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This paper presents a new technique to estimate the transition probabilities used in the Markovian-based pavement performance prediction models. The proposed technique is based on the 'back-calculation' of the discrete-time Markov model using only two consecutive cycles of pavement distress assessment. The transition probabilities, representing the pavement deterioration rates, are the main elements of the Markov model used in predicting future pavement conditions. The paper also presents a simplified procedure for evaluating the pavement state of distress using the two major pavement defect groups, namely cracking and deformation. These two defect groups are to be identified and evaluated for pavement sections using visual inspection and simple linear measurements. The extent of these two major defect groups is measured using the defected pavement areas (or lengths) and the defect severity is measured based on the average crack width and average deformation depth. A case study is presented to demonstrate the 'back-calculation' of transition probabilities. In particular, the impacts of the pavement section length on the distress rating and on the estimation of the transition probabilities have been investigated. The results have indicated that the estimated transition probabilities become highly unstable as the section length gets larger and the sample size becomes smaller.

Keywords: flexible pavement; pavement performance; distress assessment; Markovian processes; pavement management

Introduction

Pavement performance is recognised as a vital element in the analysis and design of pavement structures (Haas *et al.* 1994, Shahin 1994, Huang 2004). Pavement performance is also used in several pavement rehabilitation and management applications (Haas *et al.* 1994, Shahin 1994, Mills *et al.* 2012, Shah *et al.* 2013). Pavement performance is usually defined by a means of performance curve that depicts the trend between the pavement distress condition and service time or accumulated load applications. Figure 1 shows a typical pavement performance curve. The condition rating of a pavement structure at a given time can be assessed using three well-known performance indicators including the pavement condition index (PCI), present serviceability index (PSI) and international roughness index (IRI). In addition, pavement surface deflection measurements can be used to evaluate pavement performance using expensive instruments such as the Dynaflect or falling weight deflectometer. However, pavement deflections have mainly been used for calculating the overlay design thickness at the project-level using back-calculation of the multilayer linear elastic theory (Zaghloul and Elfino 2000, Wu *et al.* 2008).

The PCI (or rating) is estimated based on visual inspection of pavement defects and simple related measurements using a scale of 100 points with higher values indicating better pavement conditions (Shahin 1994, Mills et al. 2012). The PCI has been mainly used in pavement management applications. The PSI was originally deployed in the AASHO Road Test to assess pavement performance. It is rated using a scale of 5 points with higher values indicating better pavement. The PSI is a function of the slope variance associated with the roadway longitudinal profile and pavement cracking and deformation (AASHTO 1993, Huang 2004, Shah *et al.* 2013). The PSI was used by American Association of State Highway and Transportation Officials (AASHTO) in all its empirical pavement design guides. The IRI is developed mainly from the assessment of the roadway longitudinal profile using the riding quality test. It is typically defined using the unit of meter per lane kilometer. Several researchers have developed regression-based models to estimate the PSI using solely the IRI measurements (Paterson 1986, Al-Omari and Darter 1994). The PSI has been extensively used in pavement rehabilitation and management applications such as estimating the required overlay design thickness and performing pavement life-cycle analysis (Hall *et al.* 1992, Abaza 2002).

Assessment of pavement performance forms the basis for developing and implementing pavement performance prediction models. Pavement performance has been recognised to be probabilistic in nature, and thus, stochastic-based models have been used to predict future pavement conditions. The Markov model with discrete-time intervals (transitions) was extensively used before in

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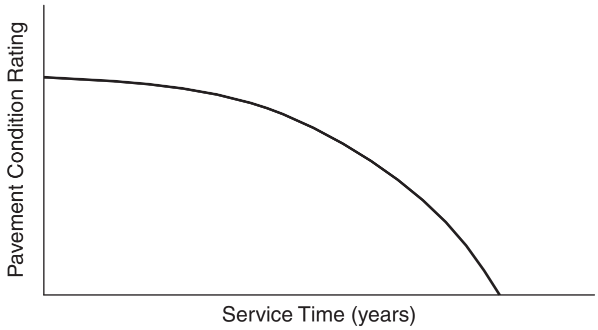

Figure 1. Typical pavement performance curve.

modelling pavement performance (Butt *et al.* 1987, Li *et al.* 1996, Abaza and Ashur 1999, Mandiartha *et al.* 2012). The outcomes of the pavement performance assessment are used in estimating the state probabilities and transition probabilities deployed by the Markov model. An effective pavement performance prediction model is considered to be an essential component of any modern pavement management system. Several advanced pavement management systems have incorporated a stochastic-based model to develop an optimum long-term pavement maintenance and rehabilitation (M&R) plan at the network level (Ferreira *et al.* 2002, Reigle and Zaniewski 2002, Abaza and Murad 2007).

Therefore, a reliable procedure for pavement performance assessment is the key for the successful implementation of an effective pavement management model. The procedure must be effective but yet simple to be applied at the network level. The proposed simplified procedure aims to assess the structural integrity of a pavement structure using mainly the two major groups of pavement defects, namely cracking and deformation. Two different assessment approaches will be presented using the length and area as two different parameters to measure the extent of prevailing pavement defects in a given pavement section. The severity of pavement defects will be assessed using severity factors to be assigned the ratings of 1, 2 and 3 for low, medium and high severity levels, respectively. The two assessment approaches will be applied to a sample pavement project with results to be analysed and compared. In particular, the impact of the pavement section length on estimating relevant stochastic-based parameters will be investigated. These stochastic parameters include the state probabilities and transition probabilities. The state probabilities at any given time can be estimated from one cycle of pavement distress assessment while two cycles are required to estimate the transition probabilities.

