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Simplified staged-homogenous Markov model for flexible pavement performance prediction

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This paper presents a simplified staged-homogenous Markov model proposed for predicting future pavement conditions at the project level with minimal time and effort. The analysis period is divided into equal staged-time periods with each represented by a unique transition probability matrix. The deterioration transition probabilities are expected to increase over time due to the progressively increasing traffic loading and gradually degrading pavement structure. Therefore, the deterioration transition probabilities associated with staged-time periods are estimated from multiplying the present deterioration transition probabilities by appropriate constants called *C* constants. The *C* constants are to be estimated from minimising the sum of squared errors (SSE) defined as the differences between the predicted and observed distress ratings (DRs). A simplified sequential trial-and-error minimisation approach is proposed for obtaining the best estimates of the *C* constants. The presented sample problem has indicated the effectiveness of the proposed staged-homogenous Markov model in predicting future pavement DRs using five-year staged-time periods. This has been validated using three key performance indicators, namely the generated best-fit performance curves, minimum SSE, and staged-performance ratios.

Keywords: pavement performance; Markovian processes; transition probabilities; pavement maintenance; pavement management

Introduction

Pavement performance is essentially concerned with predicting the future pavement conditions for the purpose of maintaining and managing the pavement network. Pavement performance has been traditionally presented using a performance curve which shows how the pavement condition declines over time in the absence of maintenance and rehabilitation (M&R) works (Huang, 2004; Shahin, 1994). The pavement condition has been historically represented by indicators such as the Present Serviceability Index, Pavement Condition Index (PCI), and distress rating (DR) (American Association of State Highway and Transportation Officials, 1993; American Standard Testing Manual, 2007; Shah, Jain, Tiwari, & Jain, 2013). The emphasis on developing effective models for predicting pavement performance had become of great importance with the merge of the pavement management science over three decades ago. Therefore, an effective pavement prediction model is a significant component of any advanced pavement management system (Abaza, Ashur, & Al-Khatib, 2004; Jorge & Ferreira, 2012; Khan, Mesbah, Ferreira, & Williams, 2014).

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Generally, there are two types of model used in predicting future pavement conditions: deterministic and stochastic models (Amin, 2015; Li, Haas, & Xie, 1997; Wang, Zaniewski, & Way, 1994). Both types of model can predict the future pavement conditions; however, the stochastic-based models have gained wider use in pavement management applications. This can be attributed to the fact that pavement performance has been identified as probabilistic which requires assigning different levels of uncertainty (i.e. probability) to different pavement condition outcomes. The stochastic model that was used by several researchers over the last three decades had mainly relied on deploying different forms of the discrete-time Markov model (Abaza, 2006; Abaza & Murad, 2009; Butt, Shahin, Feighan, & Carpenter, 1987; Hong & Wang, 2003; Mandiartha, Duffield, Thompson, & Wigan, 2012; Lethanh & Adey, 2013; Li, Xie, & Haas, 1996). The homogenous and non-homogenous chains are the two most popular forms of the deployed discrete-time Markov model with the non-homogenous being the most effective model.

The homogenous Markov model assumes steady-state transition probabilities over the entire analysis period which presents a major drawback for predicting future pavement conditions. This is because the transition probabilities, representing pavement deterioration rates, are expected to increase over time mainly due to the progressively increasing traffic loading and gradually degrading pavement structural capacity (Abaza, 2014). The non-homogenous Markov model deploys different transition probabilities for each transition which can lead to superiority in predicting future pavement conditions. However, estimating the relevant transition probabilities requires substantial time and effort. A potential alternative to the non-homogenous Markov model is the staged-homogenous Markov model wherein the requirements for transition probabilities are substantially reduced (Butt et al., 1987). The proposed simplified staged-homogenous Markov model requires one set of transition probabilities for each staged-time period with the analysis period divided into a limited number of staged-time periods. The staged-homogenous transition probabilities are to be estimated from the minimisation of SSE with the error defined as the difference between the predicted and observed DRs. A simplified sequential trial-and-error minimisation approach is proposed for reaching the best estimates of the staged-homogenous transition probabilities.

Overview of stochastic modelling of pavement performance

Pavement performance has long been recognised as probabilistic in nature, thus, several researchers over the last three decades have deployed different forms of stochastic-based models to predicting future pavement conditions (Abaza, 2006; Abaza & Murad, 2009; Hong & Wang, 2003; Lethanh & Adey, 2013; Mandiartha et al., 2012). The most popular model used is the discrete-time Markov model with both homogenous and non-homogenous chains. This section provides an overview of these two discrete-time Markov chains.

Homogenous discrete-time Markov chain

The main two elements of the discrete-time Markov model are the state probabilities and transition probabilities. The state probabilities represent the pavement proportions that may exist in the various deployed pavement condition states at any specified time interval. The transition probability denotes the probability of pavement transiting from one condition state to another during one time interval. The time interval is defined as a discrete-time period generally taken equal to one or two years, and it represents one transition. Estimation of state probabilities requires one cycle of pavement distress assessment. Generally, the pavement project is divided into small pavement sections which are surveyed for pavement distresses and assigned a DR. Based on the assigned DR, each pavement section is counted as belonging to a specific condition state defined using a range of DR. The numbers of pavement sections (N_i) assigned to various deployed condition states can then be used to estimate the state probabilities (S_i) as defined in Equation (1).

$$S_i = \frac{N_i}{N}, \tag{1}$$

where

$$N = \sum_{i=1}^{m} N_i,$$

S_i is the ith state probability, N_i is the number of pavement sections assigned to the ith condition state, N is the total number of pavement sections used in the study (i.e. sample size), and m is the number of deployed pavement condition states.

The transition probabilities can be assumed to remain constant over time, thus, indicating a steady-state condition which results in a homogenous Markov chain. In this case, the state and transition probabilities are related to each other via the discrete-time Markov model presented in Equation (2). This model can be used to predict the state probabilities associated with the kth transition provided that the initial (i.e. present) state probabilities and transition probabilities are known. The initial state probabilities for new pavement can be assumed to take on the values of (1, 0, 0, ... , 0). The homogenous Markov model was used by some researchers to model pavement performance (Abaza, 2006; Abaza et al., 2004; Butt et al., 1987). However, they indicated that the transition probabilities are most likely to change over time, thus, there is a need to use revised transition probabilities. A recent publication by Abaza (2014) has indicated that the transition probabilities are expected to increase over time due to the progressive increase in traffic loading and the gradually degraded pavement structure.

$$S^{(k)} = S^{(0)} P^{(k)} \quad (k = 1, 2, \ldots, n), \tag{2}$$

where

$$S^{(k)} = (S_1^{(k)}, \mathrm{S}_2^{(k)}, \mathrm{S}_3^{(k)}, \ldots, \mathrm{S}_m^{(k)}),$$
$$S^{(0)} = (S_1^{(0)}, S_2^{(0)}, S_3^{(0)}, \ldots, S_m^{(0)}),$$
$$\sum_{i=1}^{m} S_i^{(k)} = 1.0,$$

$S^{(k)}$ is the row vector representing state probabilities after k transitions, $S^{(0)}$ is the row vector representing initial state probabilities, $P^{(k)}$ is the transition matrix raised to the kth power, m is the number of deployed pavement condition states, and n is the number of deployed discrete-time intervals (transitions).

