Optimum Decision Policy for Management of Pavement Maintenance and Rehabilitation

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Optimum Decision Policy for Management of Pavement Maintenance and Rehabilitation

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An effective practical decision policy has been developed for use in the selection of an optimum maintenance and rehabilitation program. Its main objective is the optimization of pavement condition under constrained budgets. The developed policy utilizes a discrete-time Markovian model with five condition states labeled a, b, c, d, and f. State a represents pavements in excellent condition, and State f indicates pavements in bad condition. Several decision options have been introduced based on either maximizing the proportion of "good" pavements or minimizing the proportion of "bad" pavements. State probabilities at some desired future time have been used as the main objective functions in the development of optimum maintenance and rehabilitation programs. The unknown variables in these programs are those representing improvements to pavement condition through implementation of maintenance and rehabilitation work. The resulting optimum programs are nonlinear in form, and therefore the penalty function method with functional evaluations has been successfully used to yield optimum solutions. The optimum solution to a particular program defines the type and extent of maintenance and rehabilitation work required for annual or biennial implementation. Pavement maintenance is mainly defined as routine maintenance consisting of filling cracks, patching potholes, and other applicable techniques such as chip seal coat or slurry seal. Pavement rehabilitation is defined as major rehabilitation actions to include resurfacing (overlay), resurfacing with partial reconstruction (localized reconstruction), and complete reconstruction applied to pavements in States c, d, and f, respectively.

Substantial research has been done in the area of pavement maintenance and management in the past two decades. The emphasis has mainly been to alert the nation to the high costs of inadequately maintained pavements (*1–3*), to minimize pavement life-cycle cost through timely scheduled maintenance and rehabilitation plans (*3,4*), to predict pavement performance probabilistically (*5,6*), and to develop several pavement management systems (PMSs) (*7–9*). Pavement maintenance and management continue to be major concerns for highway officials, especially with pavements aging at much higher rates and a lack of proportional funding.

Researchers have investigated several mathematical models in their pursuit to predict future pavement performance. The Markovian model (*5*) has been demonstrated to be an effective tool in predicting future pavement performance. Its unique structure allows for the presentation of pavement deterioration and rehabilitation rates in an integrated single process.

The model presented in this research paper is based on formulating an integrated decision policy that applies the Markovian model to predict future pavement condition and on using nonlinear optimization methods to yield optimum pavement condition under constrained budgets. The decision policy is based on either optimizing pavement condition subjected to limited funding or minimizing pavement maintenance and rehabilitation costs subjected to preset pavement condition requirements. The first case is used in implementing developed optimum pavement maintenance and rehabilitation programs, whereas the latter case is to be used for planning and fund-raising purposes.

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FORMULATION OF MARKOVIAN MODEL

This research project has utilized a discrete-time Markov model with five condition states to predict the deterioration of pavements. The five condition states are labeled a, b, c, d, and f; they represent pavement sections in excellent, good, fair, poor, and bad condition, respectively. A detailed description of each state can be defined by setting limits based on either the pavement condition index (PCI) or the present serviceability index (PSI). The three main components of the Markovian predictive model are the five condition states; deterioration transition probabilities, P_{ij}, representing pavement deterioration rates; and maintenance transition probabilities, f_{ij}, representing pavement maintenance and rehabilitation rates. The transitions from one state to another are represented by a discrete-time Markov process with the transition matrix *P:*

$$P = \begin{bmatrix} P_{aa} & P_{ab} & P_{ac} & P_{ad} & P_{af} \\ f_{ba} & P_{bb} & P_{bc} & P_{bd} & P_{bf} \\ f_{ca} & f_{cb} & P_{cc} & P_{cd} & P_{cf} \\ f_{da} & f_{db} & f_{dc} & P_{dd} & P_{df} \\ f_{fa} & f_{fb} & f_{fc} & f_{fd} & P_{ff} \end{bmatrix} \tag{1}$$

The present deterioration transition probabilities, P_{ij}, represent the probabilities that a pavement section will be downgraded from State *i* to State *j* in a single time interval as a result of pavement aging and deterioration. The present maintenance and rehabilitation transition probabilities, f_{ij}, are the probabilities that a pavement section will improve from State *i* to State *j* in a single time interval as a result of currently active maintenance and rehabilitation actions. The values of f_{ij} vanish in the absence of an active maintenance and rehabilitation program.

The foregoing model is simplified by reducing the number of P_{ij} elements incorporated in the transition matrix. This simplification is accomplished by making the assumption that deteriorating transitions of pavements are only permitted in single steps (i.e., $P_{ij} = 0$ for $j > i + 1$; i,j = a, b, c, d, and f). This assumption is realistic, considering that the model only deploys five states, and a recommended

discrete time interval of usually a year or two would not practically worsen the pavement section by more than one condition state.