Overview of performance assessment procedures

Several procedures were developed and implemented in the past to evaluate flexible pavement performance for pavement management purposes. These assessment procedures attempt to evaluate the pavement condition using visual inspection of pavement distress, measurement of the roadway riding quality, or a combination of the these two parameters. Visual inspection of pavement distress typically requires selecting pavement sections that are small in length ranging from 50 to 100 m lane length. Each section is individually inspected and assigned a distress rating (DR) determined according to a specified formula that takes into consideration the ratings and weights assigned to various pavement distresses known as defects. Typically, distress data are obtained by trained engineers who can make subjective judgments about pavement condition based on predetermined factors. Distress data are usually obtained on a regular schedule about once every 1–2 years.

A pavement distress assessment procedure was developed by the World Bank and Organization for Economic Co-operation and Development to be used mainly by developing countries (World Bank 1990). The procedure involves performing what is called detailed visual inspection of several types of pavement distresses. The procedure requires identifying prevailing pavement distresses for a given pavement section; assigning a level of severity for each distress using low, medium and high, and assigning a level of extent for each distress using three classes as defined in the relevant roadway maintenance management manual. Then, for each inspected distress, a relevant chart in the manual assigns a DR on a scale of 1–5 with lower ratings indicating better pavement conditions. An overall pavement condition rating (PCR) can be obtained for each pavement section which is equal to the sum of all DRs assigned to the various prevailing pavement distresses. Shahin (1994) also outlined a procedure to obtain a similar indicator called the PCI but assigns higher ratings for better pavements.

Generally, the results obtained from the distress assessment for a particular pavement section are condensed into a single number called a DR. A perfect pavement is usually assigned a score of 100; however, if pavement distresses (defects) are observed then points are subtracted as indicated in Equation (1). Equation (1) estimates the DR for each pavement section based on the defect rating (d_i) and the corresponding relative weight (w_i) assigned to yield a section rating on a scale of 100 points (Garber and Hoel 2002). The d_i represents the number of points subjectively assigned to the ith defect using an appropriate scale.

$$\mathrm{DR} = 100 - \sum_{i=1}^{n} w_i d_i. \tag{1}$$

Currently, several highway agencies around the world are using the IRI to evaluate pavement performance. The development of the IRI was sponsored by the World Bank to provide a common basis for conducting and comparing roughness measurements. The IRI provides a summary measure of the longitudinal surface profile obtained from the surface elevation data collected using a mechanical profilometer (Sayers *et al.* 1986, Sayers 1995). The World Bank published guidelines for conducting and calibrating roughness measurements (Sayers *et al.* 1986). The IRI can be used in pavement rehabilitation and management applications; however, its use is only limited to those agencies that can acquire and calibrate a suitable mechanical profilometer (Byram *et al.* 2012, Zhou *et al.* 2013).

Current implementation of the mechanistic–empirical pavement design guide (MEPDG) requires evaluating pavement performance for the purpose of validating and calibrating the relevant performance prediction models. Several pavement performance indicators have been used in this regard as suggested by the MEPDG. For example, the state of Tennessee used the PSI, rutting and roughness (IRI) (Zhou *et al.* 2013), the state of Arkansas deployed several distresses including alligator cracking, rutting and roughness (IRI) (Byram *et al.* 2012), and the state of Louisiana adopted similar performance indicators to develop rutting and load-related fatigue cracking models for the purpose of implementing the MEPDG (Wu and Yang 2012).

Simplified distress assessment procedure

The proposed simplified procedure requires inspecting two major groups of flexible pavement defects, namely cracking and deformation. Generally, pavement defects can be classified using two main broad categories (Haas *et al.* 1994, Shahin 1994, Garber and Hoel 2002, Huang 2004). The first category of pavement defects includes those that have an immediate impact on the pavement surface condition and profile, thus affecting the roadway functional operation. They include polishing, bleeding, and ravelling and weathering of pavement which impact the roadway surface condition, consequently affecting the roadway functional operation. For example, polishing undermines the skid resistance of pavement surface which can impose a safety hazard. At advanced stages, bleeding and ravelling can weaken the structural capacity of the pavement structure, thus accelerating pavement deterioration. Both bleeding and ravelling are classified as non-load-associated defects, whereas polishing is considered as the load-associated defect. In addition, all defects associated with deformation which result in the distortion of the pavement surface profile are considered to be functional defects. They include corrugation (ripples) and depression (settlement) classified as non-load-associated, and rutting designated as load-associated.

The second category of pavement detects are those defects that directly undermine the structural integrity of the pavement structure. Undermining the structural integrity means that the pavement structure is no longer intact but it is broken into parts, and consequently, the structural capacity (soundness) of the pavement structure is reduced. Alligator or fatigue cracking is the major defect in this category and considered to be load-associated. Potholes can result from Alligator cracking once it reached an advanced stage of deterioration. Block cracking, longitudinal and transverse cracking, and slippage cracking are also examples in this category as they are all considered structural defects resulting in the weakening of the structural capacity of the pavement structure. Block cracking is considered as non-load-associated, whereas the others are classified as load-associated.

The proposed assessment procedure mainly requires performing visual inspection and simple linear measurements of all pavement defects associated with cracking and deformation. However, the procedure is simplified so that all cracking defects are considered as one group in the assessment and similarly for the deformation defects. Each defect group is evaluated based on its extent and severity. The extent of each defect group is measured using either the defected area or length to be subtracted from the area or length of the pavement section. The severity of each defect group will be evaluated using three levels, namely low, medium and high, depending on predefined criteria. The severity level for cracking is dependent on the average crack width (ACW) and it is reliant on the average deformation depth (ADD) for deformation. The severity levels can be assigned according to the criteria defined in Table 1. Table 1 also provides the values of the cracking and deformation severity factors (SF_C and SF_D) assigned according to the severity levels to be used in the calculation of the areas and lengths contributing to the estimation of the pavement section DR.