The transition matrix is a square matrix ($m \times m$) containing all relevant transition probabilities. The matrix entries ($P_{i,i}$) along the main diagonal represent the probabilities of pavements remaining in the same condition states after the elapse of one transition. Entries above the main diagonal ($P_{i,j}$; $j > i$) denote the probabilities of pavements transmitting to the worse condition states after one transition. Essentially, these probabilities represent the deterioration rates of pavements, and thus, can be called the deterioration transition probabilities. However, entries below the main diagonal ($P_{i,j}$; $j < i$) indicate the probabilities of pavements transiting to the better condition states after one transition. These probabilities stand for the pavement improvement rates and can be named the improvement transition probabilities. In the absence of any M&R works, the improvement transition probabilities are assigned zero values. An example of a simplified homogenous transition matrix is presented in Equation (3) which only contains two sets of transition probabilities ($P_{i,i}$) and ($P_{i,i+1}$) in the absence of any M&R works (Abaza, Ashur, & Al-Khatib, 2004; Butt et al., 1987). The deterioration transition probability ($P_{i,i+1}$) indicates the probability of pavement transiting into the next worse state ($i + 1$) after one transition. The assumption of using only these two sets of transition probabilities can be realistic provided that the number of deployed condition states and transition length are reasonable. A Markov model with 10 condition states and one-year transition length was concluded to be adequate in satisfying the stated assumption (Abaza & Murad, 2009; Butt et al., 1987).

$$P = \begin{pmatrix} P_{1,1} & P_{1,2} & 0 & 0 & 0 & \cdots & 0 \\ 0 & P_{2,2} & P_{2,3} & 0 & 0 & \cdots & 0 \\ 0 & 0 & P_{3,3} & P_{3,4} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & P_{m-1,m-1} & P_{m-1,m} \\ 0 & 0 & 0 & 0 & 0 & \cdots & P_{m,m} \end{pmatrix}, \quad (3)$$

where

$$P_{i,i} + P_{i,i+1} = 1.0, \quad P_{m,m} = 1.0,$$
$$0 \le P_{i,i} \le 1.0, \quad 0 \le P_{i,i+1} \le 1.0.$$

Estimation of the transition probabilities requires conducting two cycles of pavement distress assessment separated by one transition. The outcomes of these two distress assessments can be used to calculate the corresponding two sets of state probabilities using Equation (1). The deterioration transition probabilities are then estimated using Equation (4), which mainly requires two sets of state probabilities ($\mathrm{Si}^{(1)}$ & $\mathrm{Si}^{(2)}$) estimated from the two cycles of distress assessment. Equation (4) is derived from the backward solution of the homogenous Markov chain presented in Equation (2) using two consecutive transitions and the transition matrix outlined in Equation (3) (Abaza, 2014).

$$P_{1,2} = \frac{S_1^{(1)} - S_1^{(2)}}{S_1^{(1)}}, \quad S_1^{(1)} \ge S_1^{(2)}, \quad (4a)$$

$$P_{i,i+1} = \frac{S_i^{(1)} - S_i^{(2)} + S_{i-1}^{(1)} P_{i-1,i}}{S_i^{(1)}}, \quad (i = 2, 3, \ldots, m-1), \quad (4b)$$

$$P_{m-1,m} = \frac{S_m^{(2)} - S_m^{(1)}}{S_{m-1}^{(1)}}, \quad S_m^{(2)} \ge S_m^{(1)}. \quad (4c)$$

However, the first and last deterioration transition probabilities ($P_{1,2}$ and $P_{m-1,m}$) can be used to estimate the remaining deterioration transition probabilities assuming linear interpolation. Generally, there are two distinct types of pavement performance as indicated by the performance curves shown in Figures 1 and 2. Equation (5) is accordingly designed to estimate the remaining

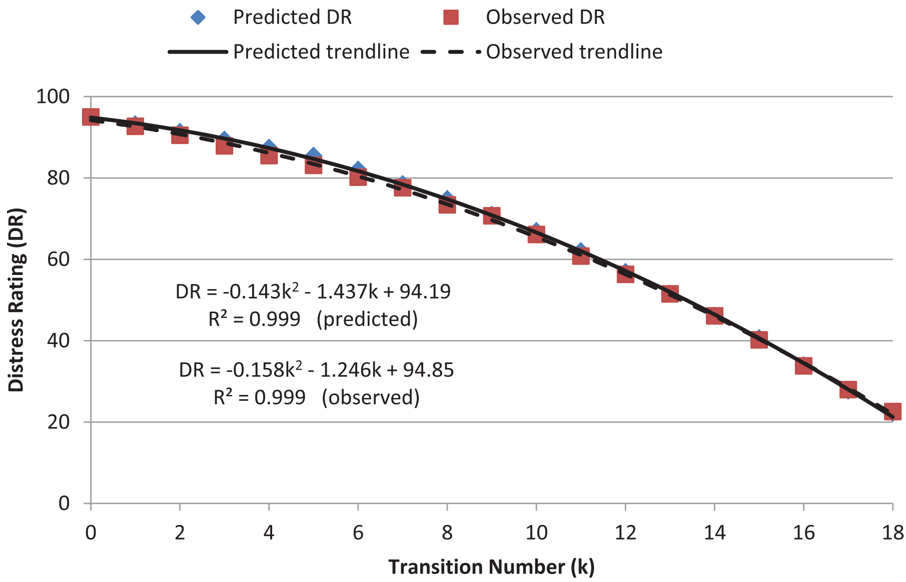

Figure 1. Predicted and observed DRs for increasingly higher deterioration rates using five-year staged–time period (SSE = 21.56).

deterioration transition probabilities associated with increasingly higher deterioration rates (i.e. transition probabilities) typically corresponding to the performance curve shown in Figure 1.

$$P_{i,i+1} = P_{1,2} + (i-1)\left(\frac{P_{m-1,m} - P_{1,2}}{m-2}\right) \quad (i = 2, 3, \ldots, m-2), \tag{5}$$

where $P_{1,2} < P_{2,3} < P_{3,4} < \cdots < P_{m-1,m}$.

