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Hot nuclear matter in the modified quark–meson coupling model with quark–quark correlations

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Abstract. Short-range quark–quark correlations in hot nuclear matter are examined within the modified quark–meson coupling (MQMC) model by adding repulsive scalar and vector quark–quark interactions. Without these correlations, the bag radius increases with the baryon density. However, when the correlations are introduced the bag size shrinks as the bags overlap. Also as the strength of the scalar quark–quark correlation is increased, the decrease of the effective nucleon mass M_N^* with the baryonic density is slowed down and tends to saturate at high densities. Within this model we study the phase transition from the baryon–meson phase to the quark–gluon plasma (QGP) phase with the latter modelled as an ideal gas of quarks and gluons inside a bag. Two models for the QGP bag parameter are considered. In one case, the bag is taken to be medium-independent and the phase transition from the hadron phase to QGP is found to occur at five to eight times ordinary nuclear matter density for temperatures less than 60 MeV. For lower densities, the transition takes place at a higher temperature, reaching up to 130 MeV at zero density. In the second case, the QGP bag parameter is considered to be medium-dependent as in the MQMC model for the hadronic phase. In this case, it is found that the phase transition occurs at much lower densities.

1. Introduction

The modified quark–meson coupling (MQMC) model has been used recently to study cold [1, 2] and hot [3, 4] nuclear matter. In this model [5–7], nucleons are assumed to be non-overlapping MIT bags interacting through scalar σ and vector ω mean fields coupled to the quarks themselves. In analogy with the non-topological soliton model [8], the bag parameter is assumed to decrease when the scalar mean field σ increases [1, 2], which makes the bag parameter medium- or density-dependent. However, as a result of introducing this medium-dependent bag parameter, it is found that as the baryonic density ρ_B increases, the nucleon bag radius increases [2, 9]. At some value of ρ_B, the bags start to overlap [10]. Since the MQMC model assumes that the bags do not overlap, the use of this model has been limited to small and moderate baryonic densities [2].

One way to extend the model is to include short-range quark–quark correlations [10] which become important when the bags overlap at high baryonic densities. These correlations are introduced by adding extra repulsive scalar and vector contact forces between the quarks in the overlapping bags to reduce the overlapping domain between the nucleons. We will follow Saito *et al* [10] and introduce these correlations in a simple geometrical way by defining a critical rigid-ball nucleonic radius R_c which, assuming close packing, can be related to the baryonic density by $R_{\mathrm{c}} = \left(1/4\sqrt{2}\rho_B\right)^{1/3}$. Hence, for a given nuclear density ρ_B, the nucleon bags are assumed to overlap only when the bag radius R is larger than R_{c}. When the nucleon bags overlap, the quarks in the bags correlate with each other by a repulsive potential to reduce the overlapping effect by shrinking the size of the bags. Within this model, we shall study the quark–quark correlations for hot nuclear matter as well as their effects on the phase transition from the hadronic phase to the quark–gluon plasma (QGP) phase. The QGP is considered as an ideal gas of non-interacting quarks and gluons inside a bubble or bag with bag parameter B [11]. It is interesting to study the variation of the phase transition with the strength of the quark–quark correlation and to examine the possibility that the bag parameter for the QGP is also medium-dependent as in the MQMC model.

The outline of the paper is as follows. In section 2, the quark–quark correlations for the overlapping bags are generalized to the case of hot nuclear matter. In section 3 we introduce a simple model for the QGP. Finally, section 3 is devoted to our results and conclusions.

