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Article in Journal of Transportation Engineering · January 2004
DOI: 10.1061/(ASCE)0733-947X(2004)130:1(24)

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Integrated Pavement Management System with a Markovian Prediction Model

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Abstract: An integrated pavement management system has been designed to provide the pavement engineers with an effective decision-making tool for planning and scheduling of pavement maintenance and rehabilitation (M&R) work. The developed system applies a discrete-time Markovian model to predict pavement deterioration with the inclusion of pavement improvement resulting from M&R actions. An effective decision policy with two major options has been used. The first option optimizes a generalized nonlinear objective function that is defined in terms of proportions of pavement sections in the five deployed condition states, and is subjected to budget constraints. The second option minimizes M&R cost which is subjected to preset pavement condition requirements in terms of state proportions at the end of a selected study period. The system applies two approaches for the selection of pavement project candidates. The first approach is based on random selection of pavement sections within the same condition state, while the second one relies on worst-first selection within the same condition state. The optimization process is performed using two different optimization methods which are the penalty function method and uniform search method.

DOI: 10.1061/(ASCE)0733-947X(2004)130:1(24)

CE Database subject headings: Pavement management; Markov process; Models; Rehabilitation; Maintenance; Decision making.

Introduction

A large number of pavement management systems have been developed and implemented in the last 2 decades ranging from the very simple ones to the most sophisticated systems. They have some elements in common such as relying mostly on pavement distress ratings, recommending appropriate maintenance and rehabilitation (M&R) strategies, a decision policy to establish priority scheduling, and dealing with fund limitations. There are two additional basic elements necessary for inclusion in a pavement management system to address the pavement management process in its totality when considering a pavement system. First, a prediction mechanism capable of predicting future pavement conditions especially in the presence of an active M&R program, and an optimization process designed to yield optimum pavement conditions based on a defined decision policy.

Some of the major developed pavement management systems deploy stochastic prediction models, which are mainly Markovian, statistical regression, and Bayesian models (Shahin and Kohn 1982; Way et al. 1982; Butt et al. 1987; Harper and Majidzadeh 1991; Abaza et al. 2001). Others simply do not use prediction models and rely mostly on experience and engineering judgment, and use simple decision trees and "prescription" procedures (Jao et al. 1985; Hill et al. 1991; Tavakoli et al. 1992). Optimization of a pavement system according to a defined effective decision policy has been attempted by some of the developed systems (Way et al. 1982; Harper and Majidzadeh 1991; Hill et al. 1991; Pilson et al. 1999; Abaza et al. 2001). Selection and integration of appropriate prediction model and optimization method into an effective decision policy are essential for the successful design of any pavement management system.

The developed integrated pavement management system (IPMS) has deployed the Markovian prediction model for its effectiveness in integrating both pavement deterioration rates and improvement rates resulting from M&R actions into a single entity that can effectively be optimized. This single entity is the transition matrix, which has been designed to only include five condition states labeled 1, 2, 3, 4, and 5 for simplicity and practicality. States 1, 2, 3, 4, and 5 represent pavement sections in excellent, good, fair, poor, and bad conditions, respectively. The use of a larger number of condition states requires a larger number of M&R variables which complicates the computational process, and requires more detailed and expensive pavement distress records for the estimation of the corresponding transition probabilities (Shahin and Kohn 1982; Way et al. 1982; Abaza and Ashur 1999). Therefore, the system applies a reasonable and practical number of M&R plans consistent with the number of deployed condition states. There are two types of improvement that are effectively integrated into the IPMS system.

The first type is maintenance, which can be applied to pavement sections in states 2, 3, 4, and 5. Pavements in state 1 usually require no maintenance work. Therefore, there are four maintenance plans represented by the four variables labeled $q_{21}$, $q_{32}$, $q_{43}$, and $q_{54}$. The expected outcome of maintenance is the improvement of a pavement section by one state. Maintenance consists of routine and preventive works such as crack sealing, pothole patching, surface treatment, and localized repair and...
resurfacing as required by the severity of pavement distress in each state. Therefore, a different maintenance plan would be required for each of the four considered states to improve each by one state. Generally, it is up to the engineer to decide the type and extent of maintenance as long as it produces an improvement by one state.

The second type is rehabilitation applied to pavement sections in states 3, 4, and 5. The expected outcome is the improvement of a pavement section from its present state to state 1. Generally, there are a total of three variables used to represent rehabilitation, namely: \( q_i, q_{i+1}, \) and \( q_{i+2} \). Rehabilitation may consist of resurfacing (plain overlay) applied to state 3, resurfacing or skin patch combined with localized reconstruction applied to state 4, and complete reconstruction applied to state 5.

### Methodology

The first part of this section provides an overview of the methodology used in the development of optimum M&R models associated with the case of random selection of pavement project candidates (Abaza and Ashur 1999). The second part presents the development of optimum M&R models for worst-first selection.

#### Overview of Random Selection Methodology

A Markovian model with \( 5 \times 5 \) discrete-time transition matrix has been used to incorporate the five condition states. The model utilizes a present transition matrix \( (P) \) and a future transition matrix \( (P') \). The elements of the present transition matrix are the present transition probabilities \( (P_{ij}) \), and the present M&R variables \( (f_{ij}) \). The elements of the future transition matrix are the future transition probabilities \( (P'_{ij}) \), and the future M&R variables \( (q_{ij}) \). The two matrices along with the deployed seven M&R variables are provided in

\[
P = \begin{pmatrix}
P_{11} & P_{12} & 0 & 0 & 0 \\
P_{21} & P_{22} & P_{23} & 0 & 0 \\
P_{31} & P_{32} & P_{33} & P_{34} & 0 \\
P_{41} & 0 & P_{43} & P_{44} & P_{45} \\
P_{51} & 0 & 0 & P_{54} & P_{55}
\end{pmatrix}
\]

\[
P' = \begin{pmatrix}
P_{11}' & P_{12}' & 0 & 0 & 0 \\
P_{21}' & P_{22}' & P_{23}' & 0 & 0 \\
P_{31}' & P_{32}' & P_{33}' & P_{34}' & 0 \\
P_{41}' & 0 & P_{43}' & P_{44}' & P_{45}' \\
P_{51}' & 0 & 0 & P_{54}' & P_{55}'
\end{pmatrix}
\]