Similarly, Equation (6) can be used to estimate the remaining deterioration transition probabilities in the case of decreasingly lower deterioration rates typically representing the performance curve depicted in Figure 2. This approach minimises the time and effort needed for pavement distress collection as only three state probabilities are required, namely those corresponding to states $(1, m-1, m)$ as indicated by Equations (4a) and (4c). Abaza and Murad (2009) deployed a similar approach and reported that the estimated deterioration transition probabilities were adequate for predicting future pavement conditions.

$$P_{i,i+1} = P_{1,2} - (i-1)\left(\frac{P_{1,2} - P_{m-1,m}}{m-2}\right) \quad (i = 2, 3, \ldots, m-2), \tag{6}$$

where $P_{1,2} > P_{2,3} > P_{3,4} > \cdots > P_{m-1,m}$.

It is to be noted that Equations (5) and (6) essentially yield the same results but they are listed as two separate equations to emphasise their corresponding distinct performance trends.

Non-homogenous discrete-time Markov chain

The non-homogenous discrete-time Markov chain for estimating the future state probabilities is presented in Equation (7). It allows for incorporating a distinct transition matrix for each transition within a specified analysis period. This requires a new set of transition probabilities for each transition, which definitely provides a much more accurate approach for predicting future pavement conditions (Abaza, 2006; Abaza & Murad, 2009; Hong & Wang, 2003;

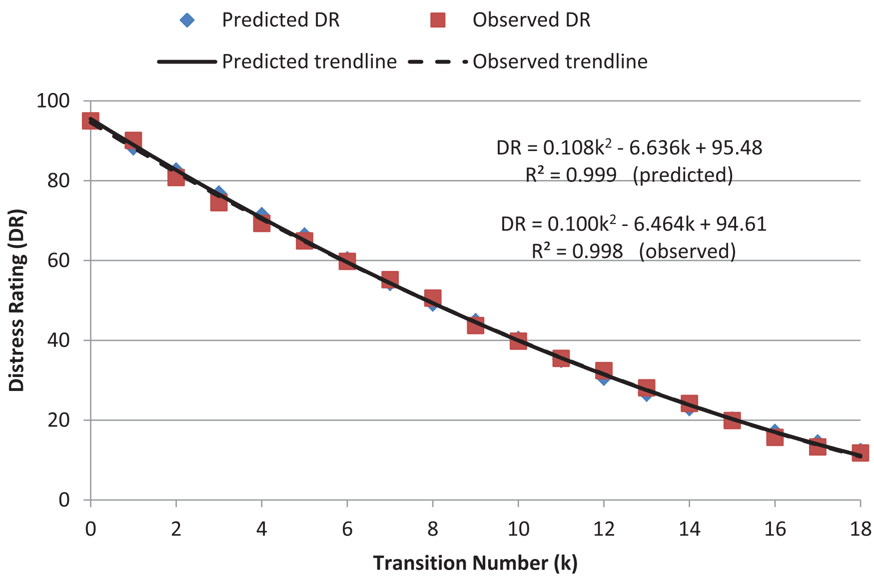

Figure 2. Predicted and observed DRs for decreasingly lower deterioration rates using five-year staged–time period (SSE = 25.43).

Li et al., 1996). However, the needed distress data are much more extensive as estimates of (n) sets of transition probabilities are required for (n) transitions. As outlined earlier, two consecutive cycles of pavement distress assessment are required to estimate one set of transition probabilities. Therefore, it requires the collection of extensive historical records of pavement distress over (n) transitions to estimate the required non-homogenous transition probabilities. The proposed staged-homogenous Markov model requires a very much reduced number of transition probability sets, which can be estimated from the pavement performance curve observed at the project level. Generation of a pavement performance curve only needs an estimate of the average DR for each transition within the analysis period at the project level.

$$S^{(n)} = S^{(0)} \left(\prod_{k=1}^{n} P(k) \right), \tag{7}$$

where

$$P(k) = \begin{pmatrix} P(k)_{1,1} & P(k)_{1,2} & 0 & 0 & 0 & \cdots & 0 \\ 0 & P(k)_{2,2} & P(k)_{2,3} & 0 & 0 & \cdots & 0 \\ 0 & 0 & P(k)_{3,3} & P(k)_{3,4} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & P(k)_{m-1,m-1} & P(k)_{m-1,m} \\ 0 & 0 & 0 & 0 & 0 & \cdots & P(k)_{m,m} \end{pmatrix},$$

$$P(k)_{i,i} + P(k)_{i,i+1} = 1.0, \quad P(k)_{m,m} = 1.0,$$

$$0 \leq P(k)_{i,i} \leq 1.0, \quad 0 \leq P(k)_{i,i+1} \leq 1.0.$$

Staged-homogenous discrete-time Markov chain

The deterioration transition probabilities are generally expected to increase over time due to the progressive increase in traffic loading, gradual degradation of the pavement structural capacity, and the impact of weather conditions. However, Butt et al. (1987) indicated that the same transition probability matrix can be used for a time period of up to five years without significantly affecting the prediction strength of the non-homogenous Markov model. This requires introducing a new transition probability matrix (P_j) for each consecutive staged-time period of a maximum of five years. The impact of traffic load increases and pavement structure degradation can be accounted for using revised sets of deterioration transition probabilities to be estimated from future pavement distress records. Therefore, a staged-homogenous discrete-time Markov model can be formulated as presented in Equation (8) wherein a new transition probability matrix (P_j) is introduced for each staged-time period (n_j).

$$S^{(n)} = S^{(0)} \left(\prod_{j=1}^{s} P_j^{(n_j)} \right), \tag{8}$$

where

$$n = \sum_{j=1}^{s} n_j,$$
$$P_1^{(n_1)} = P^{(n_1)},$$

$P^{(n_1)}$ is the present transition matrix as defined in Equation (3) raised to the power (n_1), s is the number of distinct transition matrices required over (n) transitions with each matrix raised to the power (n_j), n_j is the jth staged-time period over which the transition matrix is constant provided that the period does not exceed five years. Typically, all staged-time periods are equally selected with the exception of the last one which might be lower in value depending on the value of the analysis period composed of (n) transitions.