2. Quark–quark correlations

In dense nuclear matter, the nucleons are expected to overlap and the quarks in one nucleon correlate with the quarks in another. This correlation depends basically on how much the nucleons overlap with each other. The probability $P(R_{\mathrm{c}}/R)$ for two nucleons, each of radius R, to overlap can be estimated, using a simple geometrical approach [10, 12], to be

$$P\left(\frac{R_{\mathrm{c}}}{R}\right) = \left[1 - \frac{3}{4}\left(\frac{2R_{\mathrm{c}}}{R}\right) + \frac{1}{16}\left(\frac{2R_{\mathrm{c}}}{R}\right)^3\right]\theta\left(\frac{R_{\mathrm{c}}}{R}\right)\theta\left(1 - \frac{R_{\mathrm{c}}}{R}\right). \tag{1}$$

Since the quark–quark correlations are of short range it is reasonable to approximate them by a contact interaction [10]. In the mean-field approximation, the Dirac equation for the quark field inside a nucleon bag is given by

$$\left[\mathrm{i}\gamma\cdot\partial - (m_q - g_\sigma^q\sigma + f_s^q\langle\overline{\psi}_q\psi_q\rangle) - (g_\omega^q\omega + f_v^q\langle\psi_q^\dagger\psi_q\rangle)\beta\right]\psi_q = 0 \tag{2}$$

where m_q is the current quark mass, $f_{s(v)}^q$ is the coupling constant for scalar- (vector-) type short-range correlations, while $\langle\overline{\psi}_q\psi_q\rangle$ and $\langle\psi_q^\dagger\psi_q\rangle$ are the average values of the quark scalar density and quark density [6]. The latter, following [10], are approximated by $\langle\overline{\psi}_q\psi_q\rangle = \frac{m_\sigma^2}{g_\sigma}\sigma$ and $\langle\psi_q^\dagger\psi_q\rangle = 3\rho_B$. In this paper, as suggested by [10], the correlation potentials are taken as

$$f_s^q\langle\overline{\psi}_q\psi_q\rangle = \alpha P\,(R_{\mathrm{c}}/R)\,\sigma \tag{3}$$

and

$$f_v^q\langle\psi_q^\dagger\psi_q\rangle = \beta P\,(R_{\mathrm{c}}/R)\,\rho_B \tag{4}$$

where α and β are parameters used to control the strengths of the scalar and vector quark–quark correlations. Note that as defined here α is a dimensionless parameter, while β has the dimensions of $1/(\mathrm{energy})^2$. The coupling constants g_σ^q and g_ω^q for the scalar and vector mean fields are determined by reproducing the properties of normal nuclear matter.

The single-particle quark and antiquark energies in units of R^{-1} are given as

$$\epsilon_\pm^{n\kappa} = \Omega^{n\kappa} \pm \left[g_\omega^q\omega + f_v^q\langle\psi_q^\dagger\psi_q\rangle\right]R \tag{5}$$

where

$$\Omega^{n\kappa} = \sqrt{x_{n\kappa}^2 + R^2 m_q^{*2}} \tag{6}$$

and $m_q^* = m_q^0 - g_\sigma^q \sigma + f_s^q \langle \overline{\psi}_q \psi_q \rangle$ is the effective quark mass. The boundary condition at the bag surface is given by

$$\mathrm{i}\gamma \cdot \hat{n} \psi_q^{n\kappa} = \psi_q^{n\kappa} \tag{7}$$

which determines the quark momentum $x_{n\kappa}$ in the state characterized by specific values of n and κ. The quark chemical potential μ_q, assuming that there are three quarks in the nucleon bag, is determined through

$$\begin{aligned} n_q &= 3 \\ &= 3 \sum_{n\kappa} \left[\frac{1}{\mathrm{e}^{(\epsilon_+^{n\kappa}/R - \mu_q)/T} + 1} - \frac{1}{\mathrm{e}^{(\epsilon_-^{n\kappa}/R + \mu_q)/T} + 1} \right]. \end{aligned} \tag{8}$$

The total energy from the quarks and antiquarks is

$$E_{\mathrm{tot}} = 3 \sum_{n\kappa} \frac{\Omega^{n\kappa}}{R} \left[\frac{1}{\mathrm{e}^{(\epsilon_+^{n\kappa}/R - \mu_q)/T} + 1} + \frac{1}{\mathrm{e}^{(\epsilon_-^{n\kappa}/R + \mu_q)/T} + 1} \right]. \tag{9}$$

