such as drawing a room, constructing a revolving door, etc., they tended to use only directions to describe their turtle turns, omitting the verbs that would define the type of turn. Thus, rather than saying turn right or pitch down they would say simply right or down (or possibly use an indeterminate verb *go right* or *go down*). Such elision is compatible with everyday usage in which, as discussed, down is an absolute direction, toward the center of the earth, while right is usually taken to be relative to the vertical axis of the body and the direction in which the whole body is facing (see Figure 7). Alternatively, as seen in Episode 3, students would coordinate everyday language and gesture, using only the word *turn* while indicating the direction of the turn by a gesture.

Recognizing this difference in reference of directions in everyday and specialized discourse brings us to realize a further issue possibly raised by the initial use of a model aeroplane to introduce the idea of 3D trajectories. The model aeroplane affords imaginary positioning of the user/observer inside the plane, enabling two simultaneous possible planes of reference for “down”: relative to the floor of the aeroplane or relative to the earth. Both of these references are part of everyday discourse for those familiar with air travel. While hand and arm gestures in theory have the same duality—down relative to the inclination of the arm or down relative to the earth—not only is the former not a common usage in everyday discourse but physical constraints on the formation of gestures make it difficult to maintain a consistent “correct” inclination of the arm as sequences of gestures are constructed (see CM’s remark in Episode 2 to GD that she would find the next movement hard or T’s switch of arms in Episode 3 in order to avoid physical contortions). Students’ success in using the gestures and Logo terms in the initial aeroplane trajectory task may thus not be followed by success in using the same gestures and terms for tasks such as drawing the walls and doors of a room for which the everyday reference of down is unambiguous.

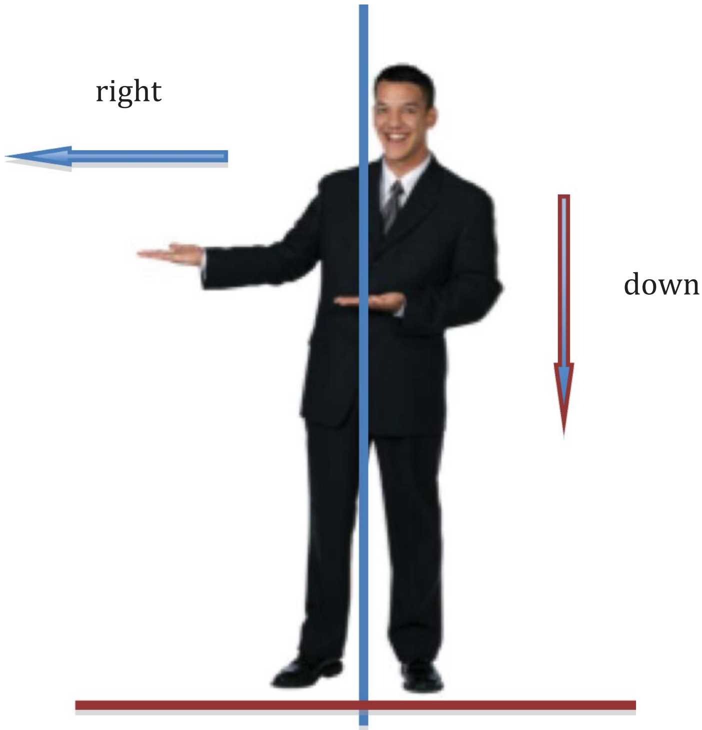


FIGURE 7 Everyday directions relative to the body and to the earth. (Color figure available online.)

A particular source of difficulty in coming to use the Logo formalism lies in the fact that the formal terms turn and roll are both modified by the everyday terms *right* and *left*. It was noticeable that, even as students became more familiar with the formal language, roll was used less frequently than either pitch or turn. This is consistent with the everyday focus on direction rather than type of movement. Students talked about their desire for the turtle to go up or down, right or left, then associated these directions with the formal pitch and turn but had no distinct everyday way of referring to the desired outcome direction of roll. In Table 5 we summarize the main areas of difference between specialized and everyday ways of experiencing and speaking about motion in 3D.