A total of 10 maintenance and rehabilitation transition probabilities, f_{ij}, are included in the initial Markov model. This number has been reduced to seven to match the number of maintenance and rehabilitation strategies recommended for implementation: there are four maintenance action plans and three major rehabilitation plans. Maintenance plans are applied to States b, c, d, and f, and no maintenance is applied to State a. The four maintenance variables consist of routine maintenance with different intensities applied to States b, c, d, and f. They can take on different forms and extents as required by the severity of pavement distress in each of the condition states. Consequently, they produce different costs. Rehabilitation plans consist of resurfacing (overlay), resurfacing with partial reconstruction (localized reconstruction), and complete reconstruction, which are applied to States c, d, and f, respectively. Therefore, a present transition matrix can contain a maximum of seven variables representing the maintenance and rehabilitation probabilities (fractions), f_{ij}. The reduced Markov model with the seven variables is given as follows:

$$P = \begin{bmatrix} P_{aa} & P_{ab} & 0 & 0 & 0 \\ f_{ba} & P_{bb} & P_{bc} & 0 & 0 \\ f_{ca} & f_{cb} & P_{cc} & P_{cd} & 0 \\ f_{da} & 0 & f_{dc} & P_{dd} & P_{df} \\ f_{fa} & 0 & 0 & f_{fd} & P_{ff} \end{bmatrix} \tag{2}$$

The main objective of this study is to determine optimum future maintenance and rehabilitation variables, labeled q'_{ij}. These optimum future variables will define the optimum maintenance program to be implemented. Estimation of the present deterioration transition probabilities, P_{ij}, has been the subject of intensive research. For example, the approach used by researchers at the U.S. Army Construction Engineering Research Laboratory (CERL) relies on minimizing the error between the observed and predicted values of the PCI. Other techniques based on the experience and judgment of pavement maintenance engineers can also be used to make appropriate estimations.

SELECTION OF PAVEMENT MAINTENANCE CANDIDATES

The future maintenance and rehabilitation variables, q_{ij}, and the future transition probabilities, P'_{ij}, have been incorporated into a future Markov model represented by the future transition matrix P' as follows:

$$P' = \begin{bmatrix} P'_{aa} & P'_{ab} & 0 & 0 & 0 \\ q_{ba} & P'_{bb} & P'_{bc} & 0 & 0 \\ q_{ca} & q_{cb} & P'_{cc} & P'_{cd} & 0 \\ q_{da} & 0 & q_{dc} & P'_{dd} & P'_{df} \\ q_{fa} & 0 & 0 & q_{fd} & P'_{ff} \end{bmatrix} \tag{3}$$

The four maintenance variables represented in this matrix and applied to States b, c, d, and f are q_{ba}, q_{cb}, q_{dc}, and q_{fd} and can only result in a single-step improvement from i to State $i + 1$. The other remaining three variables representing major rehabilitation and applied to States c, d, and f are q_{ca}, q_{da}, and q_{fa} and can result in a multiple-step improvement from State i to State a.

Certainly both the present and the future models are related to each other since the future model largely depends on the outcome of the present one, a property of the Markov model. A particular mechanism is needed to relate the future model to the present one. The present mechanism depends mainly on the utilized process for selecting pavement sections as candidates for maintenance and rehabilitation. Selection of pavement sections can be practically accomplished either by random selection or by worst-first selection. Different mathematical relations result for random and worst-first selection of candidates. Only relations for random selection are presented in this paper. "Worst-first" implies the selection of candidates on the basis of severity of pavement distress, in which pavement sections in a given condition state with more severe distress have priority for maintenance in comparison with those that exhibit less severe distress. The case of worst-first selection will be addressed later.

In the random selection of maintenance candidates, an assumption is made regarding the relation between the present transition probabilities, P_{ij}, and the future transition probabilities, P'_{ij}. This assumption states that the pair of present transition probabilities $P_{i,i}$ and $P_{i,i+1}$ will be such that their ratio will remain the same as the ratio of the future transition probabilities $P'_{i,i}$ and $P'_{i,i+1}$. Random selection implies that pavement sections are randomly scheduled for maintenance without reference to the relative severity of their condition. Random selection of maintenance candidates, all in condition state i, is a reasonable condition to apply in the absence of political or other outside influences. On the basis of this assumption, the following relation holds true:

$$\frac{P'_{i,i}}{P'_{i,i+1}} = \frac{P_{i,i}}{P_{i,i+1}} \tag{4}$$

Now, since the sum of any row in a transition matrix must add to 1, the following two conditions are necessary:

Condition 1:

$$f_i + P_{i,i} + P_{i,i+1} = 1 \tag{5}$$

where $f_i = \sum_{\forall j} f_{ij}$, for $i = \text{a, b, c, d and f}$, and

Condition 2:

$$q_i + P'_{i,i} + P'_{i,i+1} = 1 \tag{6}$$

where $q_i = \sum_{\forall j} q_{ij}$, for $i = \text{a, b, c, d and f}$.