The bag energy is given by

$$E_{\mathrm{bag}} = E_{\mathrm{tot}} - \frac{Z}{R} + \frac{4\pi}{3} R^3 B(\sigma) \tag{10}$$

where $B(\sigma)$ is the bag parameter. The medium effects are taken into account for the bag parameter [1]

$$B = B_0 \exp\left(-\frac{4 g_\sigma^B \sigma}{M_N} \right) \tag{11}$$

where B_0 corresponds to a free nucleon and g_σ^B is an additional parameter. The spurious centre-of-mass momentum of the bag is subtracted to obtain the effective nucleon mass

$$M_N^* = \sqrt{E_{\mathrm{bag}}^2 - \langle p_{\mathrm{cm}}^2 \rangle} \tag{12}$$

where

$$\langle p_{\mathrm{cm}}^2 \rangle = \frac{\langle x^2 \rangle}{R^2} \tag{13}$$

and

$$\langle x^2 \rangle = 3 \sum_{n\kappa} x_{n\kappa}^2 \left[\frac{1}{\mathrm{e}^{(\epsilon_+^{n\kappa}/R - \mu_q)/T} + 1} + \frac{1}{\mathrm{e}^{(\epsilon_-^{n\kappa}/R + \mu_q)/T} + 1} \right]. \tag{14}$$

The bag radius R is obtained by minimizing the effective nucleon mass with respect to the bag radius

$$\frac{\partial M_N^*}{\partial R} = 0. \tag{15}$$

The pressure is given by [3, 4]

$$P = \frac{1}{3} \frac{\gamma}{(2\pi)^3} \int \mathrm{d}^3 k \, \frac{k^2}{\epsilon^*} (f_B + \overline{f}_B) + \tfrac{1}{2} m_\omega^2 \omega^2 - \tfrac{1}{2} m_\sigma^2 \sigma^2 \tag{16}$$

where $\gamma = 4$ is the spin–isospin degeneracy factor and f_B and $\overline{f}_B$ are the Fermi–Dirac distribution functions for the nucleons and antinucleons

$$\begin{aligned} f_B &= \frac{1}{\mathrm{e}^{(\epsilon^* - \mu_B^*)/T} + 1} \\ \overline{f}_B &= \frac{1}{\mathrm{e}^{(\epsilon^* + \mu_B^*)/T} + 1} \end{aligned} \tag{17}$$

with $\epsilon^* = \sqrt{k^2 + M_N^{*2}}$ and $\mu_B^* = \mu_B - 3\left[g_\omega^q \omega + f_v^q \langle \psi_q^\dagger \psi_q \rangle\right]$ being the nucleonic effective energy and effective chemical potential, respectively. The chemical potential μ_B for a given density ρ_B is determined self-consistently by the subsidiary constraint [3, 4]

$$\rho_B = \frac{\gamma}{(2\pi)^3} \int \mathrm{d}^3 k \, (f_B - \overline{f}_B) \tag{18}$$

with

$$\omega = \frac{g_\omega}{m_\omega^2} \rho_B. \tag{19}$$

The scalar mean field σ is determined through maximizing the pressure $\frac{\partial P}{\partial \sigma} = 0$, which yields the self-consistency condition (SCC) for the σ field. Since the scalar-type correlation does not directly involve the σ field, the SCC is not formally modified by it and is therefore identical to that found in our earlier work [3]. The correlations do, however, affect the σ field indirectly through the quark wavefunctions.

3. The quark–gluon plasma phase

In the QGP phase we assume that we have only u and d quarks confined inside a bag with bag parameter B. This parameter can be interpreted as the energy per unit volume needed to create a bubble or bag in which the non-interacting quarks and gluons are confined. The total baryonic density is given by [11]

$$\rho_B = \frac{1}{3} \frac{\gamma_Q}{(2\pi)^3} \int \mathrm{d}^3 k \, [n_k(T) - \overline{n}_k(T)]. \tag{20}$$