Estimation of staged-homogenous transition probabilities

Several researchers had proposed different approaches to estimate the transition probabilities for various types of infrastructure (Kobayashi, Do, & Han, 2010; Mishalani & Madanat, 2002; Ortiz-Garcia, Costello, & Snaith, 2006). Generally, all deployed approaches rely on the past pavement performance as indicated by an appropriate pavement condition indicator. This paper proposes a simplified procedure to estimate the staged-homogenous transition probabilities at the project level relying mainly on the pavement distress data collected over an analysis period composed of (n) transitions. The proposed procedure attempts to estimate the staged-homogenous deterioration transition probabilities, $P_{i,i+1}^{(j)}$, as a function of the present deterioration transition probabilities ($P_{i,i+1}$) as defined in Equation (9). A constant (C_j) is to be multiplied by the present transition probabilities ($P_{i,i+1}$) to yield the staged-homogenous transition probabilities, $P_{i,i+1}^{(j)}$, associated with the jth transition matrix (P_j). This simplified approach assumes that all transition probabilities associated with a particular staged-time period will increase in the same proportion. The C constants have to be larger than one since the deterioration transition probabilities ($P_{i,i+1}$) are expected to increase over time for the reasons mentioned earlier. One approach to estimate the C constants is based on the experience and judgement of pavement experts.

$$P_{i,i+1}^{(j)} = C_j P_{i,i+1} \leq 1.0 \quad (i = 1, 2, \ldots, m-1; j = 2, 3, \ldots, s),$$
$$P_{i,i}^{(j)} = 1 - P_{i,i+1}^{(j)}, \tag{9}$$

where $C_1 = 1.0$, associated with the first staged-time period (n_1).

$$C_1 \leq C_2 \leq C_3 \leq \cdots \leq C_s.$$

Alternatively, reliable estimates of the C constants can be derived from the calibration process if pavement distress records are available over an analysis period of (n) transitions. In this approach, estimation of the C constants can be obtained based on minimising the SSE as defined in Equation (10). The SSE is defined as the difference between the predicted DR value, $\mathrm{DR_P}(k)$, and the corresponding observed value, $\mathrm{DR_O}(k)$, squared and summed over (n) transitions as presented in Equation (10). The C constants obtained from the minimisation of SSE indicate a good agreement between the predicted and observed DR values. The C constants are expected to capture the influence of increased traffic loading and degraded pavement structure on the future deterioration transition probabilities.

$$\text{Minimise}: \quad \mathrm{SSE} = \sum_{k=0}^{n} [\mathrm{DR_P}(k) - \mathrm{DR_O}(k)]^2. \tag{10}$$

The predicted DR value is estimated using Equation (11) as the mean of a compound uniform probability density function. The compound uniform probability function is defined using the state probabilities ($S_i^{(k)}$) with each being valid over a range of DR values as presented in Equation (11). The state probability coefficient (B_i) is estimated as the average of the corresponding DR range defined using lower and upper DR values, respectively. For a 10-state Markov chain ($m = 10$), an equal state range of 10 DR points is typically used assuming a 100-point total DR scale as indicated in Equation (11).

$$\mathrm{DR_P}(k) = \sum_{i=1}^{m} B_i S_i^{(k)} \quad (k = 0, 1, 2, \ldots, n),$$
$$\text{where} \quad S^{(k)} = \begin{cases} S_1^{(k)}, & 90 < DR_1 \leq 100, \quad B_1 = 95, \\ S_2^{(k)}, & 80 < DR_2 \leq 90, \quad B_2 = 85, \\ S_3^{(k)}, & 70 < DR_2 \leq 80, \quad B_3 = 75, \\ \vdots & \qquad \vdots \qquad\qquad\qquad \vdots \\ S_{10}^{(k)}, & 0 \leq DR_{10} \leq 10, \quad B_{10} = 5. \end{cases} \tag{11}$$

The state probabilities required for using Equation (11) are to be estimated from the staged-homogenous Markov model defined in Equation (8). This requires estimating the staged-homogenous transition probabilities from multiplying the present deterioration transition probabilities ($P_{i,i+1}$) by the appropriate values of the C constants as indicated in Equation (9). In the search for the best values of the C constants, a sequential trial-and-error approach can be used wherein the C values that minimise the SSE will be selected nearest to a 0.05 point. For example, a Markov chain with 20 transitions ($n = 20$), only 3 C constants (C_2, C_3, C_4) need to be estimated assuming a constant transition matrix for each five-year staged-time period (n_j). In the case of using equal staged-time periods (n_e), the staged-homogenous Markov model for estimating the state probabilities at the kth transition is presented in Equation (12).

$$S^{(k)} = \begin{cases} S^{(0)} P_1^{(n_e)}, & k \leq n_e, \\ S^{(0)} P_1^{(n_e)} P_2^{(k-n_e)}, & k \leq 2n_e, \\ S^{(0)} P_1^{(n_e)} P_2^{(n_e)} P_3^{(k-2n_e)}, & k \leq 3n_e, \\ \vdots & \vdots \end{cases} \tag{12}$$

Sequential trial-and-error minimisation approach

The estimation of the first unknown C constant (C_2) can be accomplished by simply minimising the SSE_2 as presented in Equation (13a). This is because the predicted DR ratings associated with SSE_2 are only dependent upon 2 transition matrices (P_1 & P_2) with only one unknown variable (C_2). The trial-and-error search can then proceed by assigning a value of 1.05 for (C_2) to be incrementally increased by 0.05 until no further significant reduction in SSE_2 can be obtained. This indicates that a good agreement between the predicted and observed DR values has been reached over the first two consecutive staged-time periods with length equal to ($2n_e$). Similarly, the estimation of the second unknown C constant (C_3) can be done by minimising SSE_3 as defined in Equation (13b). The SSE_3 actually depends on the two constants (C_2 and C_3), but in the first trial-and-error solution (C_2) can be kept unchanged while starting the search for (C_3) with an initial value taken equal to (C_2) as obtained from minimising SSE_2, and incrementally increased by a 0.05 point until no further reduction in the SSE_3 can be obtained. This sequential trial-and-error approach shall proceed with minimising the next SSE (i.e. SSE_4) until estimates of all involved C constants are obtained.

$$\text{Minimise :} \quad \text{SSE}_2 = \sum_{k=0}^{2n_e} [\text{DR}_\text{P}(k) - \text{DR}_\text{O}(k)]^2, \tag{13a}$$

$$\text{Minimise :} \quad \text{SSE}_3 = \sum_{k=0}^{3n_e} [\text{DR}_\text{P}(k) - \text{DR}_\text{O}(k)]^2. \tag{13b}$$

A good agreement between the predicted and observed DR values can generally be obtained by the end of the first sequential trial-and-error solution; however, refinements can be made by repeating the above outlined procedure. The sample results presented in this paper shall indicate the efficiency of this sequential approach in yielding reliable estimates of the C constants, thus, providing dependable future transition probabilities that can lead to a good prediction of the future pavement conditions. The Microsoft Office Excel has effectively been used to programme and solve the sample problem presented in a subsequent section.