The first condition corresponds to the present transition matrix P, whereas the second condition corresponds to the future transition matrix P' to be investigated for the optimum maintenance and rehabilitation policy. The objective is now to derive mathematical relations that would estimate the future transition probabilities $P'_{i,i}$ and $P'_{i,i+1}$ from other related variables. This can be achieved by equating Equations 5 and 6, which results in

$$P'_{i,i} = P'_{i,i+1} + q'_i = P_{i,i} + P_{i,i+1} \tag{7}$$

where $q'_i = q_i - f_i$.

Equations 4 and 7 are now solved simultaneously for the two unknown variables $P'_{i,i}$ and $P'_{i,i+1}$, yielding the following solutions:

$$P'_{i,i+1} = \left(\frac{P_{i,i+1}}{P_{i,i} + P_{i,i+1}}\right)(P_{i,i} + P_{i,i+1} - q'_i) \tag{8}$$

$$P'_{i,i} = \left(\frac{P_{i,i}}{P_{i,i+1}}\right) P'_{i,i+1} \tag{9}$$

These derived solutions are used to eliminate the future transition probabilities P'_{ij} and $P'_{i,i+1}$ from subsequent considerations and to replace them in terms of the remaining three variables, namely, the present transition probabilities P_{ij}, the present maintenance and rehabilitation probabilities f_{ij}, and the future maintenance and rehabilitation probabilities q_{ij}. Among those remaining variables, the future maintenance and rehabilitation probabilities q_{ij} are the only unknown variables to be investigated in the pursuit of an optimum maintenance and rehabilitation program.

DEFINITION OF DECISION POLICY

Determination of the optimum maintenance and rehabilitation variables, q'_{ij}, depends primarily on the decision policy selected. The decision policy is simply based on the state probabilities, Q_i, which represent the fraction of pavement sections expected to be in a particular state after a given number of transitions has taken place. One transition represents a discrete time interval of practically 1 or 2 years. State probabilities can be derived from the following relationship:

$$Q^{(n)} = Q^{(0)} \times P^{(n)} \tag{10}$$

where

$Q^{(0)}$ = initial state probability row vector,
$= \left[Q_a^{(0)}, Q_b^{(0)}, \ldots, Q_f^{(0)}\right]$, and
$P^{(n)}$ = transition matrix multiplied n times to present n transitions.

The decision policy includes two options. The first optimizes a particular state probability or a combination of state probabilities for a selected study period, which should not exceed 6 years, as recommended by researchers (*5*), to obtain a better estimation by satisfying the stationary requirement of the Markovian model. A 6-year period is equivalent to three transitions if a 2-year time interval between transitions is used. A total of six different optimum model selections are suggested in this option:

1. Maximizing proportion of "excellent" pavement sections in condition State a:

$$\text{Maximize } F = Q_a^{(n)} \tag{11}$$

2. Maximizing proportion of "fair" pavement sections in condition State c:

$$\text{Maximize } F = Q_c^{(n)} \tag{12}$$

3. Maximizing total proportions of "excellent" and "good" pavement sections in condition States a and b:

$$\text{Maximize } F = Q_a^{(n)} + Q_b^{(n)} \tag{13}$$

4. Minimizing proportion of "bad" pavement sections in condition State f:

$$\text{Minimize } F = Q_f^{(n)} \tag{14}$$

5. Minimizing total proportions of "poor" and "bad" pavement sections in condition States d and f:

$$\text{Minimize } F = Q_d^{(n)} + Q_f^{(n)} \tag{15}$$

6. Maximizing a weighted average pavement condition in which condition is calculated as an academic grade point average:

$$\text{Maximize } F = 4Q_a^{(n)} + 3Q_b^{(n)} + 2Q_c^{(n)} + Q_d^{(n)} \tag{16}$$

where F is a nonlinear objective function with degree n and takes on the following general form:

$$F = b_0 + \sum_{i=1}^{m} b_i x_i + \sum_{j=1}^{m}\sum_{i=1}^{m} b_{ij} x_i x_j + \sum_{k=1}^{m}\sum_{j=1}^{m}\sum_{i=1}^{m} b_{ijk} x_i x_j x_k + \ldots \tag{17}$$

where

$b_0, b_i, b_{ij}, b_{ijk}$ = constants,
m = number of maintenance variables included in the Markov model, and
X_i, X_j, X_k = variables representing the maintenance and rehabilitation transition probabilities q_{ij}.

It is clear that the state probabilities, Q_i, are functions of the maintenance and rehabilitation transition probabilities q_{ij}, which are unknown. Solutions to the foregoing optimum models will yield the corresponding optimum maintenance and rehabilitation probabilities q'_{ij}, which define the optimum maintenance and rehabilitation program to be implemented. The resulting nonlinear optimization models are subjected to budget and physical constraints.