It has been typically assumed that only two transition probabilities are present in a particular matrix row (Way et al. 1982; Butt et al. 1987; Abaza and Ashur 1999). The future transition matrix largely depends on the present one, a property of the Markovian model. Therefore, the two transition matrices have been related to each other by assuming that the ratio of the two present transition probabilities \( (P_{ij} \) and \( P_{i,j+1} \) and the ratio of the two future transition probabilities \( (P'_{ij} \) and \( P'_{i,j+1} \) are equal, and the sum of any row in either matrix is unity (Abaza and Ashur 1999).

The resulting relations are

\[
P'_{i,i+1} = \left( \frac{P_{i,i+1}}{P_{i,i} + P_{i,i+1}} \right) (P_{i,i} + P_{i,i+1} - q_i)
\]

\[
q_i = \sum_j q_{ij} - \sum_j f_{ij} \quad (i=2,3,4,5)
\]

The derived relations are then used to eliminate the future transition probabilities from any subsequent considerations and, instead, replace them in terms of the remaining variables. The total future M&R variables \( (q_i) \) represent the sum difference of future and present M&R variables \( (q_{ij} \) and \( f_{ij} \) \) applied to pavement sections in state \( i \), with each \( (q_i) \) includes one maintenance variable and a rehabilitation one, state 2 can only receive one maintenance variable as the deployed matrix shows only one entry available for improvement. The present M&R variables \( (f_{ij}) \) can be estimated by referring to an agency’s files. The subscript \( j \) designates the improved state, and equals to \((i-1)\) for maintenance variables and one for rehabilitation variables. The present transition probabilities \( (P_{i,i}) \) and \( P_{i,i+1} \) need to be estimated from historical records of pavement distress as will be later presented. Generally, the transition probabilities \( (P_{i,i+1}) \) represent the deterioration rates of pavement sections from state \( i \) to state \( i+1 \) in one transition, and the M&R variables \( (q_i \) and \( f_{ij} \) \) represent the improvement rates from state \( i \) to state \( j \) in one transition.

Unique forms of the transition matrix have been constructed to include several combinations of the seven outlined M&R variables below the matrix main diagonal. The result is 27 different combinations representing 27 distinct M&R models (matrices) that are incorporated into the IPMS system. The combinations are formed by requiring: (1) a minimum of four \( M&R \) variables in a particular model (matrix); (2) a minimum of one variable for each state with the exception of state 1 which receives none; (3) a maximum of two variables for each state (one maintenance and one rehabilitation) with the exception of state 2 (one maintenance variable); and (4) a maximum of seven variables that can only be present in one model.

There are a total of seven future M&R variables represented by the variable \( (q_{ij}) \), namely: four maintenance \( (q_{i,i-1}, i = 2,3,4,5) \), and three rehabilitation \( (q_{i,1}, i = 3,4,5) \) as presented earlier. The optimum values of the seven variables are obtained through the application of an effective decision policy that deploys two major options. The first option optimizes a generalized nonlinear objective function, which is defined as a weighted sum of state probabilities (proportions) for a selected study period of \( n \) transitions, and is subjected to budget constraints. The generalized form of the resulting nonlinear objective function is provided in

\[
\text{optimize} : \quad F = \sum_{i=1}^{5} w_i \times Q_i^{(n)}
\]

The objective in optimizing Eq. (3) is to yield the “best” pavement conditions, which can be achieved by either maximizing pavements in “good” states such as 1 and 2, or minimizing pavements in “bad” states such as 4 and 5. Therefore, the selection of appropriate weights \( (w_i) \) as required by Eq. (3) would have to be consistent with the approach being undertaken. Maximization of Eq. (3) requires assigning higher weights for the “good” states and lower weights for the “bad” ones, while minimization requires the opposite. Generally, there is a large number of weight choices for each approach, and it definitely depends on the engi-
ner’s preference. However, it turns out the two approaches are compatible in terms of their overall outcomes as demonstrated in the sample results section. In this paper, six distinct forms of Eq. (3) have been constructed using specific weight values with each form representing a nonlinear objective function with degree n.

1. Maximizing proportion of “excellent” pavement sections in state 1
   \[
   \text{Maximize } F = Q_1^{(n)} \quad (w_1 = 1, \ w_i = 0.0, \ i = 2,3,4,5) \quad (3a)
   \]

2. Maximizing proportion of “fair” pavement sections in state 3
   \[
   \text{Maximize } F = Q_3^{(n)} \quad (w_3 = 1, \ w_i = 0.0, \ i = 1,2,4,5) \quad (3b)
   \]

3. Maximizing total proportions of “excellent” and “good” pavement sections in states 1 and 2
   \[
   \text{Maximize } F = Q_1^{(n)} + Q_2^{(n)} \quad (w_1 = w_2 = 1, \ w_i = 0.0, \ i = 3,4,5) \quad (3c)
   \]

4. Minimizing proportion of “bad” pavement sections in state 5
   \[
   \text{Minimize } F = Q_5^{(n)} \quad (w_5 = 1, \ w_i = 0.0, \ i = 1,2,3,4) \quad (3d)
   \]