Pavement distress survey

The outlined sequential minimisation approach for yielding the staged-homogenous transition probabilities requires estimating the pavement DR (i.e. observed DR) for each transition within a specified analysis period composed of (n) transitions. This can be accomplished by dividing the roadway travel lanes into small sections with length ranging from 50 to 200 m in lane length. The pavement of each section is typically surveyed for pavement distresses considering both distress severity and extent. The survey of pavement distresses can be conducted by human inspection or using automated systems which mainly rely on the analysis of digital images of pavement distresses (Wang & Smadi, 2011). The outcome of the survey analysis results in assigning a DR for each pavement section, which typically deploys a scale of 100 points with higher ratings indicating better pavement. The observed DR for a particular pavement project at the kth transition, $DR_O(k)$, is estimated as the arithmetic mean of the DRs associated with all surveyed pavement sections.

Several procedures and formulas have been developed and implemented for estimating a DR for a pavement section at any given time. An example is the procedure developed by ASTM (2007) which results in the derivation of a numerical index called the PCI that describes the overall pavement condition on a scale of 100 points. Another example is the simplified formula proposed by Abaza (2014) to estimate the DR for a particular pavement section as presented in Equation (14). The simplified formula only requires identifying and assessing the localised distressed areas in terms of the two most important pavement distresses, namely cracking and deformation. A localised distressed area can only be counted as cracked or deformed with the worst case is used in the estimation of the DR. A maximum DR value of 100 is obtained when the entire pavement section is free of any distress signs and a minimum DR value of zero is assigned when the entire pavement section shows distress signs of high severity (i.e. SF = 3). Therefore, distress severity is represented by the severity factor (SF) while distress extent is indicated by the pavement surface area (A).

$$\mathrm{DR} = \left(\frac{3A_\mathrm{S} - \sum_i \mathrm{SF}_{\mathrm{C}_i} A_{\mathrm{C}_i} - \sum_i \mathrm{SF}_{\mathrm{D}_i} A_{\mathrm{D}_i}}{3A_\mathrm{S}} \right) \times 100, \tag{14}$$

where

$$\sum_i \mathrm{SF}_{\mathrm{C}_i} A_{\mathrm{C}_i} + \sum_i \mathrm{SF}_{\mathrm{D}_i} A_{\mathrm{D}_i} \leq 3A_\mathrm{S},$$

$$\sum_i A_{\mathrm{C}_i} + \sum_i A_{\mathrm{D}_i} \leq A_\mathrm{S},$$

SF_C is the SF associated with a localised cracked area taken on the values of 1, 2, or 3 for low, medium, or high severity, respectively, defined according to specified criteria, A_C is a localised cracked area (m^2) within a pavement section that contributes to the estimation of the DR, SF_D is the SF associated with a localised deformed area taken on the values of 1, 2, or 3 for low, medium, or high severity, respectively, defined according to specified criteria, A_D is a localised deformed area (m^2) within a pavement section that contributes to the estimation of the DR, and A_S is the entire area of the pavement section (m^2) to be surveyed for pavement distresses.

The observed pavement DRs, $DR_O(k)$, used in conjunction with the sample problem presented in the next section were estimated using Equation (14) based on 50 m section lane length with localised distressed areas estimated as rectangular areas. The surveys were mainly conducted using manual inspection and performing simple linear measurements. Application of Equation (14) is a straightforward task; however, sample DR calculations can be found in Abaza (2014). Also, different forms of Equation (14) can be consulted in the same reference.

Sample presentation: a case study from Palestine

This section presents a case study that involves evaluating and predicting the pavement performance associated with a major urban arterial in the city of Nablus, Palestine. The main objective is to estimate the staged-homogenous transition probabilities using the outlined sequential trial-and-error minimisation approach. Two types of pavement performance are identified and evaluated, namely superior performance associated with progressively increasing deterioration rates and inferior performance associated with progressively decreasing deterioration rates (i.e. transition probabilities). Also, two different staged-time periods have been considered in the analysis, namely three and five years.

Background and distress data

A sample problem is presented to demonstrate the implementation of the simplified staged-homogenous Markov model in predicting future pavement conditions. Distress data were annually collected during 1998–2015 on a major urban arterial paved with asphalt concrete located in the city of Nablus, West Bank, Palestine. The corresponding pavement structure consists of 13 cm (5 in.) high-stability hot-mix asphalt surface and 50 cm (20 in.) crushed limestone aggregate base. The major arterial carries an average daily traffic of 20,000 vpd which resulted in a 20-year design equivalent single axle load (ESAL) of about 5 million. The major arterial is about 7 km in length running east–west of Nablus city with two travel lanes in each direction. The major arterial under consideration was mostly constructed in mountainous terrain characterised by strong subgrade (i.e. high bearing capacity) and good drainage; however, few segments were built in level terrain characterised by weak subgrade (i.e. low bearing capacity) and poor drainage.

Pavement sections of 50-m lane length were surveyed for pavement distresses and assigned a DR on a scale of 100 points using Equation (14). The DRs associated with all surveyed pavement sections were averaged out to yield an overall annual DR called observed DR (DR_O). The results of the pavement section surveys were also used to compute the observed state probabilities (S_i) as indicated by Equation (1). Each surveyed pavement section was assigned to a pavement condition state based on its DR using the DR ranges defined in Equation (11). The total numbers of pavement sections (N_i) assigned to the 10 deployed condition states were used to estimate the observed state probabilities as presented in Equation (1). The observed state probabilities associated with two consecutive transitions were then applied to estimate the transition probabilities as indicated in Equation (4).

Alternatively, the observed DRs associated with a particular transition can be obtained from the observed state probabilities using Equation (11). Based on the observed DRs, it can be concluded that pavements built in mountainous terrain are associated with increasingly higher deterioration rates (i.e. slow initial deterioration rates), while pavements built in level terrain are associated with decreasingly lower deterioration rates (i.e. rapid initial deterioration rates). The annual observed DRs (DR_O) for both types of pavement performance are provided in Tables 1 and 2.

Sample staged-homogenous transition probabilities

A staged-homogenous Markov chain with 10 pavement condition states is used to model the long-term performance of the sample major arterial under consideration. A 10×10 transition probability matrix, similar to the one presented in Equation (7), is used in the formulation of the corresponding staged-homogenous Markov model. The first and last deterioration transition probabilities ($P_{1,2}$ and $P_{9,10}$) are estimated from the outcomes of two distress assessment cycles using Equations (4a) and (4c), respectively. The remaining first-year deterioration transition probabilities are estimated using Equations (5) and (6) for pavement performances with slow and rapid initial deterioration rates, respectively. The corresponding first-year values of

Table 1. Sample predicted and observed DRs (DR_O & DR_P) using five-year staged-time period.