The quark and antiquark distribution functions are given by

$$\begin{aligned} n_k &= \frac{1}{\exp[(k - \frac{1}{3}\mu_B)/T] + 1} \\ \overline{n}_k &= \frac{1}{\exp[(k + \frac{1}{3}\mu_B)/T] + 1} \end{aligned} \tag{21}$$

respectively, where μ_B is the baryon chemical potential, and we have assumed that the quarks have a baryon number of $\frac{1}{3}$. At finite ρ_B, equation (20) is inverted to find the baryon chemical potential μ_B. The pressure of the quark–gluon plasma is given by [11]

$$P = -B + \frac{1}{3} \frac{\gamma_G}{2} \frac{T^4 \pi^2}{15} + \frac{1}{3} \frac{\gamma_Q}{(2\pi)^3} \int \mathrm{d}^3 k \, k \, [n_k(T) + \overline{n}_k(T)] \tag{22}$$

where $k = |\vec{k}|$ and $\gamma_Q = 12$ for quarks and $\gamma_G = 16$ for gluons. The thermodynamic conditions for phase equilibrium between the baryon–meson phase and the QGP phase are satisfied by assuming mechanical, chemical and thermal equilibrium between the two phases namely, $P_{\mathrm{MQMC}} = P_{\mathrm{QGP}}$ and $\mu_{B\,\mathrm{MQMC}} = \mu_{B\,\mathrm{QGP}}$ for a given T. These conditions determine the transition line between the hadron–meson phase and the QGP phase.

We consider two cases. In the first case, the bag parameter in the QGP phase is considered as medium-independent and has the same value as that for a free nucleon. This case may be appropriate for most experimental situations aimed at producing the QGP in small chunks of hot nuclear matter produced in heavy ion collisions. In the second case, the bag parameter in the QGP is assumed to be medium-dependent, and is taken to be equal to the bag parameter of the nucleon in the MQMC model at the same density. This corresponds to the idealized case of producing the QGP in a bubble in infinite hot nuclear matter and may approximately apply to the production of the QGP in central collisions between very massive nuclei.

4. Results and discussions

We have studied nuclear matter at finite temperature using the MQMC model which takes the medium dependence of the bag parameter of the nucleon into account. We choose a direct coupling of the bag parameter to the scalar mean field σ as given in equation (11). The bag parameters are those given in [1, 3, 4] where they are chosen to reproduce the free nucleon mass M_N at its experimental value of 939 MeV and a bag radius $R_0 = 0.60$ fm. For $g_\sigma^q = 1$, the values of the vector meson coupling and the parameter g_σ^B, as fitted from the normal saturation properties of nuclear matter [1], are given as $g_\omega^2/4\pi = 5.24$ and $(g_\sigma^B)^2/4\pi = 3.69$. The current quark mass m_q is taken to be equal to zero. For the short-range quark–quark correlation strengths we use values comparable to those given in [10] which, in the present notation, are $\alpha \approx 9$ and $\beta \approx 34$ GeV^{-2}. The latter value of β is needed to reproduce the empirical value of the energy per nucleon for symmetric nuclear matter in the high-density region $\rho_B/\rho_0 = 2.5$–4, where $\rho_0 = 0.17$ fm^{-3} is normal nuclear density. We carried out calculations for the cases of $\alpha = 0, 5, 7$ and 9 as well as $\beta = 0, 30$ and 60 GeV^{-2}. The scalar quark–quark correlations affect the size and mass of the nucleon, while the vector quark–quark correlations determine the pressure and energy density of nuclear matter [10].

Figure 1 displays isotherms of the effective mass M_N^* versus the baryonic density ρ_B for various strengths of the scalar quark–quark correlations. As already mentioned, the vector correlations do not have an effect on the mass. For low values of α, M_N^* has the usual trend of decreasing with ρ_B. However, as α increases M_N^* tends to saturate and, for still larger values of α, the effective mass even starts to increase slightly at high density, especially at the lower temperatures. This novel feature is quite interesting as it is questionable that the monotonic decrease of M_N^* with density can continue unchecked for higher and higher densities.

Figure 2 displays isotherms of R/R_0 versus the baryon density for several values of α. Without correlations, i.e. $\alpha = 0$, R/R_0 increases monotonically with ρ_B. However, when the correlations are introduced, R/R_0 starts to decrease, rather abruptly, when the bags start to overlap. This abruptness is due to the simple geometrical way in which the correlations are introduced. As α is increased further, R/R_0 decreases more steeply at high densities. The repulsive nature of the quark–quark correlations shrinks the bag size.