Area-based distress assessment approach

The pavement section is to be inspected for cracking and deformation defects. The defected areas need to be identified and measured. Each defected area can only be considered either as cracked or deformed area depending on the severity of each as the one with higher severity will be counted. Each counted defected area will be multiplied by its corresponding severity factor assigned according to

Table 1. Definition of severity levels and severity factors.

Severity level	ACW (mm)	SF_C	ADD (cm)	SF_D
Low	<2	1	<2	1
Medium	2–5	2	2–5	2
High	>5	3	>5	3

the severity level as defined in Table 1 to yield the corresponding factored defected area. The total factored defected area contributing to the estimation of the pavement DR will be as defined in Equation (2). The total factored defected area cannot be more than three times the area of the pavement section as the maximum value for the severity factor is 3.

$$\mathrm{FA_T} = \mathrm{FA_C} + \mathrm{FA_D} = \sum \mathrm{SF_C} A_\mathrm{C} + \sum \mathrm{SF_D} A_\mathrm{D} \le 3A_\mathrm{S}, \quad (2)$$

where $\sum A_\mathrm{C} + \sum A_\mathrm{D} \le A_\mathrm{S}$, $\mathrm{FA_T}$ = total factored defected area that contributes to the estimation of the DR, $\mathrm{FA_C}$ = total factored cracked area that contributes to the estimation of the DR, $\mathrm{FA_D}$ = total factored deformed area that contributes to the estimation of the DR, $\mathrm{SF_C}$ = severity factor associated with cracking obtained from Table 1, A_C = a cracked area that contributes to the estimation of the DR, $\mathrm{SF_D}$ = severity factor associated with deformation obtained from Table 1, A_D = a deformed area that contributes to the estimation of the DR and A_S = area of the pavement section selected as part of a travel lane.

The two groups of cracking and deformation defects used in Equation (2) are basically given equal weights. However, different weights can be assigned, if so desired, which can be done by introducing to Equation (2) two different weights for cracking and deformation (W_C and W_D) as presented in Equation (3). In a similar way, a third defect group can be introduced to Equation (3) to include all defects related to the pavement surface texture such as polishing, bleeding, and ravelling and weathering. For example, if it is mainly desired to evaluate the pavement structural integrity, then it might be appropriate to give cracking defects higher weight than deformation defects and similarly to give deformation defects higher weight than surface texture defects.

$$\mathrm{FA_T} = W_\mathrm{C} \sum \mathrm{SF_C} A_\mathrm{C} + W_\mathrm{D} \sum \mathrm{SF_D} A_\mathrm{D} \le 3W(A_\mathrm{S}), \quad (3)$$

where W = maximum of W_C and W_D.

The pavement DR associated with a particular pavement section can now be computed for defect groups with equal weights using Equation (4). The total factored defected area is simply subtracted from its maximum value to yield a percentage value called as the pavement DR. The pavement DR as computed from Equation (4) can take on a maximum value of 100 when the entire pavement section area is free of any defects and a minimum value of 0 when the entire pavement section area is defected with a high level of severity. Equation (5) is equivalent to Equation (4) but for defect groups with unequal weights.

$$\mathrm{DR} = \left(\frac{3A_\mathrm{S} - \mathrm{FA_T}}{3A_\mathrm{S}}\right) \times 100, \quad (4)$$

where $\mathrm{FA_T}$ is as computed using Equation (2)

$$\mathrm{DR} = \left(\frac{3W(A_\mathrm{S}) - \mathrm{FA_T}}{3W(A_\mathrm{S})}\right) \times 100, \quad (5)$$

where $\mathrm{FA_T}$ is as computed using Equation (3).

Length-based distress assessment approach

Another approach for evaluating the various deployed defect groups is based on the defected pavement length. This approach is faster than the previous one as the inspector needs only to identify and measure the defected lengths. For a two-lane highway, this approach has minimal disruption to the travelling public as the inspector can do the work from the shoulder side without any lane-closure arrangements. However, for the length-based approach to be compatible to the area-based approach, the pavement lane section is to be divided into two longitudinal subsections with equal width. Each longitudinal subsection is to be individually inspected. Essentially, each longitudinal subsection represents the travel path of one wheel. It is also not uncommon practice that the pavement belonging to one of the two subsections is separately rehabilitated. The DR associated with this approach is computed for each subsection as indicated by Equation (6) which is derived in a similar way as outlined for Equation (5) but the parameters associated with the pavement section area are replaced by those related to the pavement section length.

$$\mathrm{DR} = \left(\frac{3W(L_\mathrm{S}) - W_\mathrm{C} \sum \mathrm{SF_C} L_\mathrm{C} - W_\mathrm{D} \sum \mathrm{SF_D} L_\mathrm{D}}{3W(L_\mathrm{S})}\right) \times 100, \quad (6)$$

where $\sum L_\mathrm{C} + \sum L_\mathrm{D} \le L_\mathrm{S}$.