Unlike the situation described by Arzarello and colleagues, where "the teacher uses the same gestures as the students and rephrases their sentences using precise mathematical language" and in doing so "supports the students towards a correct scientific meaning" (2009, p. 106), in the situation presented in this article the teachers/researchers themselves developed and then used a new set of gestures in an attempt to support the development of students' use of new formal language, an attempt that appears justified according to Roth's review of studies of gesture in teaching (2001). On the other hand, Roth also suggests that students may interpret teachers' metaphoric gestures as iconic, with negative consequences for their understanding of science concepts (p. 377). In the case presented here, students seem to have adopted the teachers/researchers' iconic gestures as if they were deictic.

TABLE 5
Specialized and Everyday Discourses of 3D Motion

| | Specialized Discourse | Everyday Discourse |
|---|---|---|
| Linguistic and Logo Symbolism | Turn right/leftTR/TL<br>Pitch up/downUP/DP<br>Roll right/leftRR/LR | Turn/go right/leftClockwise/anticlockwise<br>Go/turn up/down<br>Roll over/turn overturn clockwise/anticlockwise |
| | Each instruction has a single defined function. Values *right/left* may qualify both turn or roll instructions. | Terms are multivalent. *Turn*, especially may refer to any type of rotation. Generic motion words such as *go* may also be used to refer to rotations.<br>The use of some of these terms is very context-specific, depending on the kind of object and its starting orientation. |
| | *Right/left* and *up/down* are always relative to current orientation | *Right/left* is usually relative but *up/down* is usually absolute.<br>Again, there are specific contexts in which these values vary. |
| Gestures | Iconic, mimicking trajectory of the Logo turtle | Deictic, pointing in the direction of movement, or iconic, or hybrid. |
| | One-to-one relationship with specialized linguistic terms and Logo instructions | In practice, substitutable for one another, especially to overcome physical difficulties. |
| Experiences | The Logo turtle is free to move in any direction in its simulated 3D space.<br>More generally, an "ideal" object is free to move within an "ideal" 3D space. | Everyday human experience of motion in 3D space is limited primarily to motion on 2D surfaces. Free motion of a human body in 3D space is unusual, confined to very special contexts. |
| | In Logo, drawing an aeroplane trajectory and constructing a representation of a 3D object require the same types of instruction sequences. | Trajectories are experienced over time and rotations are experienced as relative to current heading.<br>3D objects are experienced as wholes with a "natural" absolute up-down orientation. |

This difference between the formal characteristics of the teachers/researchers' gesture system and students' use of its gesture components is compatible with typical differences between specialized language and everyday language use. Terms in specialized language tend to be rigorously defined and used in consistent ways across contexts, everyday language is contingent, multivalent, and changeable. For example, common everyday words such as go and get are used in a wide variety of contexts to say very diverse things (consider: I go to the paper shop every morning and I go red at the slightest provocation; I get my ticket at the station, I get cold in the winter and I get by on five dollars a day[8]) and are notoriously difficult to define.[9] While the teacher/researchers consider each gesture in the system to be unambiguously defined, there appear to be more flexible

[8]An exercise I (CM) remember being set (and enjoying) at primary school involved being required to substitute other verbs for go or get in a pageful of such sentences.

[9]The online dictionary Dictionary.com (http://dictionary.reference.com/browse/go) shows 98 distinct uses for go and 63 for get.

possibilities for the students: the same gesture may be used to represent a particular type of turn or to indicate “go in this direction.”

We have argued that students’ modes of adoption of the gestures introduced by the teachers/researchers are likely to be related to the characteristics of students’ everyday language and gesture use in the context of turning motions and mismatches between these and the formal descriptions provided by the introduced system of gestures and the formal languages of mathematics and Logo. As a consequence, the intended use of playing turtle as scaffolding to support students’ development of the formal description of motion in 3D was less effective than hoped. In Morgan and Alshwaikh (2008) we suggested that this was due to the difference in the orientations of teachers/researchers and students toward the relationship between the gesturing hand and the turtle itself, imaging the turtle trajectory or imagining its position and heading. While the physical and cognitive difficulties involved in imaging, putting oneself in the place of the turtle moving in a 3D space, may play an important part, in this article we have presented an analysis of characteristics of everyday discourses of turning and 3D directionality that suggests some strong noncognitive explanations for the ways students’ use of the introduced system of gestures differed from the formal expectations of the teachers and researchers.