Figure 3 displays the transition line between the baryon–meson phase and the QGP phase in the (T, μ_B)-plane, while figure 4 displays it in the (T, ρ_B)-plane. This phase transition line is determined by equalizing the pressure p and the chemical potential μ_B in both phases for a given temperature T. We have considered two cases for the bag parameter of the QGP bubble. In case 1, the bag parameter is taken to be medium-independent and fixed at its free-space value. As the density ρ_B is increased, the nucleon bags start to overlap and the transition line becomes sensitive to the quark–quark correlations for μ_B greater than about 950 MeV, corresponding to densities larger than about 2.5 times normal nuclear matter density. For temperatures $T < 60$ MeV, the phase transition takes place at rather large densities and large chemical potentials. The quark–quark correlations are found to move the transition line to lower chemical potentials and lower densities. For cold nuclear matter the correlations can reduce μ_B from 1850 to 1450 MeV. The corresponding change in the (T, ρ_B)-plane is more dramatic as can be seen by inspecting figure 4. The phase transition, at low temperatures, takes place at densities as high as $8\rho_0$ without correlations, but this value is reduced to about $5\rho_0$ for the strongest correlations considered. In case 2, also shown in figures 3 and 4, we have used a medium-dependent bag parameter for the QGP bubble. This bag parameter is identical to the bag parameter $B(\sigma)$ used for the nucleonic bags in the hadronic phase as