The second option aims at minimizing total maintenance and rehabilitation costs subject to predefined pavement condition levels. Pavement condition levels are set by specifying the desired state probabilities at the end of the study period, namely,

$$Q^{(n)} = Q_a^{(n)}, Q_b^{(n)}, \ldots, Q_f^{(n)} \tag{18}$$

This option provides useful information for planning purposes or a basis for seeking taxpayer support of additional funding. The resulting optimization model has an objective function representing the total cost of maintenance and rehabilitation actions over the study period. This total cost is a function of several variables including the number of transitions selected for the study period, the targeted state probabilities at the end of the study period, the unit costs of various maintenance and rehabilitation strategies, and the unknown maintenance and rehabilitation variables, q_{ij}. The optimization model is subject to equality and inequality constraints. The equality constraints are constructed to achieve the desired state probabilities, and the inequality constraints consist mainly of the physical ones placed on the variables. The outcome of the optimization process is an optimum maintenance and rehabilitation program defined in terms of the optimum maintenance and rehabilitation variables q'_{ij} that would satisfy the state probabilities requirements while minimizing maintenance and rehabilitation costs.

FORMULATION OF OPTIMIZATION MODELS

The optimum models defined in the preceding section, based on the two presented options, are mathematically formulated. In the first option, an optimum model optimizes a state probability or a combination of state probabilities as suggested in the six outlined

decision policies. The optimum model is presented in a general form as follows:

Optimize

$$\mathbf{F} = Q_i^{(k)} = f(Q_i^{(0)}, q_i, P_{ij}, f_i) \tag{19}$$

Subject to

1. $\sum_{i=b}^{f} uc_i Q_i^{(k-l)} q_i f_i L \le AB_k \ (k = 1, 2, \ldots, n)$,
2. $\sum q_i \le 1$,
3. $0 \le q_i \le 1$,

where

$Q_i^{(k)}$ = state probability i after k transitions,
q_i = total future maintenance and rehabilitation probabilities (variables),
f_i = total present maintenance and rehabilitation probabilities (variables),
AB_k = available budget during the kth transition,
uc_i = unit cost (\$/lane-kilometer) associated with q_i,
P_{ij} = present transition probabilities, and
L = total length of street network in lane-kilometers.

The first set of inequality constraints represents the annual maintenance and rehabilitation costs that must be met by the annually allocated funding. The first constraint of this set is linear in form, whereas subsequent constraints are nonlinear. The degree of nonlinearity depends on the number of transitions employed in the optimization process and therefore differs for every budget constraint. Every budget constraint is one degree higher than the one preceding it, with the last constraint having a degree n, where n is equal to the number of transitions deployed.

The physical constraints represented by $\Sigma q_i \le 1$ can consist of at most the following cases:

$$O_c + R_c \le 1$$
$$I_d + R_d \le 1$$
$$M_f + R_f \le 1$$

where R, O, I, and M denote maintenance, overlay, overlay with localized reconstruction, and complete reconstruction, respectively, as applied to applicable condition states.

The second option minimizes the total maintenance and rehabilitation cost over a specified number of transitions. The main objective in this case is to find the maintenance and rehabilitation program that would minimize the cost of implementing a particular policy compatible with the required objective. The objective function of this model is simply the summation of all cost constraints used in the first optimum option. This model has the following format:

Minimize

$$\mathbf{F} = \sum_{k=1}^{n} \sum_{i=b}^{f} uc_i \, Q_i^{(k-1)} q_i f_i L \tag{20}$$

Subject to

1. $Q^{(0)} P^{(n)} = A^{(n)}$,
2. $\sum q_i \le 1$,
3. $0 \le q_i \le 1$,

where $A^{(n)}$ is a row vector representing the desired fractions of streets in the various states at the end of the analysis period.

The foregoing model is subject to both equality and inequality constraints. The equality constraints result from specifying the desired state probabilities, whereas the inequality constraints are the same physical constraints placed on the variables.

SELECTION OF APPLICABLE OPTIMIZATION METHODS

Several optimization methods could be applied to solve problems similar to the nonlinear models presented in this paper. The choice of a particular technique depends on, among other things, the form of the objective function and its characteristics and on the type of associated constraints. The objective function associated with the presented nonlinear models can vary in complexity depending on the number of transitions (n) employed in the analysis period. Generally, those functions are multivariable polynomials of degree n as presented earlier. The associated constraints are of two types: inequality constraints associated with budgets and physical constraints placing lower and upper bounds on the variables, and equality constraints representing the desirable proportions of pavement sections in the various condition states. The inequality budget constraints are also represented by degree n multivariable polynomials with the first constraint being linear and the last one having a degree n. The physical constraints in inequality form are all linear.