Generally, the area-based approach is expected to give more reliable assessment of the pavement distress condition; however, the area measurement requires more time and effort than the length measurement. An improvement to Equation (6) can be made by introducing a second defect factor called the extent factor (EF). The EF is specified according to the defect transverse spread estimated as a percentage of the subsection width. It is assigned the value of 1, 2, or 3 for a width spread percentage of less than 1/3, 1/3–2/3 or more than 2/3, respectively. The EF along with the defected length essentially compensate for the defected area used as a measure of extent in the area-based approach. The inclusion of the EFs ($\mathrm{EF_C}$) and ($\mathrm{EF_D}$) for cracking and deformation, respectively, results in the development of Equation (7). This modified length-based equation is expected to yield results similar to those obtained from the area-based approach.

The DR value as estimated from the length-based approach for a pavement lane section is then computed as the average value of the DR values estimated for the two corresponding longitudinal subsections. The two presented performance assessment approaches are expected to minimise the role of subjectivity typically encountered with other similar performance assessment procedures.

$$\mathrm{DR} = \left(\frac{6W(L_\mathrm{S}) - W_\mathrm{C}\sum(\mathrm{EF_C}+\mathrm{SF_C})L_\mathrm{C} - W_\mathrm{D}\sum(\mathrm{EF_D}+\mathrm{SF_D})L_\mathrm{D}}{6W(L_\mathrm{S})}\right)\times 100. \quad (7)$$

Back-calculation of transition probabilities

The main elements of the homogenous discrete-time Markov model are the number of condition states, number of transitions, time interval between transitions, state probabilities and transition probabilities. The corresponding Markov model as presented in Equation (8) is generally used to determine the state probabilities after (n) transitions provided that the initial state probabilities and transition probabilities are known. The elements of the transition matrix (P) are the transition probabilities which are assumed to be equal for each transition, thus, indicating a homogenous Markovian chain. The transition matrix can be different for each transition if a non-homogenous chain is to be used (Butt *et al.* 1987, Abaza and Ashur 1999, Abaza and Murad 2007).

$$S^{(n)} = S^{(0)}P^{(n)}, \quad (8)$$

where $S^{(n)}$ = column vector representing state probabilities after (n) transitions, $S^{(0)}$ = row vector representing initial state probabilities and $P^{(n)}$ = transition matrix raised to the nth power.

Estimation of the transition probabilities has been traditionally done using two different approaches. The first approach requires collecting pavement distress data over a number of years and then using an appropriate method to estimate the corresponding deterioration rates (i.e. transition probabilities). The second approach requires the use of a panel of experts to estimate the transition probabilities. Examples among the first approach are: (1) Butt *et al.* (1987) presented a method that minimises the sum of residuals generated from PCRs collected over a number of years. The residuals were defined as the differences between the observed condition ratings and their predicted ones derived from the Markov model, (2) Mishalani and Madanat (2002) developed a probabilistic-based model to estimate the transition probabilities from the time spent (duration) in a given condition state, (3) Abaza (2004) proposed a deterministic-based approach to estimate the transition probabilities based on the time durations spent in the various deployed states as derived from the performance curve generated using the AASHTO model and (4) Ortiz-Garcia *et al.* (2006) proposed three different methods to estimate the transition probabilities essentially based on the minimisation of residuals. The first method requires the availability of original pavement condition data, the second method uses a regression curve generated from the original data and the third one assumes that the yearly distributions of pavement condition are available.

The technique proposed by this paper to estimate the transition probabilities is much simpler to apply compared to the previously outlined methods and its requirement for pavement condition data is minimal. Estimation of the state probabilities requires one cycle of pavement distress assessment. The state probabilities represent the proportions of pavement in the various deployed condition states. However, estimation of the transition probabilities requires two consecutive cycles of pavement distress assessment separated by one time interval (i.e. transition) typically chosen to be equal to one or two years. The transition probability represents the probability of pavement transition from one state to another during one time interval. The transition probabilities can then be derived using a 'back-calculation' of the discrete-time Markov model presented in Equation (8). The Markov model for two consecutive transitions is presented in Equation (9). The initial state probabilities and state probabilities after one transition are required to be able to backwardly solve the Markov model for the corresponding transition probabilities. It is typically assumed that any row in the transition matrix (P) only contains the two transition probabilities (Pi,i) and ($Pi,i+1$) in the absence of any M&R works (Butt *et al.* 1987, Abaza and Ashur 1999, Abaza and Murad 2007). It is to be noted that the sum of any row in the transition matrix must equal to one.

$$S^{(1)} = S^{(0)}P^{(1)} \quad (9)$$

where

$$S^{(1)} = \left(S_1^{(1)}, S_2^{(1)}, S_3^{(1)}, \ldots, S_m^{(1)}\right)$$

$$S^{(0)} = \left(S_1^{(0)}, S_2^{(0)}, S_3^{(0)}, \ldots, S_m^{(0)}\right)$$

$$P = \begin{pmatrix} P_{1,1} & P_{1,2} & 0 & 0 & 0 & \dots & 0 \\ 0 & P_{2,2} & P_{2,3} & 0 & 0 & \dots & 0 \\ 0 & 0 & P_{3,3} & P_{3,4} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & 0 & P_{m-1,m-1} & & P_{m-1,m} \\ 0 & 0 & 0 & 0 & 0 & \dots & P_{m,m} \end{pmatrix}$$

$$0 \le P_{i,i} \le 1.0, \quad 0 \le P_{i,i+1} \le 1.0 \quad P_{i,i} + P_{i,i+1} = 1.0$$
$$P_{m,m} = 1.0,$$

where $S^{(1)}$ = column vector representing state probabilities after one time interval (transition), $S^{(0)}$ = row vector representing initial state probabilities and $P^{(1)}$ = transition matrix with (m) condition states raised to the first power.