5. Minimizing total proportions of “poor” and “bad” pavement sections in states 4 and 5
   \[
   \text{Minimize } F = Q_4^{(n)} + Q_5^{(n)} \quad (w_4 = w_5 = 1, \ w_i = 0.0, \ i = 1,2,3) \quad (3e)
   \]

6. Maximizing a weighted average state condition (ASC) calculated similar to an academic grade point average in which higher weights are assigned to “better” states \((w_1 = 4, w_2 = 3, w_3 = 2, w_4 = 1, w_5 = 0)\). It would be comparable to minimize Eq. (3f) but with weights assigned in a reversed order \((i.e., 0,1,2,3,4)\)
   \[
   \text{Maximize } F = 4Q_1^{(n)} + 3Q_2^{(n)} + 2Q_3^{(n)} + Q_4^{(n)} + Q_5^{(n)} \quad (3f)
   \]

State probabilities after \(n\) transitions \((Q_i^{(n)})\) are obtained by multiplying the initial state probability row vector \((Q_i^{(0)})\) by the future transition matrix raised to power \(n\) \((P^{(n)})\). The optimum nonlinear model for this option is summarized below:

\[
F = \sum_{i=1}^{n} w_i \times Q_i^{(n)} = f(Q_i^{(0)}, P_{i,i}, P_{i,i+1}, q_{ij}, f_{ij}, w_i) \quad (4)
\]

Subject to:

1. \[
\sum_{i=2}^{5} uc_{ij} Q_i^{(k-1)} q_i \leq AB_k \quad (k = 1,2,...,n)
\]

2. \[
0 \leq q_i \leq 1.0 \quad \text{where } q_i = \sum_{j} q_{ij} - \sum_{j} f_{ij}
\]

where \(Q_i^{(k-1)}\) = state probability \(i\) after \((k-1)\) transitions; \(AB_k\) = available budget during the \(k\)th transition; \(uc_{ij}\) = unit cost ($/lane kilometer) associated with \(q_{ij}\); \(L\) = total length of pavement system in lane kilometer; \(i = 2,3,4,5\) for maintenance variables and 3,4,5 for rehabilitation; and \(j = (i-1)\) for maintenance variables and 1 for rehabilitation.

The second option minimizes the total M&R cost over a specified study period, and is subjected to predefined terminal state probabilities. The resulting nonlinear optimum model has the following general format:

\[
F = \sum_{k=1}^{n} \sum_{j=2}^{5} uc_{ij} Q_i^{(k-1)} q_i L \quad (5)
\]

Subject to:

1. \[Q_i^{(0)} P^{(n)} = A^{(n)}\]

2. \[0 \leq q_i \leq 1.0 \quad \text{where } q_i = \sum_{j} q_{ij} - \sum_{j} f_{ij}\]

where \((A^{(n)})\) = row vector indicating the desired state pavement proportions at the end of study period. The future matrix \((P^{(n)})\) can contain the seven M&R variables, but other matrices (models) with a minimum of four M&R variables can be generated resulting in a total of 27 distinct combinations as outlined earlier.

** Worst-First Selection Methodology **

The selection of project candidates based on the severity of pavement distress is a vital alternative to random selection. In this case, pavement sections in a given state with more severe distress have priority for M&R over sections that exhibit less severe distress. To implement the “worst-first” strategy, the engineer needs to start M&R plans on the “worst” sections in a given condition state. This can be done using field inspection procedures and developing an appropriate pavement condition indicator, which are also required for the estimation of transition probabilities. The worst-first selection can be represented mathematically by relating the total future M&R variables \((q_{ij})\) to the future transition probabilities \((P'_{i,i+1}\) and \(P'_{i,i+1})\) as indicated by

\[
q_i + P'_{i,i+1} + (P'_{i,i+1} - S_{i1}) = 1.0 \quad (6a)
\]

where

\[
0 \leq S_{i1} \leq P'_{i,i+1} \quad S_{i2} = 0.0
\]

The total M&R variables \((q_{ij})\) have been defined as the sum of two operational variables \((S_{i1}\) and \(S_{i2}\)) introduced to facilitate the description of the “worst-first” mechanism using a two-phase procedure. Eq. (6a) indicates that the sum of any row in the future transition matrix must remain one during the first phase. Eq. (6a) implies that the operational variable \((S_{i1})\) will first utilize the transition probabilities \((P'_{i,i+1})\), while the transition probabilities \((P'_{i,i+1})\) remain unchanged. The transition probability \((P'_{i,i+1})\) represents the proportion of pavement sections in state \(i\) which will transit to a “worst” state \(i+1\) after one transition. This implies that pavement sections with more severe distress will be first selected. Once all pavement sections represented by the transition probabilities \((P'_{i,i+1})\) have been selected, then the transition probabilities \((P'_{i,i+1})\) representing pavements in state \(i\) which will remain in state \(i\) after one transition, an indication of less severe distress, will be next utilized by the operational variable \((S_{i2})\).

This can be formulated using

\[
q_i + (P'_{i,i} - S_{i2}) = 1.0 \quad (6b)
\]

where

\[
0.0 \leq S_{i2} \leq P'_{i,i} \quad S_{i1} = P'_{i,i+1}
\]

The sum of any row in the future transition matrix must also remain one during the second phase as indicated by Eq. (6b). The optimum model for the worst-first selection consists of two
phases. In Phase I, the transition probabilities \(P'_{i,i+1}\) are first used. In Phase II, the transition probabilities \(P''_{i,i+1}\) are next utilized under the conditions that all transition probabilities \(P''_{i,i+1}\) have already been used, and that available budget allows for additional M&R work beyond that associated with Phase I. The nonlinear optimum model for each phase is as follows:

**Phase I**

Optimize:

\[
F = \sum_{i=1}^{5} w_i \times Q_i^{(n)} = f(Q_{i}^{(n)}, P'_{i,i+1}, q_{ij}, f_{ij}, w_i) \tag{7a}
\]