A constrained optimization problem can always be converted to an unconstrained problem. Several optimization methods can then be applied to solve the problem. Among these methods are the penalty function, the barrier function, Lagrange multipliers, and the multipliers of Hestenes (*10*). The outcome of converting a constrained problem to an unconstrained one is the formulation of an auxiliary or augmented function. Then several unconstrained optimization methods can be applied to solve the problem. All of these methods have one property in common. They are search methods in which the search for the optimal solution is initiated by specifying a starting point. These methods use derivatives, function evaluations, or conjugate directions in their search.

A solution based on derivatives or conjugate directions requires the derivation of the auxiliary function in a closed form for every possible decision policy selection. The work required to achieve this would be very tedious and would require a great deal of computer storage. A more practical approach would be to apply an optimization technique that would depend on functional evaluations rather than on derivatives. The optimization technique selected for this purpose is the iterative penalty function method with the method of Hooke and Jeeves with discrete steps used between successive iterations.

The penalty function method transforms the constrained problem into a sequence of unconstrained problems. The constraints are inserted into the objective function, by means of a penalty parameter, in such a way that any violation of the constraints is penalized. The general form of the auxiliary unconstrained problem is

Minimize

$$F(x) = f(x) + u_k A(x) \tag{21}$$

where

$f(x)$ = objective function,
$A(x)$ = penalty function, and
u_k = penalty parameter for the kth iteration.

The penalty function $A(x)$ contains both the inequality constraints $g(x)$ and the equality constraints $h(x)$ and is mathematically presented in the following form (*10*):

$$A(x) = \sum_{i=1}^{m_1} \{\max[0, g_i(x)]\}^s + \sum_{i=1}^{n_1} [h_i(x)]^s \tag{22}$$

where

m_1 = number of inequality constraints,
n_1 = number of equality constraints, and
s = positive integer taken to be 2.

An outlined algorithm of the penalty function method can be found in the text cited earlier (*10*). The method of Hooke and Jeeves with discrete steps has been applied to solve the generated sequence of unconstrained optimization problems as they result from the penalty function. The method of Hooke and Jeeves applies a simple search scheme involving functional evaluations. This method uses the coordinate axes as the search directions. It performs two types of search, namely, exploratory search and acceleration search. A detailed algorithm of this method can also be found in the same text (*10*).

A Fortran computer program has been written to solve the presented optimization models using the penalty function method. The results obtained have clearly indicated the effectiveness of the penalty method in solving all presented maintenance and rehabilitation models. Sample results based on selected input data are shown in the next section.

SAMPLE RESULTS FROM CASE STUDY

A pavement condition assessment study based on visual inspection of pavement defects was conducted on a portion of a roadway network in the city of Northwood, Ohio. A total of 120 pavement sections, part of the arterial system, were surveyed for the purpose of estimating the initial state and transition probabilities. The length of a pavement section was taken to be 150 m (500 ft). All selected sections fall in approximately the same category as far as traffic, pavement structure, and other factors are concerned. The five condition states were defined by establishing limits on various pavement defects. The survey was conducted twice, with a separation time of 2 years between the two surveys. In both surveys, a pavement section was rated on the basis of the severity of prevailing defects and was consequently assigned a corresponding condition state. The estimated initial state probabilities for States a, b, c, d, and f were found to be equal to 0.23, 0.29, 0.43, 0.043, and 0.015, respectively, and the present transition probabilities were estimated to be P_{aa} = 0.44, P_{bb} = 0.52, P_{cc} = 0.62, and P_{dd} = 0.67.

The following basic definition of transition probabilities was used to make the foregoing estimates:

$$P_{i,i} = \frac{N_f}{N_o} \qquad P_{i,i+1} = 1 - P_{i,i} \tag{23}$$

where N_o is the number of pavement sections found initially in State i, and N_f is the number of pavement sections found in State i after one transition.

It was determined that the city of Northwood did not have an active maintenance and rehabilitation program. Therefore, the present maintenance and rehabilitation variables, f_{ij}, all vanished from consideration. Maintenance and rehabilitation costs along with allocated funding for a study period of four transitions (n = 4) were estimated to be as follows:

L = 19 lane-km (12 lane-mi) (total length of arterial pavement sections inspected),
C_f = \$97,750 (reconstruction cost per lane-kilometer) (\$156,400 = reconstruction cost per lane-mile),
C_d = \$78,200 (overlay with localized reconstruction),
C_c = \$33,200 (overlay only),
C_{ba} = \$2,718 (routine maintenance per lane-kilometer applied to State b) (\$4,350 = routine maintenance per lane-mile applied to State b),
C_{cb} = \$3,625 (routine maintenance per lane-kilometer applied to State c) (\$5,800 = routine maintenance per lane-mile applied to State c),
C_{dc} = \$4,531 (routine maintenance per lane-kilometer applied to State d) (\$7,250 = routine maintenance per lane = mile applied to State d),
C_{fd} = \$5,781 (routine maintenance per lane-kilometer applied to State f) (\$9,250 = routine maintenance per lane-mile applied to State f),
n = 4 (number of transitions employed),
$AB1$ = \$80,000 (available budget during the first transition),
$AB2$ = \$85,000 (available budget during the second transition),
$AB3$ = \$90,000 (available budget during the third transition), and
$AB4$ = \$95,000 (available budget during the fourth transition). In addition, one transition = 2 years.