Slow initial deterioration rates				Rapid initial deterioration rates			
Transition no. (k)	$DR_P(k)$	$DR_O(k)$	PR_J	Transition no. (k)	$DR_P(k)$	$DR_O(k)$	PR_J
0	95	95.0		0	95	95.0	
1	93.18	92.7		1	88.5	90.1	
2	91.31	90.5		2	82.38	82.8	
3	89.40	87.9		3	76.62	74.5	
4	87.44	85.5		4	71.20	69.3	
5	85.43	83.1	1.013	5	66.10	64.9	1.012
6	82.03	80.2		6	60.10	59.8	
7	78.48	77.6		7	54.54	55.2	
8	74.79	73.4		8	49.38	50.6	
9	70.95	70.7		9	44.61	43.7	
10	66.94	66.1	1.016	10	40.18	39.8	1.000
11	62.02	60.8		11	35.27	35.5	
12	56.87	56.3		12	30.81	32.4	
13	51.53	51.5		13	26.77	28.1	
14	46.08	46.1		14	23.16	24.2	
15	40.62	40.2	1.009	15	19.98	19.9	0.974
16	33.95	33.8		16	16.86	15.7	
17	27.73	28.0		17	14.24	13.3	
18	22.24	22.6	1.012	18	12.10	11.8	1.004
Average	66.11	65.37		Average	47.78	47.61	

Table 2. Sample observed and predicted DRs (DR_O & DR_P) using three-year staged-time period.

Slow initial deterioration rates				Rapid initial deterioration rates			
Transition no. (k)	$DR_P(k)$	$DR_O(k)$	PR_J	Transition no. (k)	$DR_P(k)$	$DR_O(k)$	PR_J
0	95.00	95.0		0	95.00	95.0	
1	93.18	92.7		1	88.50	90.1	
2	91.31	90.5		2	82.38	82.8	
3	89.40	87.9		3	76.62	74.5	
4	86.56	85.5		4	70.93	69.3	
5	83.61	83.1	1.009	5	65.59	64.9	1.010
6	80.55	80.2		6	60.58	59.8	
7	77.06	77.6		7	54.98	55.2	
8	73.42	73.4		8	49.80	50.6	
9	69.64	70.7		9	44.99	43.7	
10	65.70	66.1	0.997	10	40.23	39.8	1.006
11	61.61	60.8		11	35.90	35.5	
12	57.37	56.3		12	31.98	32.4	
13	51.65	51.5		13	27.42	28.1	
14	45.79	46.1		14	23.35	24.2	
15	39.93	40.2	1.005	15	19.77	19.9	0.991
16	33.57	33.8		16	16.51	15.7	
17	27.66	28.0		17	13.80	13.3	
18	22.42	22.6	0.992	18	11.62	11.8	1.026
Average	65.55	65.37		Average	47.89	47.61	

the nine deterioration transition probabilities are provided in Table 3 for both types of pavement performance ($C_1 = 1.0$). The staged-homogenous Markov model presented in Equation

Table 3. Sample staged-homogenous transition probabilities using five-year staged-time period.

Deterioration transition probabilities ($P_{i,i+1}$)	Slow initial deterioration rates				Rapid initial deterioration rates			
	C Constants (C_1–C_4)				C Constants (C_1–C_4)			
	1.00	1.65	1.95	2.45	1.00	1.25	1.50	1.75
$P_{1,2}$	0.182	0.300	0.355	0.446	0.650	0.812	0.975	1.000
$P_{2,3}$	0.207	0.342	0.404	0.508	0.591	0.739	0.887	0.914
$P_{3,4}$	0.232	0.384	0.454	0.569	0.532	0.666	0.799	0.829
$P_{4,5}$	0.258	0.425	0.503	0.631	0.474	0.592	0.711	0.743
$P_{5,6}$	0.283	0.467	0.552	0.693	0.415	0.519	0.622	0.658
$P_{6,7}$	0.308	0.509	0.601	0.755	0.356	0.445	0.534	0.572
$P_{7,8}$	0.334	0.550	0.650	0.817	0.298	0.372	0.446	0.486
$P_{8,9}$	0.359	0.592	0.670	0.879	0.239	0.298	0.358	0.401
$P_{9,10}$	0.384	0.634	0.749	0.941	0.180	0.225	0.270	0.315

Table 4. Sample staged-homogenous transition probabilities using three-year staged-time period.

Deterioration transition probabilities ($P_{i,i+1}$)	Slow initial deterioration rates					
	C Constants (C_1–C_6)					
	1.00	1.45	1.60	1.60	2.10	2.35
$P_{1,2}$	0.182	0.264	0.291	0.291	0.382	0.428
$P_{2,3}$	0.207	0.300	0.332	0.332	0.435	0.487
$P_{3,4}$	0.232	0.337	0.372	0.372	0.488	0.546
$P_{4,5}$	0.258	0.374	0.412	0.412	0.541	0.606
$P_{5,6}$	0.283	0.410	0.453	0.453	0.594	0.665
$P_{6,7}$	0.308	0.447	0.493	0.493	0.647	0.724
$P_{7,8}$	0.334	0.484	0.534	0.534	0.700	0.784
$P_{8,9}$	0.359	0.520	0.574	0.574	0.753	0.843
$P_{9,10}$	0.384	0.557	0.614	0.614	0.806	0.902

(8) has been used to predict the pavement DRs for 18-year analysis period (i.e. 18 transitions) considering two alternatives: three- and five-year staged-time periods.

The staged-homogenous transition probabilities have been estimated using the sequential trial-and-error minimisation approach which mainly relies on estimating the C constants needed to compute the corresponding transition probabilities as defined in Equation (9). The sequential trial-and-error approach for minimising the SSE as described in Equation (13) has been used to estimate the C constants for the two alternatives considered. Table 3 provides the best obtained C-constant values, namely (C_2–C_4) for slow and rapid initial deterioration rates considering equal five-year staged-time periods. Similarly, Tables 4 and 5 provide the best obtained C constants, namely (C_2–C_6) for both types of pavement performance considering equal three-year staged-time periods. The C constants associated with slow initial deterioration rates are consistently lower than the corresponding values associated with rapid initial deterioration rates considering the same staged-time periods. The C constants are mainly used to calculate the first and last deterioration transition probabilities ($P_{1,2}$ and $P_{9,10}$) using Equation (9) with the remaining transition probabilities being estimated using Equations (5) and (6). The nine staged-homogenous transition probabilities associated with the best C-constant values are provided in Tables 3–5.

Table 5. Sample staged-homogenous transition probabilities using three-year staged-time period.