Subject to:

1. \[
\sum_{j=2}^{5} u_{ij} Q_{i}^{(k-1)} q_{ij} = L \leq AB_k \quad (k = 1, 2, ..., n)
\]

2. \[
0 \leq q_{ij} \leq P'_{i,i+1} \quad \text{where} \quad q_i = \sum_{j} q_{ij} - \sum_{j} f_{ij}
\]

**Phase II**

Optimize:

\[
F = \sum_{i=1}^{5} w_i \times Q_i^{(n)} = f(Q_{i}^{(n)}, P''_{i,i+1}, q_{ij}, f_{ij}, w_i) \tag{7b}
\]

Subject to:

1. \[
\sum_{j=2}^{5} u_{ij} Q_{i}^{(k-1)} q_{ij} = L \leq AB_k \quad (k = 1, 2, ..., n)
\]

2. \[
P'_{i,i+1} \leq q_{ij} \leq 1.0 \quad \text{where} \quad q_i = \sum_{j} q_{ij} - \sum_{j} f_{ij}
\]

The two-phase optimization model uses similar objective functions and budget constraints; but the two phases require different physical constraints. Phase II will be executed for the \(k\)th transition (time interval) if the associated optimum cost is smaller than the budget available for that particular transition. Minimization of total M&R cost can be extended to the “worst-first” selection using a two-phase procedure, but it is not presented in this paper.

**System’s Structure**

The IPMS system has been designed using Fortran as the programming language. The system can be operated from a personal computer using either online input data for a small pavement system or electronically fed data from a computerized database for a larger one. The IPMS system operates using several modules designed to perform specific tasks as described below.

**Pavement Inventory Module**

The inventory database is programmed to store, update and retrieve information related to the pavement system such as pavement type, pavement structural section properties, traffic condition such as the average daily traffic, pavement section coding number, pavement section length and width, M&R history, and pavement condition rating as obtained from pavement distress assessment.

**Pavement Condition Assessment Module**

Periodic assessment of the pavement condition is essential to the successful implementation of any pavement management system.

This can be accomplished economically through visual inspection of pavement defects annually or biennially. The goal is estimating a pavement condition rating for each pavement section. This module stores distress data obtained from field inspections, and then calculates a weighted pavement condition rating (PCR) for each pavement section according to selected defect weights.

**Pavement Maintenance Effectiveness Module**

This module deals with the data pertaining to the effectiveness of the specified M&R plans. The system provides for four maintenance plans, one for each of the states 2, 3, 4, 5; and three rehabilitation plans, one for each of the states 3, 4, and 5. The system provides general guidelines for defining the seven M&R plans, but the user has the freedom to select the type and extent of M&R work involved in each plan. The user has to provide the system with a reliability index value for each M&R plan. The reliability index reflects the level of confidence an agency has in its design and construction practices so that the selected M&R plans will perform as expected (Abaza et al., 2001), and it is related to the seven M&R variables as indicated by

\[
q'_{i,j-1} = \alpha_{i,j-1} q_{i,j-1}; \quad q_{11} = q'_{11} + q'_{12} \\
q'_{11} = \alpha_{11} q_{11}; \quad q'_{12} = (1 - \alpha_{11}) q_{11} \tag{8}
\]

where \(q'_{i,j-1}\) is proportion of pavement sections that has actually improved from state \(i (2, 3, 4, 5)\) to state \((i-1)\) by an application of maintenance plan; \(q_{i,j-1}\) is proportion of pavement sections that has been maintained in state \(i (2, 3, 4, 5)\) with the intention of improvement to state \((i-1)\) by an application of maintenance plan; \(\alpha_{i,j-1}\) is reliability index reflecting the confidence level that the applied maintenance plan will result in improving a pavement section from state \(i\) to state \((i-1)\). Its value is generally lower for maintenance than rehabilitation. For rehabilitation, it could reach up to 99% whereas for maintenance it may not exceed 80% (Abaza et al., 2001); and \(q_{11}\) is proportion of pavement sections that has been rehabilitated in state \(i (3, 4, 5)\) with the intention of improvement to state 1. The outcome is either placement in state 1 \((q'_{11})\) with \(\alpha_{11}\) reliability or placement in state 2 \((q'_{12})\) with \((1 - \alpha_{11})\) reliability.

The introduction of the reliability index is consistent with the probabilistic performance of pavements. A higher reliability index implies a better quality M&R plan, which means a higher cost. Therefore, the reliability index value can also be related to the type of highway facility being considered in which a major one would be justified for a higher M&R expenditure. The introduced reliability index is similar to the reliability concept used by the American Association of State Highway and Transportation Officials (AASHTO) in the design of pavements in which a higher reliability level \(R\) results in a higher strength pavement (AASHTO 1993). Initially, the reliability index values can be estimated based on experience and engineering judgment. However, it is recommended that a highway agency establishes a mechanism for evaluating and monitoring the maintained and rehabilitated pavements to ensure that actual performance outcomes do confirm to the expected ones. Otherwise, the reliability index values used would have to be revised accordingly.

**Pavement Condition Prediction Module**

This module performs the prediction process using the presented Markovian model. It applies the estimated transition probabilities and the initial state probabilities along with other relevant input
data, and works interchangeably with the optimization module in the search for optimal solutions. It simply performs matrix multiplications to predict the state and transition probabilities for a specified number of transitions.

**Optimization Module**

This module applies two different optimization methods. The first method is the penalty function using the method of Hooke and Jeeves between successive iterations which has been effectively applied to the case of random selection (Abaza and Ashur 1999). The penalty function method has been applied to the worst-first selection in this paper. The second method is simultaneous search which has been applied, in this paper, to both random and worst-first selections. The two selected methods apply functional evaluations rather than derivatives, thus eliminating the need to derive the involved functions in closed forms, which would have been a very cumbersome task.