The first transition probability ($P_{1,2}$) can be obtained from multiplying the initial state probability row vector by the first column in the transition matrix. The result of this multiplication product is equal to the state probability $\left(S_1^{(1)}\right)$ associated with state (1) at the end of the first transition. The transition probability ($P_{1,2}$) is then derived as indicated by Equation (10a). Equation (10c) is similarly derived to estimate the last transition probability ($P_{m-1,m}$) from multiplying the initial state probability row vector by the last column in the transition matrix. The other remaining transition probabilities ($P_{i,i+1}$) can be derived in a similar way using a recursive approach with the final result presented in Equation (10b). Equation (10b) is recursively solved as the transition probability ($P_{i,i+1}$) is dependent on the preceding transition probability ($P_{i-1,i}$). Equation (10b) is also valid for determining the last transition probability ($P_{m-1,m}$). The initial state probability $\left(S_1^{(0)}\right)$ associated with state (1) is typically larger than its value after one transition $\left(S_1^{(1)}\right)$ in the absence of any M&R works. Similarly, the initial state probability $\left(S_m^{(0)}\right)$ associated with state (m) is smaller than its value after one transition $\left(S_m^{(1)}\right)$ since all pavements will eventually terminate in this trapping state.

$$P_{1,2} = \frac{S_1^{(0)} - S_1^{(1)}}{S_1^{(0)}}, \quad S_1^{(0)} \ge S_1^{(1)}, \tag{10a}$$

$$P_{i,i+1} = \frac{S_i^{(0)} - S_i^{(1)} + S_{i-1}^{(0)} P_{i-1,i}}{S_i^{(0)}}, \quad (i = 2, 3, \dots, m-1), \tag{10b}$$

$$P_{m-1,m} = \frac{S_m^{(1)} - S_m^{(0)}}{S_{m-1}^{(0)}}, \quad S_m^{(1)} \ge S_m^{(0)}. \tag{10c}$$

The initial state probabilities and state probabilities after one transition can be estimated using Equation (11a) and (11b), respectively. The state probability can simply be defined as the ratio of the number of pavement sections in state (i) to the total number of pavement sections used in the study. Once the state probabilities are estimated from Equation (11), then the corresponding transition probabilities can be computed using Equation (10).

$$S_i^{(0)} = \frac{N_i^{(0)}}{N_\mathrm{T}} \quad (i = 1, 2, \dots, m), \tag{11a}$$

$$S_i^{(1)} = \frac{N_i^{(1)}}{N_\mathrm{T}} \quad (i = 1, 2, \dots, m), \tag{11b}$$

where $N_\mathrm{T} = \sum_{i=1}^{m} N_i^{(0)} = \sum_{i=1}^{m} N_i^{(1)}$, $S_i^{(0)}$ = initial state probability associated with state (i), $S_i^{(1)}$ = state probability associated with state (i) after one transition, $N_i^{(0)}$ = initial number of pavement sections in state (i), $N_i^{(1)}$ = number of pavement sections in state (i) after one transition and N_T = total number of pavement sections used in the study (i.e. sample size).

Alternatively, the transition probabilities can directly be obtained from Equation (12) using the numbers of pavement sections associated with the two consecutive cycles of pavement distress assessment. Equation (12) is simply derived from substituting in Equation (10) the values of the state probabilities as defined in Equation (11).

$$P_{1,2} = \frac{N_1^{(0)} - N_1^{(1)}}{N_1^{(0)}}, \quad N_1^{(0)} \ge N_1^{(1)}, \tag{12a}$$

$$P_{i,i+1} = \frac{N_i^{(0)} - N_i^{(1)} + N_{i-1}^{(0)} P_{i-1,i}}{N_i^{(0)}}, \quad (i = 2, 3, \dots, m-1), \tag{12b}$$

$$P_{m-1,m} = \frac{N_m^{(1)} - N_m^{(0)}}{N_{m-1}^{(0)}}, \quad N_m^{(1)} \ge N_m^{(0)}. \tag{12c}$$

The transition probabilities are expected to be different for every successive transition. In particular, the transition probabilities ($P_{i,i+1}$) representing the pavement deterioration rates are expected to increase overtime due to the growth in traffic loading and weakening of the pavement structure. However, Butt *et al.* (1987) indicated that a constant transition matrix can be used for a period up to five years without significantly affecting the prediction strength of the homogenous Markov model. This means at least a new set of transition probabilities should be estimated for every five-year period based on new pavement distress records. In this case, a staged-homogenous Markov model can be used as indicated by Equation (13) wherein a new transition matrix is introduced every time period (a).

$$S^{(n)} = S^{(0)} P_1^{(a)} P_2^{(a)}, \ldots, P_k^{(a)}, \tag{13}$$

where $k = n/a =$ number of transition matrices required over (n) transitions with each matrix raised to the ath power, and a = time period in years over which the same transition matrix is used provided that the period does not exceed 5 years.

The application of the staged-homogenous Markov model as presented in Equation (13) requires considerable time to develop adequate number of transition matrices for a given pavement structure. Therefore, the author is proposing a simple alternative to estimate the transition probabilities associated with the required transition matrices as defined in Equation (14) based on the first set of transition probabilities ($P_{i,i+1}$). The transition probabilities associated with the jth transition matrix, $P_{i,i+1}^{(j)}$, are estimated by multiplying the first set of transition probabilities by an appropriate constant (C). This constant has to be larger than one since the deterioration transition probabilities ($P_{i,i+1}$) are expected to increase overtime for the reasons mentioned earlier. For example, the values of the C constants can be $C_1 = 1.1$, $C_2 = 1.2$, $C_3 = 1.3$ and so on. Estimation of the C constants in this way requires the judgment and experience of the pavement engineer. Alternatively, reliable estimates of the C constants can be derived from the calibration process if pavement distress records are available over (n) transitions.

$$P_{i,i+1}^{(j)} = C_{j-1} P_{i,i+1} \quad (j = 2, 3, \ldots, k), \tag{14}$$

where $C_1 \leq C_2 \leq C_3 \leq \cdots \leq C_{j-1}$.