Deterioration transition probabilities ($P_{i,i+1}$)	Rapid initial deterioration rates					
	C Constants (C_1–C_6)					
	1.00	1.05	1.25	1.35	1.75	1.90
$P_{1,2}$	0.650	0.682	0.812	0.878	1.000	1.000
$P_{2,3}$	0.591	0.621	0.739	0.798	0.914	0.918
$P_{3,4}$	0.532	0.559	0.666	0.719	0.829	0.836
$P_{4,5}$	0.474	0.497	0.592	0.640	0.743	0.753
$P_{5,6}$	0.415	0.436	0.519	0.560	0.658	0.671
$P_{6,7}$	0.356	0.374	0.445	0.481	0.572	0.589
$P_{7,8}$	0.298	0.312	0.372	0.402	0.486	0.506
$P_{8,9}$	0.239	0.251	0.298	0.322	0.401	0.424
$P_{9,10}$	0.180	0.189	0.225	0.243	0.315	0.342

Results validation

The final estimated staged-homogenous transition probabilities have then been used to calculate the predicted DRs (DR_P) using Equation (11) with the corresponding state probabilities computed from Equation (12). The final predicted DRs are provided in Table 1 for both types of pavement performance considering five-year staged-time period. Similarly, Table 2 provides the final predicted DRs for three-year staged-time period. The predicted and observed DRs are both plotted in Figures 1–4 for comparison purposes. It can generally be concluded that there is a very good agreement between the predicted and observed DR values. It can also be concluded that using three-year staged-time period has resulted in modest improvements compared to the five-year staged-time period. For example, the associated SSE has decreased from 21.56 to 8.48 in the case of increasingly higher deterioration rates, and decreased from 25.43 to 18.52 in the case of decreasingly lower deterioration rates. Also, the figures depict the corresponding best-fit performance curves along with their regression equations generated for both predicted and observed DR values, which are very similar considering the two types of pavement performance.

Another performance indicator used to check the conformity between the predicted and observed DRs is the performance ratio (PR) as indicated in Equation (15). Pavement performance can be defined by the area falling under the performance curve (Abaza & Murad, 2009; Huang, 2004). Therefore, the predicted and observed performances as required in Equation (15) can be computed from the predicted and observed DRs. A similar staged-time PR_j is presented in Equation (16) which requires the staged-time areas under the predicted and observed performance curves (AP_j and AO_j), respectively. The closer is this ratio to one, the better is the conformity between the two sets of distress data.

$$\mathrm{PR} = \frac{\mathrm{Performance}_{\mathrm{predicted}}}{\mathrm{Performance}_{\mathrm{observed}}}, \tag{15}$$

$$\mathrm{PR}_j = \frac{\mathrm{AP}_j}{\mathrm{AO}_j} \quad (j = 1, 2, \ldots, s). \tag{16}$$

The corresponding staged-time PRs (PR_j) provided in Tables 1 and 2 indicate that there is a very good conformity between the predicted and observed DRs as all PRs are very close to one. In addition, there are generally minor improvements gained from using the three-year staged-time period over five-year staged-time period.

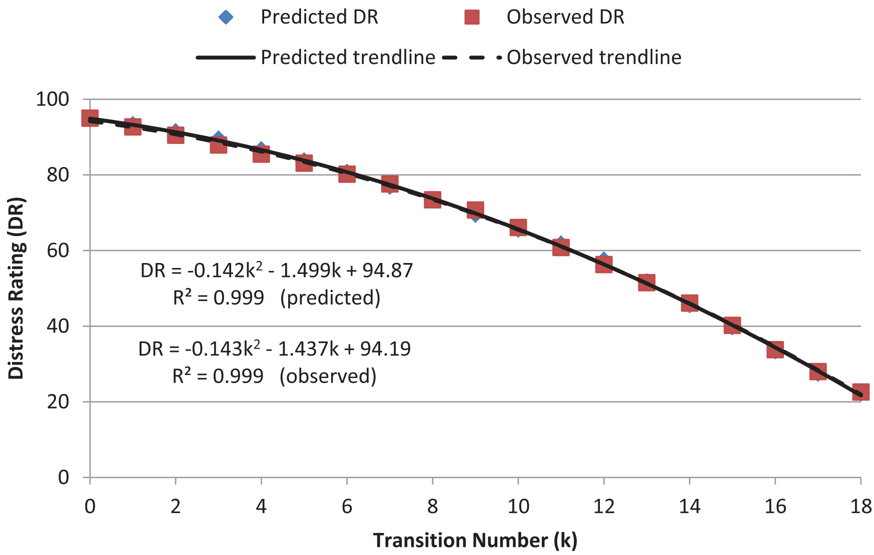

Figure 3. Predicted and observed DRs for increasingly higher deterioration rates using three-year staged-time period (SSE = 8.48).

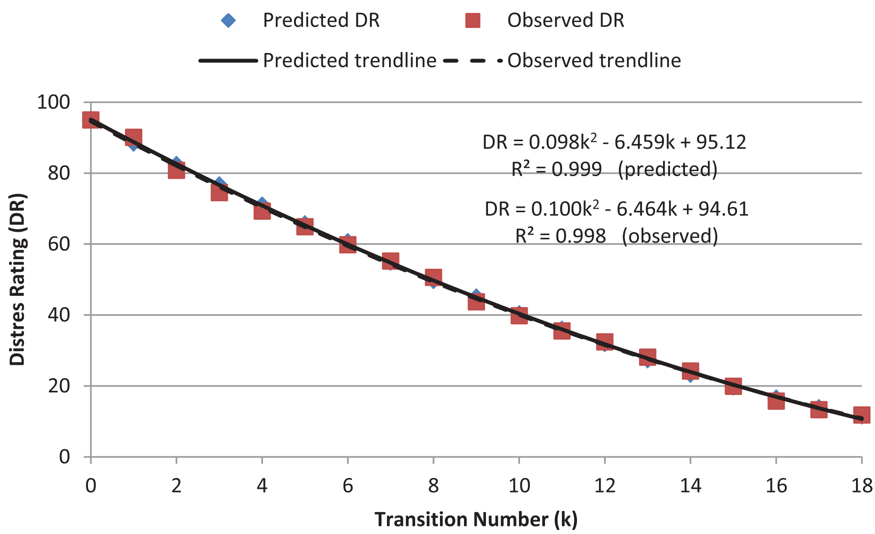

Figure 4. Predicted and observed DRs for decreasingly lower deterioration rates using three-year staged-time period (SSE = 18.52).