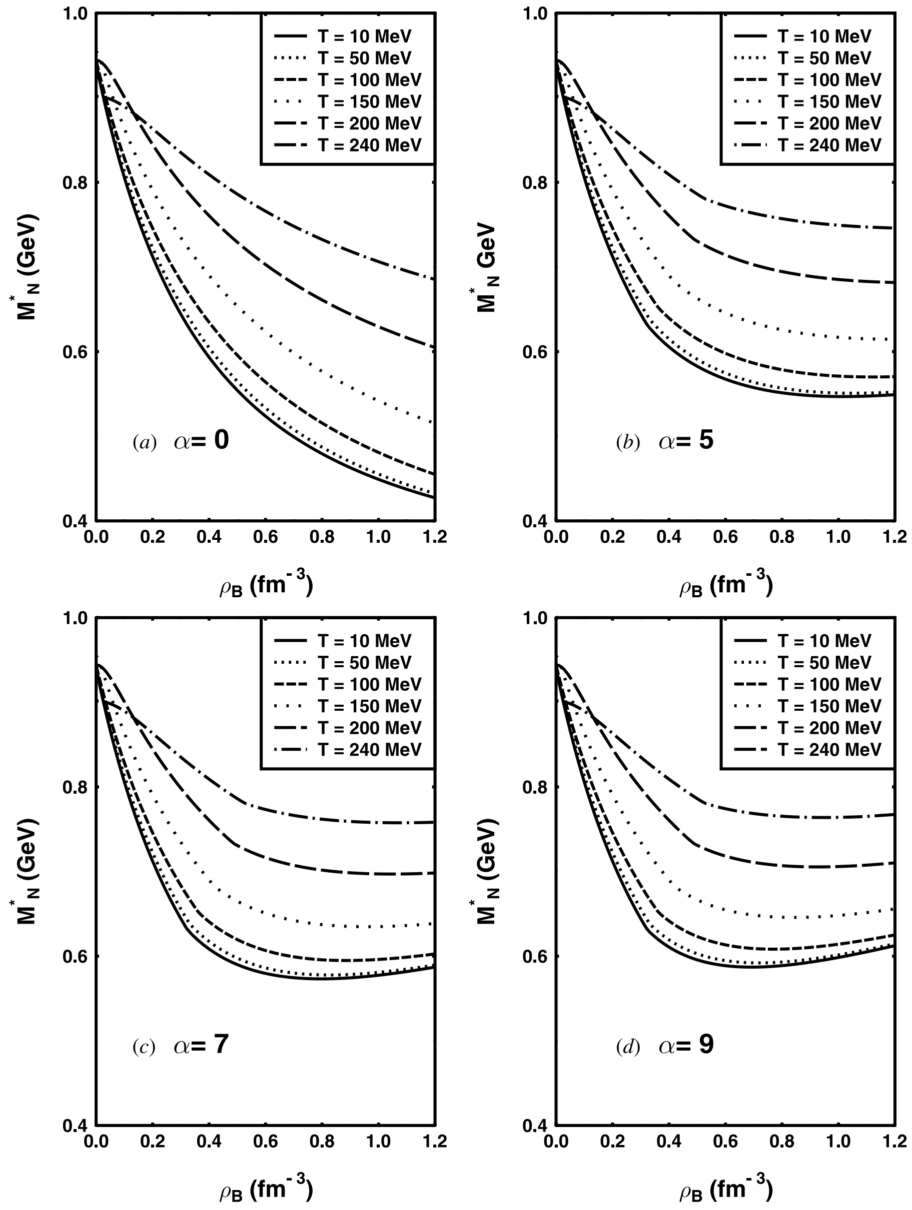

Figure 1. Isotherms of the effective nucleon mass M_N^* as a function of the baryonic density ρ_B for different strengths of the scalar quark–quark correlations.

given in equation (11). This medium dependence is appropriate for the production of a QGP bubble in infinite nuclear matter and may be appropriate for its production in the heart of the participant region in central collisions between very massive nuclei. In this case it is found that the phase transition from the baryon–meson phase to the QGP phase takes place at much lower densities so that the nucleons do not overlap and the quark–quark correlations do not play a role in determining the transition line. The transition temperature falls rapidly with density and the phase transition at low temperatures occurs at a comparatively low chemical potential