These explanations are related to the different structures of the field of motion in 3D space as it is realized in everyday discourse and in specialized discourse, summarized in Table 5. Whereas most of the studies of differences between specialized and everyday language in mathematics education cited in the review of literature earlier in this article have focused on the reference of individual words or sets of words, our analysis attempts to explain differences between teacher and student communications by considering the different ways of experiencing the field that are construed by the specialized and everyday discourses. This approach is consonant with Walkerdine’s (1988) explanation of differences in the use of more and less. In that case, the specialized discourse assumes that these terms are symmetrically opposite and that their use is essentially the same in all contexts and for all language users. Walkerdine argued that these assumptions are unfounded: most young children’s experience of more is primarily in the context of interactions about distribution of goods, a context in which, unlike in school mathematics, more is opposed to no more rather than to less and in which the material and affective significance of such interactions is likely to be stronger for children and parents from less affluent families. Similarly in the context of 3D motion considered in this article, it has been important to identify not just differences between the uses of key terms but also differences in the structures of students’ everyday experiences of the field, recognizing that, unlike specialized discourse, which assumes a high level of generalizability across contexts, everyday discourse varies significantly between different concrete contexts.

In addressing Kress and colleagues’ (2001) challenge to determine the interests motivating students’ use of particular forms of representation, we believe that our analysis of the differences between the structures of specialized and everyday discourses in the field of 3D motion offers an explanation of student “errors” that does not assume cognitive difficulties. We contend that such an approach, studying the discourse of the field as a whole rather than the use of isolated signs, has potential to offer insights into difficulties in other areas of mathematics. Of course, in this study we have not addressed the question of how teachers might address the kinds of differences the analysis identifies between specialized and everyday discourses. However, developing knowledge of differences between discourses and understanding the complexities of communication seem likely to support the development of strategies for improving teacher-student communication and student acquisition and use of specialized discourses. The need for teachers to be aware of differences between the specialized vocabulary of mathematics and the everyday language students bring to the classroom is well established as an essential part of pedagogic content knowledge. The analysis we have offered here demonstrates that there can also be differences between specialized mathematical and everyday use of a wider range of multimodal forms of communication. Awareness of this broader conception of communication may provide teachers with additional means of interpreting students' interactions when these do not conform with the expected specialized discourse.

Recent interest in the gestural mode as part of teaching and learning mathematics is certainly to be welcomed as part of our increased awareness of the multimodal nature of communication and representation. The study we have reported here has attended to a very small part of the potential use of gestures, located in the specific small field of movement in 3D space. Nevertheless, we feel that issues raised by our analysis of the ways differences between teachers/researchers' and students' use of gestures appear to relate to differences between everyday and specialized discourses are likely to be of wider concern. Whereas there is already a body of work alerting us to differences between everyday and mathematical use of verbal language, our knowledge of the ways gestures may be used is much less developed and demands more investigation. While there are some commonly recognized (conventional and shared) sets of gestures (e.g., deictic gestures used in the context of measurement, see Alshwaikh, 2011), many others, including those used in mathematical contexts, appear to be developed idiosyncratically and with only local applicability. Whether developed self-consciously like the system of gestures discussed in this article or developed more organically as part of student-teacher interaction, it is not safe to assume that students' and teachers' uses of apparently similar gestures arise from the same "interests" and have the same relationship to mathematical discourse. By paying close attention to the ways students use gestures and integrate them into their multimodal communication (alongside speech, writing, drawing, etc.) it is possible to appreciate more fully how they are drawing on the resources of everyday or of mathematical discourse and hence to understand the effectiveness or ineffectiveness of the gestures as a means of mathematical communication.

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