Sample presentation

The sample presentation section includes two subsections. The first one presents sample pavement distress assessment results obtained using both distress assessment approaches. The second one investigates the impact of pavement section length on stochastic parameters.

Sample pavement distress assessment results

The two presented pavement distress assessment approaches were applied to a 1.5 km lane segment that is part of a two-lane highway in the District of Nablus, Palestine. A pavement section of 10 m lane length and 3.5 m width was used in the distress assessment resulting in a total of 150 pavement sections. The field work involved was carried out by a trained civil engineer which took him one working day (8 h shift) to perform the assessment using the two approaches. Equation (4) was used to compute the DR values using the area-based approach with equal defect weights. Sample DR calculations are provided in Table 2 for a selected number of pavement sections. The ith defected area (A_i) within a particular pavement section is estimated as a rectangular area with length (L_i) and width (W_i). A defected area can either be counted as cracked (C) or deformed (D) area with the corresponding severity factor (SF_i) is assigned 1, 2 or 3 for low, medium or high severity rating, respectively, as defined in Table 1. Equation (2) was used to calculate the total factored defected area (FA_T) assuming equal defect weights.

Similarly, Equation (7) was used to determine the DR values using the length-based approach but with equal defect weights. According to the length-based approach, each pavement section was evaluated as two longitudinal subsections with equal width. The DRs for the left subsection and right subsection are averaged out to yield the DR for the corresponding pavement section. The average DRs associated with the left subsections and right subsections are 46.4 and 38.6, respectively, indicating that the right wheel path is worse than the left wheel path which might be contributed to deficiencies in the roadside drainage. Also, as one would expect, Alligator cracking and depression were often encountered to occupy the same pavement spots and the one with a higher severity was counted.

Figure 2 shows the DR values obtained from both approaches for the 150 inspected pavement sections.

Table 2. Sample DR calculations using area-based assessment approach ($A_s = 35\ \mathrm{m}^2$).

Station (m)		Defected area details						
From	To	Defect type (i)	L_i (m)	W_i (m)	A_i (m^2)	SF_i	FA_T	DR
50	60	C	5.25	2.55	13.39	3	40.17	61.74
240	250	C	3.45	3.35	11.56	2	61.56	41.37
		C	4.6	3	13.8	2		
		D	1.75	3.1	5.42	2		
450	460	C	5.3	3.5	18.55	3	88.91	15.32
		D	2.25	2.85	6.41	3		
		D	0.9	2.25	2.02	2		
		C	1.55	2.15	3.33	3		
690	700	C	2.65	1.15	3.05	1	5.33	94.92
		D	1.2	0.95	1.14	2		
970	980	C	5.15	2.6	13.39	3	69.86	33.47
		C	1.2	2.85	3.42	2		
		C	1.35	3.5	4.72	3		
		D	0.85	1.75	1.49	1		
		D	1.5	1.6	2.4	3		

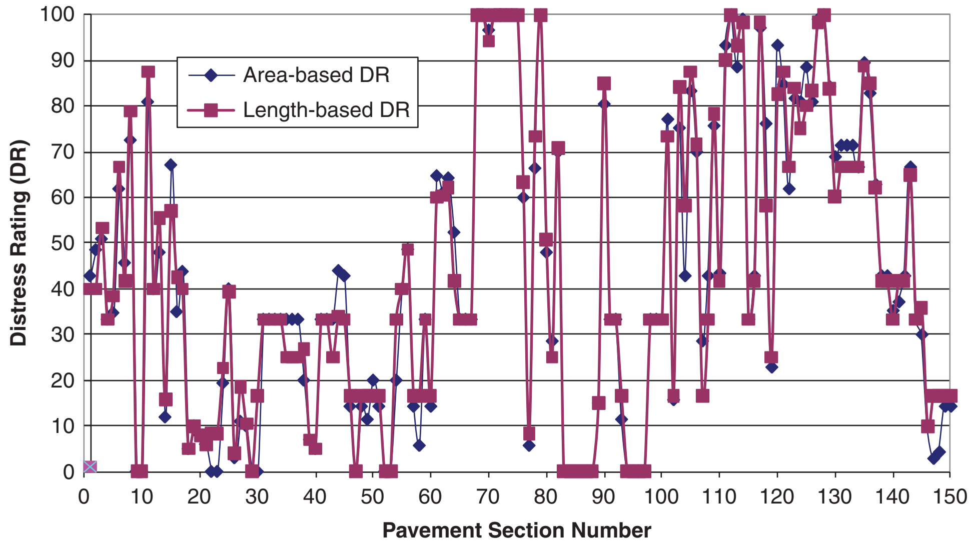

Figure 2. Sample pavement DR using area-based and length-based approaches.

Generally, Figure 2 shows the pavement DR to be highly variable over the entire surveyed pavement lane segment which supports the claim that pavement performance is probabilistic in nature. It can also be noticed that there is a close agreement between the DR values obtained from the area-based approach and length-based approach. The overall average DR values for the 150 sections as obtained from the area-based approach and length-based approach are 42.1 and 42.5, respectively; however, the individual section DR values estimated from both approaches are not that close.