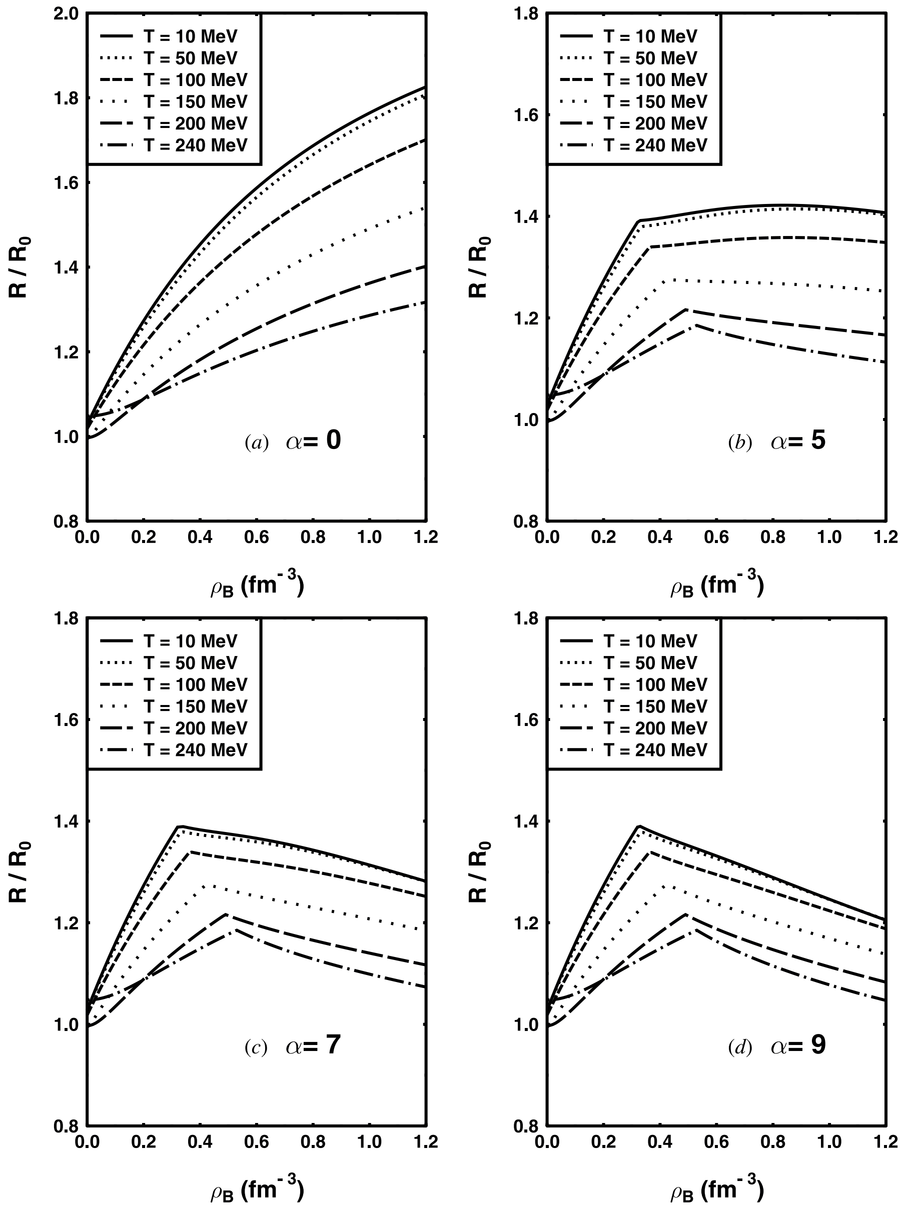

Figure 2. Isotherms of $\frac{R}{R_0}$ versus the baryonic density ρ_B for different strengths of the scalar quark–quark correlations.

$\mu_B = 950$ MeV and a correspondingly low density $\rho_B/\rho_0 = 1.35$. This density is obviously too low for the production of the QGP in heavy ion collisions and is strictly appropriate only for infinite nuclear matter as it does not include any finite-size effects. It does, however, hint at the sizeable reduction in the compression needed to produce the QGP in collisions between very heavy systems.

In conclusion, we have investigated the effect of short-range quark–quark correlations on the properties of hot nuclear matter and the phase transition to the QGP. We have found that

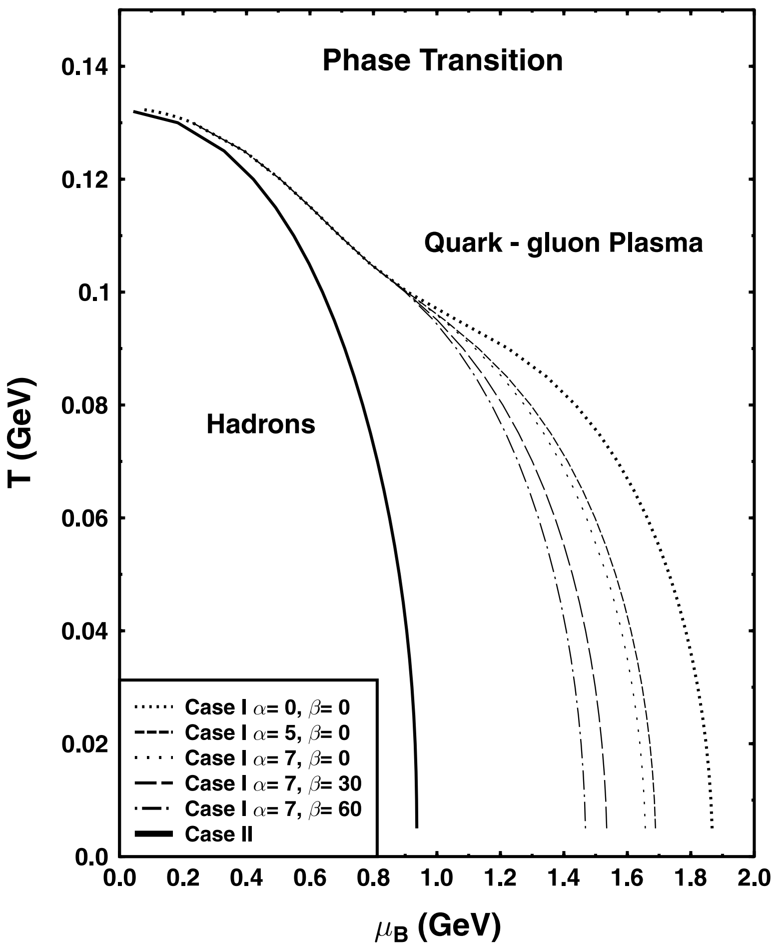

Figure 3. The phase transition line, in the (T, μ_B)-plane, between the hadronic phase and the QGP phase for different strengths of the scalar and vector quark–quark correlations.