All initialization parameters for both the penalty function method and the method of Hooke and Jeeves were built into the computer program after they had been tested and found to be very effective in yielding accurate optimum solutions.

The computer solution for the first option optimum models, which maximizes total proportions of States a and b, is shown in Table 1. Model number 1 simply represents the "do-nothing" alternative. Models 2 through 9 contain only four distinct improvement variables, whereas models 10 through 21 are constructed by including five variables. These distinct variables are any combination of the seven maintenance and rehabilitation variables defined earlier.

Generally, there are seven improvement variables to be considered in the Markov model, as presented earlier. The different models are formed by requiring a minimum of one improvement plan to be applied to each of the condition States b, c, d, and f. In addition, it is required that a maximum of two improvement plans, consisting of both routine maintenance and major rehabilitation, be applied to States c, d, and f. Table 2 shows similar results for Models 22 through 28. Models 22 through 27 include a maximum of six variables, whereas Model 28 contains all seven variables.

The variables X_1 through X_7 are used to represent the seven maintenance and rehabilitation variables, namely, R_b, R_c, R_d, and R_f for routine maintenance and O_c, I_d, and M_f for major rehabilitation (Table 3). Reference to Table 4 is required to identify the corresponding maintenance and rehabilitation plans associated with a particular model. For example, the variables X_1 through X_4 represent the four routine maintenance variables R_b, R_c, R_d, and R_f used in Model 2. The values of the optimum variables represent the fractions of repair work to be applied to pavement sections in various condition states. For example, a value of unity implies that all pavement sections in the corresponding state should receive the designated treatment plan.

Results in Tables 1 and 2 show that the majority of models yielded optimum solutions, as is evident from the values of the corresponding objective functions provided in the second column. A maximum objective value of 0.984 was reached by Model 5, with several other models reaching very close results. This result indicates that the optimization process has successfully converged with optimum solutions very close to the absolute maximum value of unity. Of course, not all models can necessarily yield optimum solutions with similar results. It can also be concluded that those models reaching close optimum

TABLE 1 Optimum Maintenance and Rehabilitation Programs for Maximizing Total Proportions of Pavement Sections in Condition States a and b: Models 1–21

Model No.	Q_A+Q_B	X_1	X_2	X_3	X_4	X_5	Cost ($1000)
1	.0872	.000	.000	.000	.000	.000	000.0
2	.9806	1.000	1.000	1.000	1.000	_#	132.0
3	.8073	.956	.356	.812	.056	_	219.5
4	.9766	1.000	.824	.646	.512	_	230.2
5	.9840	1.000	.929	.882	.953	_	199.1
6	.8020	.962	.326	.169	.145	_	236.9
7	.8386	.969	.302	.337	.756	_	288.3
8	.9835	1.000	1.000	1.000	1.000	_	159.5
9	.8134	.942	.358	.944	.672	_	223.2
10	.9652	1.000	.071	.682	.474	.475	248.6
11	.9833	1.000	.213	.787	1.000	1.000	167.1
12	.9616	1.000	.128	.514	.651	.870	253.5
13	.9353	1.000	.230	.412	.859	.376	207.6
14	.7502	.825	.334	.132	.001	.126	251.7
15	.9753	1.000	.773	.669	.001	.532	239.0
16	.9843	1.000	1.000	.500	.500	1.000	163.1
17	.7887	.872	.366	.001	.799	.707	222.2
18	.9835	1.000	1.000	1.000	.999	.001	159.5
19	.9783	1.000	.896	.565	.552	.001	220.9
20	.8075	.956	.356	.816	.055	.001	219.6
21	.8020	.962	.326	.169	.144	.001	263.5

#: Representing models with four variables.
_: Not applicable

solutions are compatible with each other in terms of their expected performance.

The results also show several optimum improvement variables with values close to unity. These variables represent routine maintenance and were selected because of their relatively low cost. In contrast, other variables with low close-to-zero values, representing major rehabilitation, carry low weight because of their relatively high cost. Obviously, pavement maintenance engineers are presented with a wide variety of optimum repair programs. The choice can be made considering several factors including predicted objective outcome, associated cost, and confidence level in the expected performance of treatment plans. The predicted objective outcome may not necessarily agree with the actual one considering the probabilistic nature of pavement deterioration. Therefore, revised repair programs should be sought on the basis of newly refined input parameters. This revision would provide an opportunity to improve the effectiveness of the models presented.