Simultaneous search is a method of optimization that depends on functional evaluations of predetermined points. An example of simultaneous search is uniform search, wherein we decide initially the points at which the functional evaluations are to be made. The interval of uncertainty [0,1] associated with each M&R variable is divided into \( r \) grid points, and the objective function is evaluated at each of those grid points. The highest objective function value is selected in the case of maximization over all evaluated grid points, and the lowest value is selected in the case of minimization. Two levels of accuracy have been programmed into the system. Level 1 divides the entire range [0,1] into ten increments (i.e., 0.0, 0.1, 0.2,..., 1.0), whereas level 2 divides it into 20 increments (i.e., 0.0, 0.05, 0.10, 0.15,..., 1.0). Level 2 provides more confidence than level 1 since it searches more points and still includes all solutions generated by level 1. Computer time has been the critical factor for selecting the two levels of accuracy.

**Optimum Decision Policy Module**

This module guides through the various available IPMS selections as shown in Fig. 1. The user makes the desired selections regarding the optimum decision policy options (two options): optimizing state proportion functions for option one [six suggested selections as provided in Eqs. (3a)–(3f)] or minimizing total M&R cost, selection methods of M&R candidates (random or worst-first), optimization methods (penalty or uniform search), and accuracy levels for uniform search (two levels).

**Estimation of Transition Probabilities Module**

There are several methods that can be used to estimate the present transition probabilities \( (P_{i,j} \text{ and } P_{i,j+1}) \). These methods are based either on the experience and judgment of pavement experts or on sound engineering principles. Application of engineering principles requires feedback on pavement performance as obtained from field assessment of pavement distress. Three methods are presented in this section with different requirements.

The first method is to apply the very basic definition of transition probabilities; that is, if \((N_0)\) pavement sections are initially found in state \(i\), and \((N_f)\) sections existed in state \(i\) after one transition, the transition probabilities can be estimated using

\[
P_{i,j+1} = \frac{N_0 - N_f}{N_0}
\]

\[
P_{i,j+1} = 1 - P_{i,j+1} = \frac{N_f}{N_0}
\]

The values of the transition probabilities given by Eq. (9) provide realistic estimates if large number of pavement sections is inspected. Application of Eq. (9) requires two cycles of pavement distress assessment.

The second method is based on estimating the service periods \((D_i)\) in years that a pavement section is going to stay in state \(i\). Let \((t)\) be the length of time interval in years between successive transitions. Then, one simple equation can estimate the transition probabilities as follows:

\[
P_{i,j+1} = \frac{t}{D_i} \leq 1.0
\]

where

\[
\sum D_i = T
\]

where \((T)\) is either the service life estimated from actual pavement performance records or the analysis period used in the design of pavement. Researchers have used performance curves generated using the AASHTO design method of flexible pavement to estimate the service periods \((D_i)\) (Abaza et al. 2001). The present serviceability index (PSI) has been used to define the five condition states. State 1 corresponds to a PSI range of 4.5–4.0, while state 5 represents a PSI range of 2.5–2.0. For example, the five state service periods \((D_i)\) are estimated from the constructed performance curve to be 6, 5, 4, 3, and 2 years for states 1, 2, 3, 4, and 5, respectively. These estimates are obtained for a 20 year design analysis period and a particular set of design input parameters as required by the AASHTO design method of flexible pavements. A better alternative to design analysis period is service life, which requires extensive pavement performance historic records, and it is highly dependent on pavement type and current states.

---

**Fig. 1.** Integrated pavement management system optimum decision policy selections

\[
P_{i,j+1} = 1 - P_{i,j+1} = \frac{N_f}{N_0}
\]
The resulting transition probabilities, based on a 2-year time interval \((t = 2\text{ years})\), are as follows: \(P_{12} = 0.33, P_{23} = 0.40, P_{34} = 0.50, P_{45} = 0.67, \) and \(P_{45} = 1.0\).

The third method is based on the model adopted by researchers at the United States Army Construction Engineering Research Laboratory (CERL) (Butt et al. 1987). It minimizes the error defined as the difference between the observed pavement condition index (PCI) value and its predicted one. The predicted PCI value is obtained by applying the Markovian model. The objective function to be minimized takes on the following form:

\[
F = \sum_{i=1}^{n} \sum_{j=1}^{m} \left| Y(i,j) - E(X(i,p)) \right|
\]

where \(n\) = total number of transitions for which PCI versus age data are available; \(m(i,j)\) = total number of data points recorded at the \(i\)th transition; \(Y(i,j)\) = observed PCI value for each sample taken at the \(i\)th transition; and \(E[X(i,p)]\) = predicted PCI value at the \(i\)th transition which is a function of the unknown transition probabilities \(P_{i,i+1}\) that are represented by the vector \(p\).

Eq. (11) minimizes the sum of all residual absolute values. Minimizing the sum of squared values of all residuals can also be used. In predicting the PCI values, the state probabilities associated with the five condition states have been defined in terms of PCI as a uniform probability density function that takes on the following general form:

\[
F = \begin{cases} 
Q_1^{(1)}, & k_2 \leq PCI \leq k_1 \\
Q_2^{(1)}, & k_3 \leq PCI \leq k_2 \\
Q_3^{(1)}, & k_4 \leq PCI \leq k_3 \\
Q_4^{(1)}, & k_5 \leq PCI \leq k_4 \\
Q_5^{(1)}, & k_6 \leq PCI \leq k_5 
\end{cases}
\]