Figure 3 shows the differential DR for each pavement section computed as the difference between the DR estimated from the area-based approach (DR_A) and the corresponding one obtained from the length-based approach (DR_L). Figure 3 shows that while many pavement sections are assigned the same DR from both approaches, there are several others that are associated with different DR ratings. The same DR values have been mainly obtained for pavement sections with either perfect condition (DR = 100) or worst condition (DR = 0.0). These are the two cases wherein the two approaches will be perfectly compatible. The differential DR as indicated by Figure 3 varies from about −17 to 18 points. However, the overall average differential DR is −0.27 with a 5.12 standard deviation. These two values become equal to 3.19 and 4.00 when the absolute values of all differential DRs are considered. Even these values are still substantially low when considering the large sample size of 150 pavement sections. A relevant test of statistical hypothesis

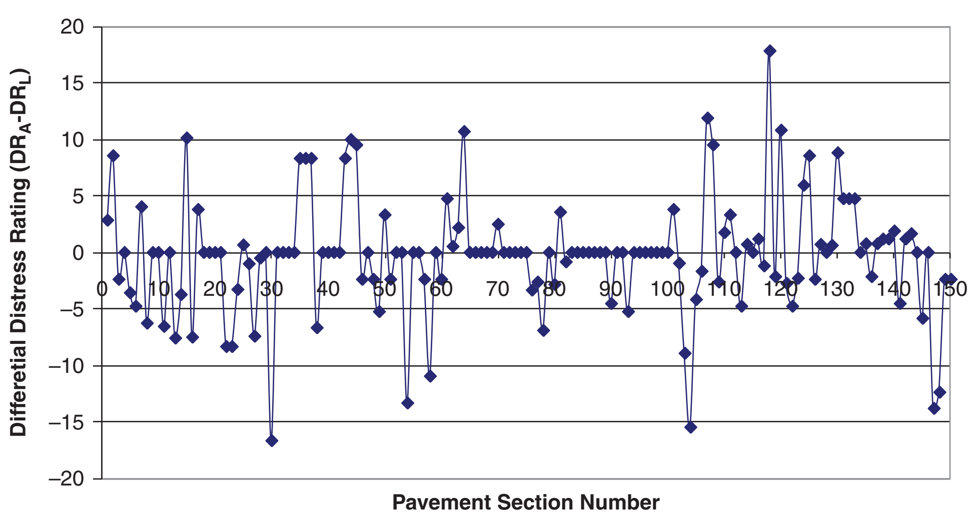

Figure 3. Sample differential DR obtained from the area-based and length-based approaches.

Table 3. Numbers of pavement sections assigned to five DR ranges using variable pavement section length (L_s).

DR range	Number of pavement sections (N_i)	$L_s = 10$ (m)		$L_s = 30$ (m)		$L_s = 50$ (m)		$L_s = 100$ (m)	
		M_A	M_L	M_A	M_L	M_A	M_L	M_A	M_L
100–80	N_1	28[a]	28	5	5	4	4	1	1
		23[b]	22	4	4	2	2	1	1
80–60	N_2	20	20	8	8	3	3	4	4
		18	18	7	7	3	3	3	3
60–40	N_3	23	23	11	12	8	8	1	1
		21	22	9	10	7	6	1	1
40–20	N_4	33	33	14	15	6	7	5	6
		30	34	13	13	7	6	4	4
20–00	N_5	46	46	12	10	9	8	4	3
		58	54	17	16	11	13	6	6
Total number of sections (N_T)		150	150	50	50	30	30	15	15
		150	150	50	50	30	30	15	15
Overall average DR		42.1	42.5	42.1	42.5	42.1	42.5	42.1	42.5
		37.3	37.6	37.3	37.6	37.3	37.6	37.3	37.6

Notes: M_A: area-based approach; M_L: length-based approach.
[a] Results from the first cycle.
[b] Results from the second cycle.

would result with very high confidence level in the decision that there is no difference in the DR means associated with the two distress assessment approaches. This can mainly be attributed to the use of the EFs in the modified length-based equation, making it quite compatible to the area-based equation.

Impact of section length on stochastic parameters

The main stochastic parameters used in Markovian prediction models are the state probabilities and transition probabilities. The state probabilities can be estimated from one cycle of pavement distress assessment; however, two assessment cycles are required to estimate the transition probabilities. The Markov model requires specifying the number of condition states to be used in the prediction process. The condition states are typically defined using equal ranges of DR values. For example, the DR ranges of 100–80, 80–60, 60–40, 40–20 and 20–00 can be used for a Markov model with five condition states. Then, each pavement section will be assigned to a specific condition state based on its DR.

The numbers of pavement sections assigned to each condition state (i.e. DR range) have been determined using variable pavement section length (L_s). Two cycles of pavement distress assessment were conducted with the first cycle taken place in April 2011, and the second cycle a year later. The sample pavement distress assessment was carried out using pavement sections with 10 m lane length. The DRs for pavement sections with larger length can be essentially computed from the DRs associated with the 10-m pavement sections. For example, the DRs associated with five 10-m consecutive sections are averaged out to yield the DR for the corresponding section with 50 m lane length. Table 3 provides the numbers of pavement sections (N_i) assigned to the five outlined DR ranges using variable pavement section length (L_s). Table 3 indicates that the sample size (N_T) becomes smaller as the section length becomes larger. The numbers of pavement sections are obtained for both the area-based (M_A) and length-based (M_L) approaches, which are quite similar. Table 3 also indicates that the overall average DR value for each cycle remains unchanged for the entire pavement segment regardless of the pavement section length.