The computer solution for the second option optimum models, which minimizes total maintenance and rehabilitation cost over the study period, is provided in Table 4. The specified state probabilities at the end of the study period are 0.35, 0.30, 0.20, 0.10, and 0.05 for States a, b, c, d and f, respectively. Table 4 shows the optimum improvement programs for Models 1 through 21. Examination of the cost column in Table 4 reveals a wide variation in the total cost. Again, this result is attributed to the low cost of performing routine maintenance in comparison with major rehabilitation.

Table 5 shows results that are associated with the optimum solutions presented in Table 4, but they are provided only for Models 1 through 9 for the purpose of demonstrating the values of the state probabilities at the end of each transition.

The values of the state probabilities at the end of the fourth transition clearly demonstrate the convergence of the optimization process to the required specified state probabilities for all indicated models.

TABLE 2 Optimum Maintenance and Rehabilitation Programs for Maximizing Total Proportions of Pavement Sections in Condition States a and b: Models 22–28

Model No.	Q_A+Q_B	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Cost ($1000)
22	.9318	.997	.257	.394	.001	.859	.212	_#	201.8
23	.9514	1.000	.221	.436	.240	.742	.925	_	207.2
24	0.9649	1.000	.069	.651	.493	.486	.001	_	254.7
25	0.9353	1.000	.230	.412	.859	.376	.000	_	207.6
26	0.7583	.839	.339	.136	.001	.127	.001	_	253.8
27	0.9577	.916	.824	.655	.001	.526	.001	_	230.5
28	0.9143	.994	.299	.322	.021	.714	.001	.801	195.0

#: Models that contain only six maintenance variables.
_: Not applicable

TABLE 3 Identification of Maintenance and Rehabilitation Variables

Model No.	R_b	R_c	R_d	R_f	O_c	I_d	M_f
1	-	-	-	-	-	-	-
2	X_1	X_2	X_3	X_4	-	-	-
3	X_1	-	X_2	-	X_3	-	X_4
4	X_1	X_2	-	-	-	X_3	X_4
5	X_1	X_2	-	X_3	-	X_4	-
6	X_1	-	-	-	X_2	X_3	X_4
7	X_1	-	-	X_2	X_3	X_4	-
8	X_1	X_2	X_3	-	-	-	X_4
9	X_1	-	X_2	X_3	X_4	-	-
10	X_1	X_2	-	-	X_3	X_4	X_5
12	X_1	X_2	-	X_3	X_4	X_5	-
13	X_1	X_2	X_3	-	X_4	-	X_5
14	X_1	-	X_2	-	X_3	X_4	X_5
15	X_1	X_2	X_3	-	-	X_4	X_5
16	X_1	X_2	X_3	X_4	-	X_5	-
17	X_1	-	X_2	X_3	X_4	X_5	-
18	X_1	X_2	X_3	X_4	-	-	X_5
19	X_1	X_2	-	X_3	-	X_4	X_5
20	X_1	-	X_2	X_3	X_4	-	X_5
21	X_1	-	-	X_2	X_3	X_4	X_5
22	X_1	X_2	X_3	-	X_4	X_5	X6
23	X_1	X_2	X_3	X_4	X_5	X6	-
24	X_1	X_2	-	X_3	X_4	X_5	X6
25	X_1	X_2	X_3	X_4	X_5	-	X6
26	X_1	-	X_2	X_3	X_4	X_5	X6
27	X_1	X_2	X_3	X_4	-	X_5	X6
28	X_1	X_2	X_3	X_4	X_5	X6	X7

_: Not applicable

TABLE 4 Optimum Maintenance and Rehabilitation Programs for Minimizing Maintenance and Rehabilitation Cost

Model No.	X_1	X_2	X_3	X_4	X_5	Cost ($1000)
1	.0000	.0000	.0000	.0000	_#	.0
2	.7468	.3100	.6012	.2387	_	94.6
3	.3775	.3850	.2937	.4918	_	314.4
4	.4612	.0962	.6712	.1318	_	357.9
5	.4649	.0925	.7099	.0837	_	346.7
6	.3550	.1562	.5718	.2512	_	393.9
7	.3718	.1412	.6562	.1525	_	370.5
8	.7062	.2900	.5043	.3306	_	156.8
9	.4112	.4337	.4206	.3744	_	231.1
10	.3874	.1075	.0306	.6031	.2150	382.8
11	.7406	.0075	.3049	.5981	.2419	97.1
12	.3712	.1431	.0006	.6531	.1562	369.9
13	.7062	.0006	.2893	.5043	.3306	157.0
14	.3718	.3218	.1637	.2100	.4337	335.0
15	.6175	.2206	.2619	.3043	.2637	229.6
16	.6581	.2412	.2500	.3881	.1900	174.4
17	.4037	.3612	.1774	.3093	.3326	262.6
18	.7462	.3106	.6006	.0006	.2393	94.7
19	.4612	.0968	.6693	.1343	.0006	357.8
20	.4012	.4150	.3743	.1800	.2362	260.4
21	.3556	.1562	.5718	.2500	.0006	393.7