The uniform probability density function \(F\) has been applied since the state probabilities are not explicit functions of the PCI. The state probabilities are functions of the initial state probability vector, and unknown transition probabilities \(P_{12}, P_{23}, P_{34}, \) and \(P_{45}\) which need to be estimated from the minimization process. The expected PCI value at the \(i\)th transition is obtained by

\[
E[PCI^{(i)}] = c_1 Q_1^{(i)} + c_2 Q_2^{(i)} + c_3 Q_3^{(i)} + c_4 Q_4^{(i)} + c_5 Q_5^{(i)}
\]

where \((c_k)\) = expected PCI value for a particular state probability. Each \((c_k)\) is the mean of a uniform probability density function. For example, \(c_1 = (k_1 + k_2)/2, c_2 = (k_2 + k_3)/2, \) etc. The constants \((k_i)\) are defined using appropriate PCI values. The presented CERL model requires extensive historic records of pavement distress to yield reliable estimates of the transition probabilities. Therefore, highway agencies that have recently initiated a pavement distress survey program cannot use the CERL method.

System’s Requirements

The pavement engineer needs to prepare in advance the input data required to utilize the IPMS system. The pavement system under the jurisdiction of an agency needs to be broken down into subsystems with similar pavement structures and traffic conditions. As a minimum, there are two types of pavement structures consisting of rigid and flexible, and four levels of traffic conditions corresponding to the four major road classes of highways, arterials, collectors, and locals. Therefore, a minimum of eight input data files are required for a given road network. The following requirements are needed for each pavement subsystem to make an effective use of the IPMS system:

1. Breaking down the pavement subsystem into sections of equal lane length and coding the subsystem for computer identification. The typical section length is 50 m. The coding system should be based on node and lane numbers, and post kilometers. It is the same system used in establishing the pavement inventory database.

2. Estimating the anticipated funds that are expected to be available annually or biennially for M&R work during a study period of \(n\) transitions for each pavement subsystem. The general M&R budget can be divided among the various subsystems in proportion to the existing traffic volumes as represented by the average daily traffic \((V_j)\) and length of a pavement subsystem in lane kilometers \((L_j)\). The allocated budget \((AB_j)\) for subsystem \(j\) can be calculated in relation to total available budget \((\sum AB_i)\) during the \(i\)th transition using

\[
AB_j = \frac{V_j \times L_j}{\sum V_i \times L_i} AB_i
\]

Eq. (14) provides fair mechanism for allocating M&R funds in the absence of political and other influential factors. A vital alternative to Eq. (14) is optimal allocation of M&R funds based on pavement distress condition in each pavement subsystem. The only foreseeable drawback of this alternative is the increased number of M&R variables which would make the nonlinear optimization extremely difficult.

3. Estimating the present M&R proportions \((f_{ij})\) as obtained from an agency’s files. In the absence of an active M&R program, then a set of zeros can be assigned.

4. Estimating the reliability indices \((\alpha_{ij})\). These indices are products of the experience and judgment of the pavement engineers, which reflect their expectations of the outcomes of their M&R actions. In general, there is a higher level of reliability in rehabilitation when compared to maintenance. The reliability index may reach 99% for rehabilitation work, but it may not exceed 80% for maintenance.

5. Estimating the unit costs of various M&R plans in dollars per lane kilometer. The system requires seven unit costs corresponding to the seven deployed M&R plans.

6. Specifying the desired study period in terms of the equivalent number of transitions. It is recommended that the study period does not exceed 6 years to satisfy the “stationary” property of the Markovian model (Butt et al. 1987). This recommendation assures that the transition probabilities will remain practically unchanged by normal changes in traffic loadings. However, 6 years is adequate to plan ahead since estimation of anticipated M&R funds can only be foreseeable for a limited number of years. Therefore, the maximum number of transitions can vary from 3 to 6 depending on the length of time interval used between successive surveys of pavement distress. The length of this time interval is typically 1 or 2 years.

7. Estimating the initial state probabilities. This step requires field inspection of the pavement system prior to the application of the IPMS system. It requires a complete cycle of
pavement distress survey to obtain an estimate of the PCR for each pavement section. The five condition states can be defined in terms of PCR limits. Then, the initial state probability ($Q_i^{(0)}$) for state $i$ can be estimated from the initial number of pavement sections ($N_i^{(0)}$) found in each of the five states using

$$Q_i^{(0)} = \frac{N_i^{(0)}}{\sum_{j=1}^{5} N_j^{(0)}}$$  \hspace{1cm} (15)

8. Estimating the present transition probabilities ($P_{ij}$ and $P_{ij+1}$) associated with the five condition states. This step can be performed using the methods presented in a preceding section. A minimum of two cycles of pavement distress surveys is required. Otherwise, rough estimates can be provided based on personal judgment and experience. Once sufficient historical records of pavement distress become available, then the IPMS system can be applied to obtain refined estimates of the transition probabilities using the CERL method.

9. Required optimization parameters for the search process such as starting point, step size, termination constant, and penalty parameter have been extensively tested and programmed in the IPMS system.

### Analysis of Sample Results

Sample results are presented for the purpose of making certain comparisons among the different options available to the IPMS users. The input data for the sample presentation is summarized below.

#### Table 1. Identification of Maintenance and Rehabilitation Variables

<table>
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<tr>
<th>Model No.</th>
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<th>$P_{33}$</th>
<th>$P_{34}$</th>
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<th>$P_{41}$</th>
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</table>

Note: — not applicable.

#### Table 2. Minimizing State 5 Proportion for Random Selection Using Both Optimization Methods

<table>
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<tr>
<th>Model No.</th>
<th>$Q_5$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>Cost ($$1,000$)</th>
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<td>3</td>
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<td>$318.00$</td>
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<td>0.472</td>
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<td>0.300</td>
<td>0.300</td>
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<td>$284.30$</td>
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<td>0.379</td>
<td>0.993</td>
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</tr>
<tr>
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<td>1.000</td>
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<td>1.000</td>
<td>$159.50$</td>
</tr>
<tr>
<td>14</td>
<td>0.0001</td>
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<td>1.000</td>
<td>1.000</td>
<td>$159.50$</td>
</tr>
<tr>
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</table>

aOptimization by uniform search method.
bOptimization by penalty function method.