these correlations cure the problem usually encountered in the MQMC model of a very large nucleonic bag radius. They also lead to the saturation of the effective mass at high densities. Moreover, these correlations affect the properties of the phase transition at low temperatures for the case of a medium-independent bag parameter for the QGP bubble in vacuum (case 1). Such a situation arises in experimental situations attempting to produce the QGP in small finite hot nuclear systems. In such a case, the present results indicate that the phase transition occurs at very high densities five to eight times normal nuclear matter density. The only exceptions occur at very high temperatures, greater than 100 MeV, in which case the transition occurs at arbitrarily small densities. In case 2 we have used a medium-dependent bag parameter for the QGP bubble, and the phase transition is found to occur at much lower densities than in case 1. The phase transition occurs before the nucleons overlap and so the quark–quark correlations do not play a role in determining the transition line. This case, strictly speaking, corresponds to producing a QGP bubble in infinite nuclear matter, but may be approximately approached in central collisions involving two very heavy nuclei. The comparatively low compressions

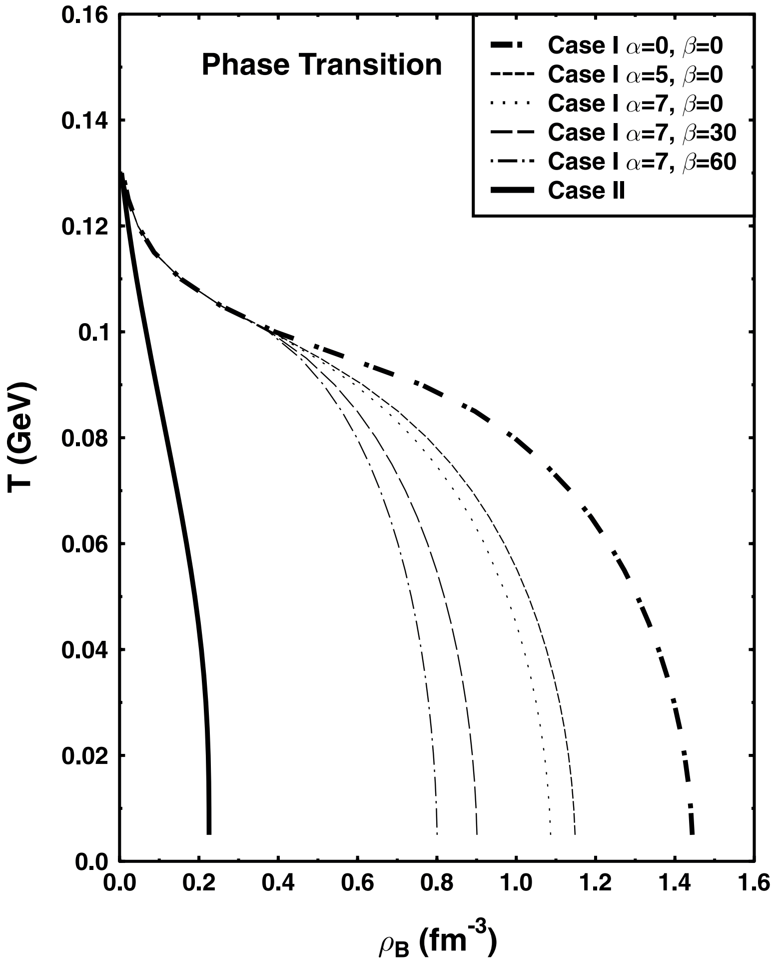

Figure 4. The onset phase transition line, in the (T, ρ_B)-plane, between the hadronic phase and the QGP phase for different strengths of the scalar and vector quark–quark correlations.

required for the phase transition in such collisions would thus offer the best chance of producing the QGP.

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