#: Models that contain only four maintenance variables.
_: Not applicable

TABLE 5 State Probabilities at End of Each Transition Associated with Minimizing Maintenance and Rehabilitation Cost

Model No.	K^{th} Transition	State Probabilities for the Kth transition				
		Q_a	Q_b	Q_c	Q_d	Q_f
1	1	0.1012	0.2796	0.4058	0.1922	0.0292
	2	0.0445	0.2021	0.3858	0.2830	0.0926
	3	0.0196	0.1300	0.3362	0.3362	0.1860
	4	0.0086	0.0786	0.2708	0.3530	0.2970
2	1	0.2961	0.2982	0.2611	0.1343	0.0183
	2	0.3307	0.2895	0.2363	0.1168	0.0347
	3	0.3401	0.3005	0.2144	0.1081	0.0449
	4	0.3516	0.3015	0.2016	0.1016	0.0516
3	1	0.3640	0.2370	0.2672	0.1217	0.0181
	2	0.3468	0.2908	0.2092	0.1224	0.0387
	3	0.3460	0.2991	0.2043	0.1092	0.0494
	4	0.3516	0.3016	0.2016	0.1016	0.0516
4	1	0.2509	0.2558	0.3249	0.1587	0.0177
	2	0.3200	0.2519	0.2558	0.1478	0.0326
	3	0.3436	0.2831	0.2156	0.1213	0.0443
	4	0.3516	0.3016	0.2016	0.1016	0.0516
5	1	0.2516	0.2538	0.3254	0.1593	0.0180
	2	0.3243	0.2503	0.2558	0.1457	0.0319
	3	0.3457	0.2838	0.2153	0.1198	0.0434
	4	0.3517	0.3016	0.2016	0.1016	0.0516
6	1	0.2846	0.2362	0.3197	0.1502	0.0173
	2	0.3339	0.2500	0.2444	0.1456	0.0342
	3	0.3503	0.2819	0.2095	0.1201	0.0462
	4	0.3516	0.3016	0.2017	0.1016	0.0515
7	1	0.2827	0.2336	0.3216	0.1523	0.0178
	2	0.3406	0.2464	0.2458	0.1425	0.0327
	3	0.3541	0.2824	0.2096	0.1175	0.0443
	4	0.3517	0.3014	0.2018	0.1015	0.0516
8	1	0.2902	0.2962	0.2673	0.1365	0.0178
	2	0.3216	0.2887	0.2362	0.1250	0.0365
	3	0.3364	0.2970	0.2155	0.1121	0.0470
	4	0.3516	0.3017	0.2016	0.1015	0.0516
9	1	0.3857	0.2331	0.2549	0.1155	0.2311
	2	0.3610	0.2979	0.2037	0.1093	0.0361
	3	0.3530	0.3042	0.2029	0.1015	0.0464
	4	0.3515	0.3017	0.2016	0.1016	0.0516

_: Not applicable

CONCLUSIONS AND RECOMMENDATIONS

A discrete-time Markov model with five condition states has been presented. Seven pavement improvement variables representing maintenance and rehabilitation actions have been incorporated into the developed model. Four of these variables are intended to be taken by routine maintenance and the other three by major rehabilitation. Two major decision policies have been outlined to optimize the formulated Markov model to yield an optimum maintenance and rehabilitation program based on random selection of pavement improvement candidates. The first major policy optimizes a state probability or a combination of several state probabilities subjected to budget constraints and physical constraints placing upper and lower bounds on the variables. The second major policy minimizes total maintenance and rehabilitation cost subjected to specified state probabilities to be achieved at the end of the study period.

The optimum models presented have been successfully solved using a specially designed computer program. The computer program applies nonlinear optimization algorithms to solve the constrained problem by converting it to an unconstrained auxiliary function. The penalty function method is then used, with successive iterations, to solve the unconstrained problem. The method of Hooke and Jeeves with discrete steps was applied to solve the generated sequences of unconstrained optimization problems as they result from the penalty function. The sample results presented demonstrated the convergence accuracy of this optimization technique. Computer time to solve a particular model was found to be minimal. The results also demonstrated that the variables representing routine maintenance dominated the optimum solutions. This result is to be

expected, since the cost of routine maintenance is substantially less than the cost of major rehabilitation.

On the basis of the developed maintenance and rehabilitation models and the corresponding successful solutions, it is recommended that the approach outlined in this paper be built into an Integrated Pavement Maintenance Management System (IPMMS) and made available for public use. The proposed IPMMS will be designed to take major influencing factors into consideration, such as varying traffic and loading conditions and differing pavement structures. The system is mainly intended to serve as an effective decision-making tool for planning and scheduling of pavement repair work. It will provide pavement maintenance engineers with a wide variety of options and guide them step by step through the entire interactive session. The system will also be designed to offer self-learning demonstrations so that the engineers can receive on-the-job training sessions, eliminating the need for formal training. The system can easily be operated using a personal computer.

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