1. The total length of surveyed pavement sections, all with similar traffic and pavement structures, was about 20 lane-kilometers with each section about 50 m long.
2. The initial state probabilities ($Q_i^{(0)}$) for states 1, 2, 3, 4, and 5 were estimated using Eq. (15) based on pavement distress survey. The estimates were $Q_1^{(0)} = 0.23$, $Q_2^{(0)} = 0.29$, $Q_3^{(0)} = 0.43$, $Q_4^{(0)} = 0.043$, and $Q_5^{(0)} = 0.015$.
3. The present transition probabilities ($P_{ij}$) were estimated using Eq. (9) based on two cycles of distress survey of the same pavement system. The two cycles were separated by a 2 year period. The estimates were $P_{11} = 0.44$, $P_{22} = 0.52$, $P_{33} = 0.62$, and $P_{44} = 0.67$.
4. It was determined that the local agency having jurisdiction over the surveyed pavement system did not have an active M&R program. Hence, the present M&R variables ($f_{ij}$) vanished.
5. The reliability indices ($\alpha_{ij}$) for maintenance and rehabilitation are specified at 80 and 95%, respectively.
6. The study period is considered for 6 years consisting of three transitions ($n = 3$) with each transition representing a 2 year interval. The length of transition period has to be consistent with the time period used in the estimation of transition probabilities. Two cycles of distress surveys separated by 2 years were conducted to estimate the transition probabilities.
7. Maintenance unit costs as applied to states 2, 3, 4, and 5 are estimated in dollars per lane-kilometer to be $4,350, $5,800, $7,250, and $9,250, respectively.
8. Rehabilitation unit costs as applied to states 3, 4, and 5 are also estimated in dollars per lane-kilometer and found to be $33,200, $78,200, and $156,400, respectively.
9. Allocated funding for the study period of three transitions has been specified for the first, second, and third transition to be $80,000, $85,000, and $90,000, respectively.
The analysis of results is confined to optimizing state proportions, and for models containing four M&R variables (Models 2–9) and seven M&R variables (Model 28). Guidelines for matching the variables \( (X_1, X_2, X_3) \) with the corresponding M&R plans for the 28 models are provided in Table 1. In Model 8 for example, the variables \( (X_1, X_2, X_3, X_4) \) represent the maintenance variables \( (q_21, q_32, q_43) \), while \( (X_4) \) represents the rehabilitation variable \( (q_5) \). Model Number 1 is the do-nothing M&R program. The value of an optimum variable represents the fraction of pavement sections in a condition state to receive the corresponding M&R plan. For example, a 1.0 value implies that all pavement sections present in the corresponding state during two-year transition within the study period of 6 years shall receive the designated M&R plan.

Table 2 contains the optimum M&R programs obtained by minimizing the proportion of pavement sections in state 5 using both uniform search and penalty function methods. Inspection of the optimum state 5 proportion \( (Q_5) \) reveals that both methods have converged with similar results. Uniform search has reached improved solutions in two models, the penalty method has yielded improved solutions in three models, and identical solutions are obtained in three models. Improvements in either case are not substantial.

A comparison between random and worst-first selections can be made by inspecting the optimal solutions presented in Table 3 for the case of maximizing ASC. Worst-first selection has yielded improved optimal solutions, in terms of the objective function value (ASC), in almost all models. Associated M&R costs are generally lower for the case of worst-first selection with even improved solutions. This conclusion evidently supports the worst-first selection of M&R candidates all in the same condition state.

Investigation of the effect of selecting a particular starting point to initiate the search process, as required by the penalty function method, is illustrated in Table 4. Two different starting points have been used to maximize the total proportions of states 1 and 2. Table 4 shows that identical solutions have been reached in six models, while minor variations resulted in two models. Therefore, it is concluded that the effect of selecting a starting point is seen to be negligible; therefore, a value of zero has been programmed in the IPMS system for all variables.

Table 5 provides results obtained by maximizing state 1 proportion using uniform search method with the two defined levels of accuracy. Accuracy level 2 has resulted in minor improvements in some of the optimal solutions. The minor improvements obtained by applying accuracy level 2 are gained at a cost of substantial increase in required computer time. Therefore, accuracy level 2 is not deemed practical from an economic point of view.

Table 6 provides optimum M&R programs for model Number 28, which contains all seven M&R variables. The solutions are provided for optimizing the six suggested state proportion functions using worst-first selection and penalty function method. All seven M&R variables have contributed to the optimum solutions in three of the presented M&R programs, five variables contributed to one program, and four variables contributed to the remaining two programs (a contribution of 0.001 is considered zero). Therefore, using a larger number of M&R variables does not necessarily imply that they would all contribute to the optimal solution. In addition, there are no substantial improvements gained in the objective function values from the inclusion of more M&R variables. However, increasing the number of M&R variables provides additional M&R program choices, which may come at an increased cost as the additional variables could prod-

Table 3. Maximizing Average State Condition Using Penalty Function Method for Both Random and Worst-First Selections

<table>
<thead>
<tr>
<th>Model No.</th>
<th>ASC</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( X_3 )</th>
<th>( X_4 )</th>
<th>Cost ($1,000)</th>
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\(^a^\)Optimum solutions using random selection.

\(^b^\)Optimum solutions using worst-first selection.