The state probabilities associated with the two cycles of pavement distress assessment are estimated using Equation (11) based on the numbers of pavement sections provided in Table 3. The corresponding state probabilities (S_1 through S_5) are provided in Table 4 for the five previously outlined condition states considering variable pavement section length. It can be noticed that the state probabilities estimated using the area-based (M_A) and length-based (M_L) approaches are very much similar when considering the same pavement section length. The first state probability (S_1) values associated with the second cycle are smaller than the corresponding ones associated with the first cycle as it would be expected in the absence of M&R works. Similarly, the fifth state probability (S_5) values associated with the second cycle are larger than the equivalent ones associated with the first cycle. It can also be noticed that the state probabilities become relatively unstable as the section length increases. Generally, substantial differences have often occurred in the state probability values when increasing the section length.

Table 4. State probabilities obtained using variable pavement section length.

	S_1		S_2		S_3		S_4		S_5	
Pavement section length (L_s)	M_A	M_L	M_A	M_L	M_A	M_L	M_A	M_L	M_A	M_L
10	0.187[a]	0.187	0.133	0.133	0.153	0.153	0.22	0.22	0.307	0.307
	0.153[b]	0.147	0.12	0.12	0.14	0.147	0.2	0.226	0.387	0.36
30	0.1	0.1	0.16	0.16	0.22	0.24	0.28	0.3	0.24	0.2
	0.08	0.08	0.14	0.14	0.18	0.2	0.26	0.26	0.34	0.32
50	0.133	0.133	0.1	0.1	0.267	0.267	0.2	0.233	0.3	0.267
	0.067	0.067	0.1	0.1	0.233	0.2	0.233	0.2	0.367	0.433
100	0.067	0.067	0.267	0.267	0.067	0.067	0.333	0.4	0.267	0.2
	0.067	0.067	0.2	0.2	0.067	0.067	0.267	0.267	0.4	0.4

Notes: M_A: area-based approach; M_L: length-based approach.
[a] Results from the first cycle.
[b] Results from the second cycle.

Of course, one would expect a higher reliability in the estimated state probabilities when using a shorter section length.

The transition probabilities can be estimated using a back-calculation of the Markov model as presented earlier. The state probabilities obtained from the two conducted cycles of pavement distress assessment have been used to estimate the transition probabilities as indicated by Equation (10). Table 5 provides the estimated four transition probabilities obtained from the area-based (M_A) and length-based (M_L) approaches with variable pavement section length. It can be noticed that there are now some notable differences in the estimated transition probabilities when considering the area-based (M_A) and length-based approaches (M_L) for the same section length, which can be attributed to the law of error propagation. However, the differences in the transition probabilities become much larger when using larger section length. This is because the transition probabilities become highly unstable with the increase in the section length as the sample size (N_T) involved in the study becomes small. Examples of that are when using 50 and 100 m section lengths. In these two cases, it can be observed that the corresponding transition probabilities are highly different from the corresponding values obtained using 10 m section length. Therefore, it is recommended that a small section length be used for reliable estimates of the transition probabilities. The impact of using transition probabilities estimated from long section length can result in totally erroneous pavement rehabilitation and management decisions especially when the deployed sample size is small. An increase in the section length can be compensated by an increase in the sample size; however, this will require additional efforts in conducting the distress assessment as the overall network size becomes larger.

Summary and conclusion

The presented pavement performance assessment procedure is simple to implement but yet effective in yielding reliable pavement DRs to be used in pavement rehabilitation and management applications. The presented assessment procedure has mainly relied on using the extent and severity factors for cracking and deformation so that the visual inspection of related pavement defects becomes more reliable and less subjective. The sample results obtained from the area-based and length-based approaches have indicated that the two approaches are quite compatible. However, the length-based approach can be performed faster than the area-based approach with less interruption to the travelling public especially in the case of a two-lane highway. In addition, the application of the back-calculation of transition probabilities from the discrete-time Markov model has been effective in estimating the transition probabilities. Only two cycles

Table 5. Transition probabilities obtained using variable pavement section length.

	P_{12}		P_{23}		P_{34}		P_{45}	
Pavement section length (L_s)	M_A	M_L	M_A	M_L	M_A	M_L	M_A	M_L
10	0.182	0.214	0.354	0.399	0.393	0.386	0.364	0.241
30	0.200	0.200	0.250	0.250	0.364	0.333	0.357	0.400
50	0.496	0.496	0.660	0.660	0.374	0.498	0.334	0.712
100	0.000	0.000	0.251	0.251	1.000	1.000	0.399	0.500

Notes: M_A: area-based approach; M_L: length-based approach.

of pavement distress assessment are required to be able to use this straightforward technique. A highway agency interested in the back-calculation approach to estimate the transition probabilities can use any preferable distress assessment procedure as it is not required to deploy the one proposed in this paper.

The sample results have also indicated the significant impact of the pavement section length on the estimation of the state probabilities and transition probabilities used in the Markovian-based performance prediction models. An effective pavement performance prediction model is an essential component of any advanced pavement management system. The presented sample results have indicated that the project overall average DR is independent of the pavement section length. The vast majority of current pavement assessment procedures recommend the use of a pavement section length in the range of 100–200 m. Therefore, this relatively large section length is appropriate if the objective is to yield an overall average pavement DR for the purpose of recommending an appropriate rehabilitation strategy at the project level. However, if estimates of the state probabilities and transition probabilities are to be obtained from the distress assessment, then the pavement section length ought to be as small as possible. Otherwise, the use of long pavement section length can result in unreliable estimates of the transition probabilities especially when the corresponding sample size is not large enough. This can cause any pavement management system with Markovian-based performance prediction model to yield unreliable optimal M&R plans.

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