Conclusions

The presented sample problem has indicated the effectiveness of the proposed staged-homogenous Markov model in predicting the future pavement DRs. This has been validated using three different indicators, namely the generated best-fit performance curves, minimum SSE, and staged-PRs (PR_j). All three indicators have overwhelmingly supported the high predictive strength of the proposed staged-homogenous Markov model. A major component of the proposed staged-homogenous Markov model is the simplified sequential trial-and-error minimisation approach for estimating the corresponding deterioration transition probabilities. The simplified approach, as applied to the sample problem, has proven to be an invaluable tool in yielding reliable estimates of the C constants used to compute the staged-homogenous transition probabilities. In addition, the sample results obtained using five-year staged-time periods are very similar to those derived from deploying three-year staged-time periods. Therefore, it would be sufficient to use five-year staged-time periods leading to reduction in the time and effort required to develop the corresponding transition probabilities.

The simplified approach can be used to estimate the staged-homogenous transition probabilities at the project level with minimal data requirements. It requires estimates of only the first and last transition probabilities ($P_{1,2}$ and $P_{9,10}$) associated with the first-year (i.e. present year) assuming 10 pavement condition states. It also requires an annual estimate of the pavement DR for each year (i.e. transition) within an analysis period of (n) transitions. The proposed simplified sequential trial-and-error approach can then be used to yield reliable estimates of the C constants using five-year staged-time periods. This requires the estimation of the three corresponding C constants, namely (C_2, C_3, C_4). However, this can be further simplified if the analysis period is chosen to be equal to 10 years (i.e. 10 transitions), which is typically the maximum analysis period used in most pavement management applications. In this case, it requires the estimation of only one C constant, namely (C_2) which can be obtained from one simple trial-and-error solution.

It is to be pointed out that the presented sample problem did not investigate the impact of certain key factors related to the formulation of the staged-homogenous Markov model. For example, the impact of the form of the transition probability matrix in relation to the number of deployed condition states has not been investigated. Also, only two transition probabilities ($P_{i,i}$ and $P_{i,i+1}$) have been used to represent two pavement transitions for each condition state, but it is worthwhile to investigate the impact of using additional state transitions, for example, adding a third state transition to be represented by the transition probability ($P_{i,i+2}$). However, it is to be reminded that there is an inverse relationship between the number of pavement condition states and number of state transitions. In addition, the sample presentation has mainly investigated two distinct types of performance trend that can be represented by polynomials with second degree; therefore, the conclusions drawn from this study are mainly restricted to the investigated types of pavement performance. Finally, the sample problem has mainly dealt with an unmaintained pavement structure; however, the same approach can be used in the case of maintained pavements provided that relevant historical records of pavement distress are available.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Abaza, K. A. (2006). Iterative linear approach for non-linear non-homogenous stochastic pavement management models. *Journal of Transportation Engineering*, *132*(3), 244–256.

Abaza, K. A. (2014). Back-calculation of transition probabilities for Markovian-based pavement performance prediction models. *International Journal of Pavement Engineering*, published online. doi:10.1080/10298436.2014.993185

Abaza, K. A., Ashur, S. A., & Al-Khatib, I. A. (2004). Integrated pavement management system with a Markovian prediction model. *Journal of Transportation Engineering*, *130*(1), 24–33.

Abaza, K. A., & Murad, M. M. (2009). *Predicting flexible pavement remaining strength and overlay design thickness with stochastic modeling*. Transportation Research Record 2094, Washington, DC, 62–70.

American Association of State Highway and Transportation Officials. (1993). *AASHTO guide for design of pavement structures*, Washington, DC: AASHTO.

American Standard Testing Manual. (2007). *Standard practice for roads and parking lots pavement condition index surveys*, D6433-07, Philadelphia. Retrieved from http://www.astm.org/Standards/D6433.htm

Amin, Md Shohel Reza. (2015). The pavement performance modeling: deterministic vs. stochastic approaches. In S. Kadry & A. El Hami (Eds.), *Numerical methods for reliability and safety assessment* (pp. 179–196). Switzerland: Springer International.

Butt, A., Shahin, M., Feighan, K., & Carpenter, S. (1987). *Pavement performance prediction model using the Markov process*, pp. 12–19. Transportation Research Record 1123, Washington, DC: TRB.

Hong, H., & Wang, S. (2003). Stochastic modeling of pavement performance. *International Journal of Pavement Engineering*, *4*(4), 235–243.

Huang, Y. (2004). *Pavement analysis and design*. 2nd ed. Upper Saddle River, NJ: Pearson/Prentice Hall.

Jorge, D., & Ferreira, A. (2012). Road network pavement maintenance optimization using the HDM-4 pavement performance prediction models. *International Journal of Pavement Engineering*, *13*(1), 39–51.

Khan, M. U., Mesbah, M., Ferreira, L., & Williams, D. J. (2014). Development of road deterioration models incorporating flooding for optimum maintenance and rehabilitation strategies. *Road & Transport* Research*: A Journal of Australian and New Zealand Research and Practice*, *23*(1), 3–24.

Kobayashi, K., Do, M., & Han, D. (2010). Estimation of Markovian transition probabilities for pavement deterioration forecasting. *KSCE Journal of Civil Engineering*, *14*(3), 342–351.

Lethanh, N., & Adey, B. (2013). Use of exponential hidden Markov models for modelling pavement deterioration. *International Journal of Pavement Engineering*, *14*(7), 645–654.

Li, N., Haas, R., & Xie, W.-C. (1997). Investigation of relationship between deterministic and probabilistic prediction models in pavement management. *Transportation Research Record*, *1592*, 70–79.

Li, N., Xie, W.-C., & Haas, R. (1996). Reliability-based processing of Markov chains for modeling pavement network deterioration. *Transportation Research Record*, *1524*, 203–213.

Mandiartha, P., Duffield, C., Thompson, R., & Wigan, M. (2012). A stochastic-based performance prediction model for road network pavement maintenance. *Road and Transport Research*, *21*(3), 34–52.

Mishalani, R., and Madanat, S. (2002). Computation of infrastructure transition probabilities using stochastic models. *Journal of Infrastructure Systems*, *8*(4), 139–148.

Ortiz-Garcia, J., Costello, S., & Snaith, M. (2006). Derivation of transition probability matrices for pavement deterioration modeling. *Journal of Transportation Engineering*, *132*(2), 141–161.

Shah, Y., Jain, S., Tiwari, D., & Jain, M. (2013). Modeling the pavement serviceability index for urban roads in Noida. *International Journal of Pavement Research and Technology*, *6*(1), 66–72.

Shahin, M. (1994). *Pavement management for airports, roads, and parking lots*. New York, NY: Chapman and Hall.

Wang, K. C., & Smadi, O. (2011). *Automated imaging technologies for pavement distress surveys*. Transportation Research E-Circular (E-C156), Transportation Research Board, 22p.

Wang, K. C. P., Zaniewski, J., & Way, G. (1994). Probabilistic behavior of pavements. *Journal of Transportation Engineering*, *120*(3), 358–375.