Table 4. Maximizing Total Proportions of States 1 and 2 for Random Selection and Penalty Function Method Using Two Different Starting Points

<table>
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<th>( X_2 )</th>
<th>( X_3 )</th>
<th>( X_4 )</th>
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<td>6</td>
<td>0.9766</td>
<td>1.000</td>
<td>0.824</td>
<td>0.646</td>
<td>0.512</td>
<td><strong>$230.20</strong></td>
</tr>
<tr>
<td>7</td>
<td>0.9766</td>
<td>1.000</td>
<td>0.824</td>
<td>0.646</td>
<td>0.512</td>
<td><strong>$230.20</strong></td>
</tr>
<tr>
<td>8</td>
<td>0.9840</td>
<td>1.000</td>
<td>0.929</td>
<td>0.882</td>
<td>0.953</td>
<td><strong>$199.10</strong></td>
</tr>
<tr>
<td>9</td>
<td>0.9840</td>
<td>1.000</td>
<td>0.926</td>
<td>0.886</td>
<td>0.947</td>
<td><strong>$199.70</strong></td>
</tr>
</tbody>
</table>

\(^a^\)Optimization search started at 0.0 for all variables.

\(^b^\)Optimization search started at 0.5 for all variables.


**Table 7.** Final State Proportions Associated with Model 28 Optimum Maintenance and Rehabilitation Programs

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Final State Proportions (n = 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Q_1^{(3)})</td>
</tr>
<tr>
<td>Maximum (Q_1)</td>
<td>0.6334</td>
</tr>
<tr>
<td>Maximum (Q_3)</td>
<td>0.0978</td>
</tr>
<tr>
<td>Minimum (Q_5)</td>
<td>0.5274</td>
</tr>
<tr>
<td>Maximum (Q_1 + Q_2)</td>
<td>0.6187</td>
</tr>
<tr>
<td>Minimum (Q_1 + Q_3)</td>
<td>0.6245</td>
</tr>
<tr>
<td>Maximum ASC</td>
<td>0.6321</td>
</tr>
</tbody>
</table>

**Table 6.** Optimum Maintenance and Rehabilitation Programs for Model 28 Using Worst-First Selection and Penalty Function Method

<table>
<thead>
<tr>
<th>Objective function value</th>
<th>(q_21)</th>
<th>(q_32)</th>
<th>(q_43)</th>
<th>(q_54)</th>
<th>(q_13)</th>
<th>(q_41)</th>
<th>(q_51)</th>
<th>Cost (S1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum (Q_1)</td>
<td>0.6334</td>
<td>1.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.999</td>
<td>0.001</td>
<td>0.833</td>
<td>0.944</td>
</tr>
<tr>
<td>Maximum (Q_3)</td>
<td>0.5180</td>
<td>0.000</td>
<td>0.599</td>
<td>0.500</td>
<td>0.500</td>
<td>0.000</td>
<td>0.001</td>
<td>0.241</td>
</tr>
<tr>
<td>Minimum (Q_4)</td>
<td>0.0004</td>
<td>0.824</td>
<td>0.575</td>
<td>0.215</td>
<td>0.785</td>
<td>0.170</td>
<td>0.417</td>
<td>0.425</td>
</tr>
<tr>
<td>Maximum (Q_1 + Q_2)</td>
<td>0.9817</td>
<td>0.876</td>
<td>0.775</td>
<td>0.273</td>
<td>0.001</td>
<td>0.264</td>
<td>0.381</td>
<td>0.001</td>
</tr>
<tr>
<td>Minimum (Q_1 + Q_3)</td>
<td>0.0009</td>
<td>1.000</td>
<td>0.794</td>
<td>0.120</td>
<td>0.880</td>
<td>0.138</td>
<td>0.862</td>
<td>0.206</td>
</tr>
<tr>
<td>Maximum ASC</td>
<td>3.6390</td>
<td>1.000</td>
<td>0.832</td>
<td>0.094</td>
<td>0.906</td>
<td>0.144</td>
<td>0.856</td>
<td>0.168</td>
</tr>
</tbody>
</table>

\[\text{All seven M&R variables contributed to the optimum solution.}\]
lowest percentage of “bad” pavements. Associated M&R cost can also serve as an important guide in the selection of a potential M&R program.

The computer time required to solve a particular M&R program by either optimization methods can vary considerably. The computer time required by the penalty function method has been found superior to that required by uniform search. The computer time required to solve a particular M&R program by uniform search method is at least 20 times more than the time required by the penalty method. Required computer time increases at a very fast rate as the number of deployed M&R variables and transitions increases when uniform search is applied, while the corresponding time required by the penalty method increases at a much slower rate.

The IPMS system is expected to be applied as often as updated input data become available, especially pavement distress data. Whenever a new set of data becomes available, it is recommended that an agency should apply the system to obtain updated optimal solutions reflecting any recent changes that might have taken place in the pavement system. This may lead to the implementation of a revised optimum M&R program.

### Table 8. Verification of Multi-Optimal Solutions

<table>
<thead>
<tr>
<th>Model No. (Table)</th>
<th>Objective function value</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 (Table 2)</td>
<td>0.000</td>
<td>0.500</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.849</td>
<td>0.846</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>7 (Table 3)</td>
<td>0.815</td>
<td>1.000</td>
<td>0.278</td>
<td>0.391</td>
<td>0.706</td>
</tr>
<tr>
<td></td>
<td>0.820</td>
<td>0.480</td>
<td>0.380</td>
<td>0.330</td>
<td>0.000</td>
</tr>
<tr>
<td>4 (Table 5)</td>
<td>0.629</td>
<td>1.000</td>
<td>1.000</td>
<td>0.630</td>
<td>0.300</td>
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<tr>
<td></td>
<td>0.630</td>
<td>1.000</td>
<td>1.000</td>
<td>0.580</td>
<td>0.400</td>
</tr>
</tbody>
</table>

### References


